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A Quantitative Approach to Central Bank Haircuts and Counterparty Risk Management

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A Quantitative Approach to Central Bank Haircuts and Counterparty Risk Management
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ABSTRACT: This paper presents a comprehensive framework for determining haircuts on collateral used in central bank operations, quantifying residual uncollateralized exposures, and validating haircut models using machine learning. First, it introduces four haircut model types tailored to asset characteristics—marketable or non-marketable—and data availability. It proposes a novel model for setting haircuts in data-limited environment using a satallite cross-country model. Key principles guiding haircut calibration include non-procyclicality, data-drivenness, conservatism, and the avoidance of arbitrage gaps. The paper details model inputs such as Value-at-Risk (VaR) percentiles, volatility measures, and time to liquidation. Second, it proposes a quantitative framework for estimating expected uncollateralized exposures that remain after haircut application, emphasizing their importance in stress scenarios. Illustrative simulations using dynamic Nelson-Siegel yield curve models demonstrate how volatility impacts exposure. Third, the paper explores the use of Variational Autoencoders (VAEs) to simulate stress scenarios for bond yields. Trained on U.S. Treasury data, VAEs capture realistic yield curve distributions, offering an altenative tool for validating VaR-based haircuts. Although interpretability and explainability remain concerns, machine learning models enhance risk assessment by uncovering potential model vulnerabilities.

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Introduction

Since the 2008 global financial crisis, central banks around the world have been actively reviewing and strengthening their collateral management frameworks. It is now widely recognized that establishing appropriate haircuts is essential for central banks to protect against potential losses. For instance, a report by the BIS Market Committee highlights that central bank collateral frameworks have evolved significantly since mid-2007.[1] During the financial crisis, many central banks broadened the range of eligible collateral to support liquidity and subsequently introduced stricter risk controls and more granular approaches to setting haircuts. The report underscores that collateral policies play a critical role in liquidity provision, financial stability, and the development of financial markets.

To determine haircuts in a systematic way, it is essential to adopt quantitative models. This is because such models ensure internal consistency in haircuts across different types of collateral and enhance public accountability. Also, quantitative models reduce reliance on subjective judgements by applying verifiable assumptions for determining haircuts. Such quantitative models are inherently data-driven and thus strengthen public accountability by providing justifiable decisions.

Determining haircuts using quantitative models may appear challenging in low-income countries with limited data availability but this is not necessarily the case. As will be discussed later, this challenge can be addressed by combining cross-country models with standard models. To the best of our knowledge, this is the first study to propose a quantitative model for determining haircuts in data-limited environments.

This paper has three objectives. The first objective is to provide haircut models depending on the type of eligible assets and the availability of data. The second objective is to present a framework for quantifying expected uncollateralized exposure under haircut models. The third objective is to simulate the uncollateralized exposure and validate haircut models using a Machine Learning technique.

The first objective is to comprehensively discuss the selection of quantitative models for determining haircuts on collateral pledged in central bank operations. We classify a given situation into four cases by considering: (i) whether the target asset which haircuts need to be determined is marketable or non-marketable. Non-marketable assets are primarily credit claims such as loan portfolios; and (ii) whether relevant data is available or not. In the case of marketable assets, the historical data on the asset prices is needed. In the case of loan portfolios, the probability of default (PD) and loss given default (LGD) are needed. Additionally, we discuss several variants of these main models. For example, we explain how to model haircuts on illiquid securities when relevant liquid securities are traded.

When setting haircuts, several principles must be adhered to. The first is non-procyclicality. Haircuts should not be procyclical. Increasing haircuts during a crisis could amplify financial institutions' incentives to liquidate assets and the broader economy through fire sales. The second is data-drivenness. Haircuts should be informed by data sets that capture risks embedded in the target assets. While this can be challenging in developing or low-income countries due to limited capacity, it is crucial to ensure that haircuts reflect prevailing risks. Following this principle ensures accountability. The third is conservatism. Haircuts should protect the central bank from potential losses by benchmarking against losses during historical stress episodes. Ideally,

[1] Please refer to Bank for International Settlements (2013) for further details.

the sample period for estimating model parameters should include historical stress episodes such as financial crises. The fourth principle is no arbitrage gap. If a central bank's collateral risk management is overly lenient, it may lead to an adverse selection problem, whereby lower-quality assets are disproportionately pledged as collateral.

We begin by presenting a model for determining haircuts on marketable assets in data-rich environment, introducing the Duration Approximation with Stressed Volatility (DASV) model, particularly suited for fixed income instruments like government bonds across various maturities. This approach is based on a widely used duration-based approximation of bond price change combined with EGARCH model for stressing the bond yield volatility. It offers two key benefits: simplicity and modularity. Its simple formula makes it easy to implement, while its modular modeling structure allows each component to be independently assessed and validated. To stress the bond yield volatility, we adopt the EGARCH model because it is a simple time-varying volatility model. Notably, the DASV model supports the calculation of haircut rates using both Value-at-Risk (VaR) and Expected Shortfall (ES). We apply the DASV model for the US treasury bonds for illustrative purposes.

We then explore haircut models for marketable assets in data-limited environments, particularly when default rates are unavailable. We propose the Duration-Approximation with Cross-country Regression (DACR) model that combines a duration-based approximation with a satellite cross-country regression to construct hypothetical sovereign spreads. The strength of the DACR model is that it allows a smooth transition: once a sufficiently long time series of the bond prices becomes available in the future, the satellite cross-country model can be phased out and replaced with actual data.

To support central bank practitioners, we outline key parameters that must be determined. These include: (i) VaR percentile for the bond yield shocks; (ii) the time to liquidation, which is often difficult to observe and estimate; (iii) the VaR percentile for parameter uncertainty; and (iv) risk-free interest rate volatility based on the time series of the actual policy rate set by the central bank or short-term rate in the country. In cases where such time series data are unavailable, the modeler must make reasonable assumptions. (i) and (ii) are common in both DASV and DACR models. Note that these two VaR percentiles reflect the central bank's risk tolerance.

Next, we address the haircut models to non-marketable assets such as loan portfolios in data-rich environments. We adopt the Asymptotic Single Risk Factor model with Adjustments (ASRFA) to produce stressed probabilities of default (PDs) using the historical data, building on the foundational work of Vasicek (1991) and its adoption in the Basel II Internal Ratings-Based (IRB) framework, with theoretical support from Gordy (2003). The model calculates stressed PDs as either the Value-at-Risk (VaR) or Expected Shortfall (ES) of PD at a specified percentile. The model is both practical and parsimonious, requiring only three inputs—PD, risk tolerance (VaR percentile), and a correlation parameter—and benefits from a closed-form solution. We adjust the correlation parameter to ensure a conservative estimate, particularly when the quality of historical default data is uncertain.

Regarding non-marketable assets, the authority needs to quantify the following key parameters: (i) VaR percentile for stressing the default probability; (ii) the average PD; (iii) LGD; (iv) the correlation parameter; (v) the time to liquidation; and (6) risk tolerance parameter for parameter uncertainty. These two VaR percentiles should reflect the central bank's risk tolerance. The average PD and LGD are estimated using available data. The correlation parameter can be estimated from joint defaults across loans within the same type of loan portfolio, but it may be estimated as the sensitivity of credit quality of firms to a systematic factor such as macroeconomic environment. The time to liquidation should be also determined when defining recovery rate, or equivalently, loss given default.

Similar to marketable assets, we also explore haircut models for non-marketable assets in data-limited environments, particularly when default rates are unavailable, by examining three representative cases. In the first case, where firm-level balance sheet data exists, structural credit risk models like Merton (1974) can be used to estimate a proxy for PD, with stress scenarios applied by adjusting asset volatility or growth rates. In the second case, we address the absence of balance sheet data, proposing a cross-country regression model using macroeconomic and sector-level variables to estimate sector-level credit spreads and infer PDs. In the third case, we deal with short data periods, suggesting the use of satellite models to extrapolate PDs under stress. Reliance on such models will diminish as the data period becomes long enough to cover at least one business cycle.

The second objective is to develop a quantitative framework for counterparty risk management. It is important for central banks to calculate and continuously monitor residual uncollateralized exposures that persist even after applying haircuts for loss mitigation. These remaining exposures can pose counterparty risks—particularly during periods of market stress—and therefore warrant careful evaluation. By adopting a quantitative framework, central banks can strengthen their ability to assess both the magnitude and the underlying drivers of these residual risks.

As illustrative examples, we analyze uncollateralized exposure under two cases: (i) there is one collateral asset which follows a geometric Brownian motion. (ii) Bond portfolios which yield curves are modeled via a conventional dynamic Nelson-Siegel model. We report how volatility of underlying yield curve factors impacts the expected uncollateralized exposure.

The third objective is to demonstrate the effectiveness of Machine Learning techniques to strengthen the central bank's counterparty risk management. Specifically, we employ Variational Autoencoders (VAEs) to generate stress scenarios for bond yields. VAEs are interpretable generative neural networks that learn latent variables from historical yield curves and reconstruct realistic yield curve shapes through a stochastic process. Their ability to simulate diverse, non-normal yield distributions makes them well-suited for stress testing.

As an illustrative application, we train a VAE on U.S. Treasury yield curve data to simulate yield curves. The results show that the VAE captures realistic yield distributions and can be used to validate VaR-based haircuts. The analysis underscores that, although machine learning models like VAEs may not be directly used for setting collateral policy due to interpretability concerns, they offer valuable support for model validation—such as identifying potential add-ons to account for gaps in risk estimates.

This paper is structured as follows. The first section is an introduction. The second section is literature review. The third section explains a quantitative framework for determining haircuts and counterparty credit risk management. The fourth section describes a haircut model for marketable securities with a focus on the government bonds. The fifth section proposes a haircut model for government bonds under data-limited environments. The sixth section introduces a haircut model for non-marketable assets with a focus on loan portfolios. The seventh section proposes a haircut model for loans under data limited environments. The eighth section develops simulation analysis of counterparty credit risk and model validation with a Machine Learning technique. The nineth section concludes.

Related Literature

This study contributes to two strands of the literature, one of which focuses on haircut modeling from the perspective of central bank risk management. Official publications detailing central banks' haircut models are notably scarce.

Alder et al. (2023) explain the European Central Bank (ECB)'s approach for calibrating haircuts for marketable assets that involves three main elements: time to liquidation, market risk, and jump-to-default risk.[2] Market risk is captured by the duration approximation formula which takes an instrument's cash flows and the yield volatility of their discounting curve into account. Note that jump-to-default risk is intended to capture the credit risk not contained in the asset prices.

Rule (2012) and Breeden and Whisker (2010) report that the Bank of England used to characterize the base haircuts to narrow sovereign and supranational securities based upon a 99th percentile five-day VaR.

Different from these previous studies, this paper aims to provide a comprehensive guide to modeling haircuts under different data availability and target assets and then discuss a quantitative framework of counterparty risk management by using relevant concepts such as expected uncollateralized exposure.

It is noteworthy that previous studies have empirically shown that central bank haircut policies influence the supply and demand dynamics of eligible bonds, thereby affecting their pricing (e.g., Cassola and Koulischer, 2019; Kaldorf and Poinelli, 2024). In addition, theoretical research has explored various trade-offs central banks face when designing collateral policies (e.g., Chapman et al., 2011; Choi et al., 2021). This study, however, focuses specifically on modeling haircuts from a risk management perspective.

This study also contributes to an emerging literature on the application of generative neural networks to finance. There are several previous studies applying VAEs to finance.

Caprioli et al (2025) employ VAEs to create an interpretable latent space and generate realistic synthetic correlation matrices by sampling from that space. This approach helps identify key factors influencing portfolio diversification, especially in understanding how credit portfolios respond to changes in asset correlations. By training VAEs on historical correlation data, their model simulates correlation matrices that align with stylized facts.

Bergeron et al. (2022) apply VAEs estimate missing data on volatility surfaces, which are essential for pricing and hedging derivatives. A volatility surface maps implied volatility as a function of an option's strike price and maturity. Given the frequent incompleteness of market, the authors propose a two-step method: first, use VAEs to learn latent variables that generate realistic synthetic surfaces; second, select the synthetic surface that best matches the available market data. These synthetic surfaces can also support stress testing, market simulation, and exotic option valuation. They demonstrated the performance of this method using foreign exchange market data.

[2] For a broader perspective on the ECB's collateral framework, see Nyborg (2017).

Sokol (2020) introduces a variational autoencoder (VAE)-based latent factor model for yield curves across multiple currencies, offering a parsimonious representation and revealing a latent space "world map" where curve shapes cluster geographically. In the second part, he presents autoencoder market models (AEMMs) that enhance classical models by replacing state variables with VAE-derived latent variables, improving robustness, realism, and calibration simplicity in both risk-neutral and physical-measure frameworks.

The previous papers suggest that VAEs offer valuable tools for risk management or pricing. Different from the previous studies, the focus of this study is on validating haircut models with VAEs.

Setting Haircuts with a Tail Risk Measure

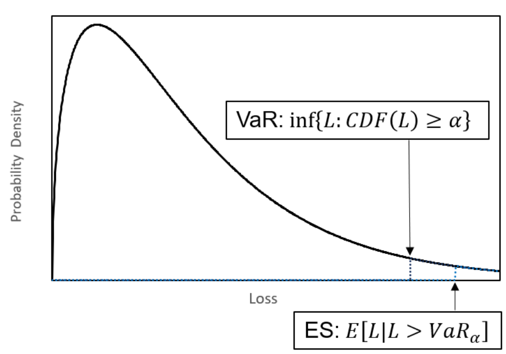
Protecting Central Bank's Balance Sheet with Tail Risk Measures

The core idea is to set haircuts that cover potential losses, as quantified by a tail risk measure, under stressed market conditions. This allows the central bank to mitigate losses resulting from a decline in the value of collateral with a pre-defined probability. In the context of monetary policy operations—where marketable securities are posted as collateral—stressed conditions refer to periods of financial market turmoil. Potential losses are assessed using tail risk measures, which are explained in detail below. This approach has been adopted by banks, central banks, and central clearing counterparties (e.g., Garcia and Gencay, 2012; Rule, 2012; Alder et al., 2023).

It is essential to ensure that haircuts applied to securities used as collateral are not procyclical. Raising haircut rates during periods of market stress can compel market participants to liquidate assets in order to meet additional collateral requirements, thereby amplifying market instability. To avoid this, haircut models should incorporate stressed market conditions in their design and calibration.

Tail risk can be defined using two widely adopted measures: Value-at-Risk (VaR) and Expected Shortfall (ES). VaR estimates the maximum potential loss over a specified holding period at a given confidence level, while ES represents the average loss in scenarios where losses exceed the VaR threshold, as illustrated in Figure 1.

Figure 1. Potential Losses, Value-at-Risk and Expected Shortfall



CDF: Cumulative Distribution Function.
Source: Author.

Value-at-Risk vs. Expected Shortfall

Value-at-Risk (VaR) is generally easier to understand and validate, while Expected Shortfall (ES) provides a more accurate reflection of potential losses. Each measure has its own advantages and limitations, as summarized in Table 1 below. The primary strength of VaR lies in its simplicity: it is conceptually and computationally more straightforward than ES, which is defined as the conditional expectation of losses exceeding the VaR threshold.

Table 1. Pros and Cons of Value-at-Risk and Expected Shortfall

	Value-at-Risk (VaR)	Expected Shortfall (ES)
Definition	Maximum loss at a given confidence level	Average of losses exceeding VaR
Advantages	Easy to compute & understand	More relevant to potential losses
	Relatively easy to validate the statistical performance of VaR forecasting model	Sensitive to the extreme losses by considering the losses beyond VaR
	Widely used in financial institutions and recognied by regulators	Theoretically, capturing portfolio diversification effect (sub-additivite)
Disadvantages	Ignoring losses beyond the VaR threshold	More complex to compute when there is no analytical formula
	Theoretically, not capturing portfolio diversification effect (non sub-additive)	Relatively difficult to validate the statistical performance of ES forecasting model

Source: Author.

Both VaR and ES are effective tools for measuring tail risk, provided that the VaR percentile—representing the institution's risk tolerance—is appropriately calibrated. By definition, ES is always larger than VaR at the same confidence level, which is also referred to as the risk tolerance parameter. However, in practical applications, the choice between VaR and ES is not critical. It is possible to obtain a VaR that is equal to a given ES by adjusting the risk tolerance parameter accordingly.

Mathematical Definition of Haircuts

Our goal is to model haircuts on the government bonds from the view of the central bank. We assume that the government bond prices reflect all relevant risks—namely, interest rate risk, sovereign default risk, and liquidity risk—to which the central bank is exposed. Accordingly, our analysis focuses on the risks stemming from fluctuations in bond prices. We also discuss a potential extension of the model to account for risks that may not be priced into asset values, as well as a credit rating-based approach.

We denote variables parameters as follows. h is haircut rate. Δt is the time to liquidation. τ^D is the time when the counterparty bank defaults.

To determine haircut based on tail-risk measure, we need to assess VaR or Expected Shortfall (ES) of the uncollateralized exposure. The loss per unit of the exposure is given by $Loss = 1 - \frac{1}{1-h}\frac{A_{t+\Delta t}}{A_t}$.

The value of collateral should be equal to the exposure after applying haircut. Following Lou (2017), the mathematical definition of VaR-based haircuts is given by

$$h \text{ such that } VaR_\alpha = 0 \text{ where } VaR_\alpha = \inf\{L^* \in \mathbb{R}: \Pr(Loss_{t+\Delta t} \geq L^* \mid t = \tau^D) \geq 1-\alpha\} \quad (1)$$

This condition implies that the haircut is chosen so that the probability of experiencing a loss (i.e., the collateral losing value) is no greater than $1-\alpha$.

Similarly, the definition of ES-based haircuts is given by

$$h \text{ such that } E[Loss_{t+\Delta t} \mid Loss_{t+\Delta t} \geq VaR_\alpha, t = \tau^D] = 0 \quad (2)$$

This condition implies that the haircut is set such that the expected loss, conditional on exceeding the Value-at-Risk at the designated confidence level is zero.

Determining VaR Percentile

The Value-at-Risk (VaR) percentile should be benchmarked against the historically most severe shock. Ideally, it should be set more conservatively than the percentile implied by that shock, enabling the central bank to safeguard its balance sheet against adverse market conditions. This approach helps mitigate the need for rising haircuts during periods of stress, thereby reducing the risk of fire sales triggered by margin calls induced by increased haircuts.[4] Ultimately, however, the final decision rests with the central bank.

External Factors Not Accounted for Within the Modeling Framework

Our discussion centers on a quantitative framework for setting haircuts. However, it is also important to compare the central bank's collateral policy with that of market participants. If the central bank's risk management is relatively more lenient—for example, if its haircuts are lower than those applied by banks—then banks may prefer to pledge collateral to the central bank to obtain liquidity rather than using it in the markets. This is called an adverse selection problem and could expose the central bank to increased risk.[5]
Also, designating a security as eligible collateral or applying a lower haircut can increase its attractiveness to financial institutions. This enhanced value—often referred to as the collateral eligibility premium or specialness—can increase demand for the asset, potentially leading to price appreciation. As noted by Cassola and Koulischer (2011), such price effects may influence market participants' behavior, altering their incentives and collateral composition choices. Understanding this dynamic is essential for central banks when designing collateral frameworks. However, this mechanism is not currently captured in our modeling framework.

[4] According to FSB (2015), "Haircut methodologies should be designed to limit potential procyclical fluctuations in haircuts, specifically by moderating the extent to which they decline in benign market environments (for example characterized by low market volatility and rising asset prices) and thus mitigate the magnitude of the potential increase in volatile markets."

[5] Here, the term "adverse selection" is used in a broad sense. In a narrower sense, adverse selection refers to situations where under information asymmetry, the more-informed agent gains an advantage from a contract or a trade with the less-informed agent. In our context, adverse selection refers to a situation in which the central bank receives collateral of lower quality than it has assessed.

An Illustrative Toy Model

To better understand how the framework above operates, consider a simplified setting where the collateral asset value A_t follows a geometric Brownian motion with constant drift μ_A and constant volatility σ_A. We assume that the exposure is constant ($L_t = L$). Let us derive the haircut based on VaR.

The α-percentile VaR of the collateral asset over time horizon Δt, which is called time to liquidation, is given by $A_{t+\Delta t} = A_t \exp\left(\left(\mu_A - \frac{\sigma_A^2}{2}\right)\Delta t - \Phi(\alpha)\sigma_A\sqrt{\Delta t}\right)$. Accordingly, the haircut is expressed as:

$$h = 1 - \exp\left(\left(\mu_A - \frac{\sigma_A{}^2}{2}\right)\Delta t - \Phi(\alpha)\sigma_A\sqrt{\Delta t}\right) \tag{3}$$

When the exponent is small, the linear approximation can be used. Assuming that the drift term $(\mu_A - \sigma_A^2/2)\Delta t$ is small, the haircut simplifies to:

$$h = \Phi(\alpha)\sigma_A\sqrt{\Delta t} \tag{4}$$

This simplified formula highlights the three main drivers of the haircut. The first driver is the collateral asset volatility. The second one is time to liquidation Δt. The third one is the confidence level multiplier, which maps the volatility to VaR.

It is important to note that the assumption of constant volatility is made here for illustrative purposes. In practice, volatility is time-varying and may be correlated with counterparty default risk, which should be accounted for in more robust models.

Dependance Between Collateral Volatility and Counterparty Default Risk

It is important to note that parameters such as volatility and time to liquidation may be influenced by the default risk of the counterparty bank. While modeling this dependency is often complex and challenging, it remains a critical consideration.6

In the context of this paper, two cases warrant attention. First, as commonly observed in the sovereign-bank nexus, when a counterparty bank holds a significant amount of government bonds, a correlation may exist between the bank's default risk and the value of those bonds. This exemplifies a general form of wrong-way risk. Second, when a bank uses its own issued bonds as collateral, a direct link arises between its default risk and the value of those bonds—this is referred to as specific wrong-way risk.

To illustrate the impact of this dependency, consider the case where the volatility of collateral value is correlated with the credit quality of the counterparty. In this framework, the counterparty's credit quality is proxied by the value of its total assets. For illustrative purposes, we assume that the total asset value follows a geometric Brownian motion. Additionally, we model the logarithm of squared volatility as a mean-reverting

[6] There are many studies on modeling the dependence between default risk of a counterparty and exposure. Such studies are discussed in the literature under the key word "wrong-way risk".

process to ensure its positivity, similar in spirit to the EGARCH model. Following structural credit risk model of Merton (1974), the default occurs when the total asset value X_t falls below the level of total liabilities D at time T.

$$d \log \sigma_t^2 = \kappa\left(\log \overline{\sigma^2} - \log \sigma_t^2\right)dt + \omega dW_{\sigma,t} \tag{5}$$

$$dX_t = X_t\left(\mu_X dt + \sigma_X dW_{X,t}\right) \tag{6}$$

The key parameter is correlation $\rho = dW_{\sigma,t}\, dW_{X,t}/dt$. We are interested in understanding the impact of this correlation parameter. For the parameters of the collateral volatility, we set $\bar{\sigma} = \sigma_0 = 10\%.\ \kappa = 0.01,\ \omega = 0.01.$ For the counterparty's credit quality, $X_0 = 1,\ D = 0.9,\ \mu_X = 0,\ \sigma_X = 20\%.$

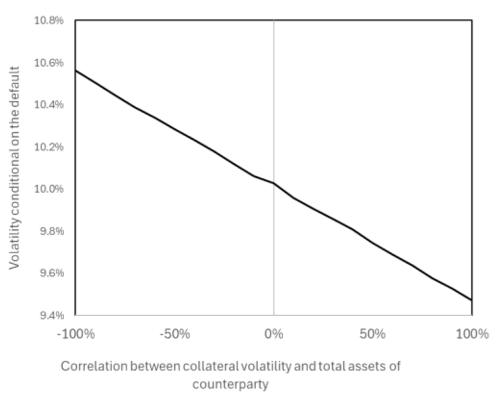
Figure 2-A illustrates the relationship between the volatility of the collateral asset and the correlation with the counterparty's credit quality. As the correlation becomes more negative, the volatility conditional on default increases. This occurs because a lower credit quality of the counterparty is more likely to coincide with higher collateral asset volatility.

Figure 2-B depicts the relationship between the haircut and the correlation. As the correlation becomes more negative, the haircut increases, reflecting the rise in volatility.

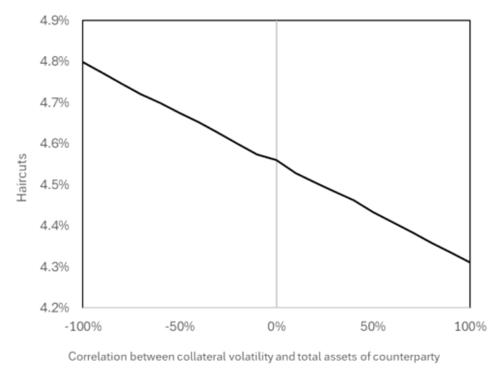
The results highlight the potential importance of adjusting model parameters to account for the dependence between default risk and those parameters, even though modeling and estimating such dependencies is inherently challenging. This consideration becomes particularly relevant in cases involving counterparties with significant market presence, or when the collateral asset is directly linked to the counterparty—for example, corporate bonds issued by the counterparty itself.

Figure 2. The Impact of Correlation Between Collateral Volatility and Counterparty Default Risk

A. Correlation and Volatility Conditional on the Counterparty Default



B. Correlation and Haircuts



Source: Author.

Central Bank's Uncollateralized Exposure

It would be valuable for a central bank's counterparty credit risk management team to compute and monitor the residual uncollateralized exposure that remains after applying haircuts for loss mitigation. Such exposure may pose hidden risks, particularly under stressed market conditions, and therefore warrants careful assessment. A quantitative approach to counterparty risk management can help central banks better understand the magnitude and underlying drivers of these residual risks.

Let us denote the loan from the central bank to a counterparty bank or the central bank's exposure to a counterparty bank as L, and the collateral value as C_t. At the start of the transaction, the collateral amount is set equal to the exposure after applying haircut (C_0=$L/(1-h)$).

The residual uncollateralized exposure E_t^u is given by:

$$E_{t+\Delta t}^{U} = L - C_{t+\Delta t} = L\left(1 - \frac{1}{1-h}\frac{A_{t+\Delta t}}{A_t}\right) \tag{7}$$

We are interested in computing the expected uncollateralized exposure conditional on the default of the counterparty bank. Note that we take the maximum of the exposure and zero, as our focus is solely on losses.

$$\mathrm{E}[\max(E_{t+\Delta t}^{U}, 0)\,|t = \tau^D] = \frac{L}{1-h}\cdot \mathrm{E}\left[\max\left(1 - h - \frac{A_{t+\Delta t}}{A_t}, 0\right)|t = \tau^D\right] \tag{8}$$

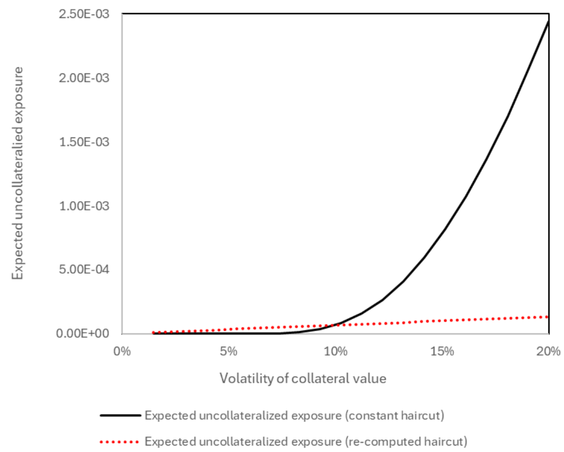
The equation above shows that the expected uncollateralized exposure can be interpreted as a put option where the strike is $1-h$ and the maturity corresponds to the time to liquidation Δt.[8] If we assume the collateral value follows a geometric Brownian motion as described earlier, the well-known Black-Scholes formula becomes applicable.

Figures 3 and 4 illustrates how the expected uncollateralized exposure varies with key input parameters. In the base case, we assume that $\mu = 0\%$, $\sigma = 10\%$, and $\Delta t = 10/252$. Haircuts are computed with 99th percentile VaR ($\alpha = 0.99$). For each parameter, we show two cases: (i) haircut is held constant (solid line); (ii) haircut is re-computed given a new parameter value (dotted line)

Figure 3 shows that the expected uncollateralized exposure increases nonlinearly with the volatility of the collateral asset, as indicated by the solid line. Notably, the solid and dotted lines intersect at a volatility of 10 percent, which corresponds to the baseline assumption. In regions where volatility exceeds 10 percent, the expected uncollateralized exposure is underestimated when the haircut is not adjusted, since the actual volatility is higher than the one used to set the haircut

[8] Note that there is no discount factor.

Figure 3. Expected Uncollateralized Exposure and Volatility

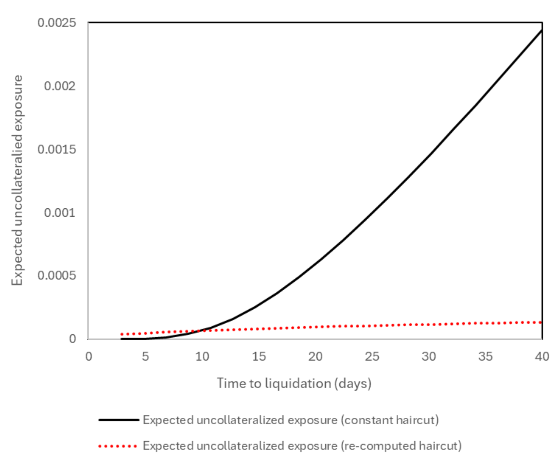


Source: Author.

Figure 4 demonstrates a similar nonlinear relationship between expected uncollateralized exposure and time to liquidation. Again, the solid and dotted lines intersect at 10 business days, consistent with the baseline assumption. When the actual time to liquidation exceeds this baseline, the exposure is underestimated if the haircut is not recalibrated accordingly.

These results underscore the significance of conservatively setting the collateral asset volatility and time to liquidation, especially under stressed market conditions.

Figure 4. Expected Uncollateralized Exposure and Time to Liquidation



Source: Author

So far, we have discussed the expected uncollateralized exposure for a single counterpart bank at the transaction level. Naturally, this concept can be extended to multiple counterparties at the portfolio level.

When portfolio-level expected exposure (PPE) for the i-th counterparty is netted out across transactions, it is defined as follows.

$$\mathrm{PEE}_{\mathrm{i,t}} = \mathrm{E}\left[\max\left(\sum_{k=1}^{K_i} E^{U}_{i,k,t+\Delta t}\,, 0\right) | t = t_i^D\right] \tag{9}$$

Where K_i is the number of the exposures. A more specific case is discussed in the section of simulating counterparty risk exposure.

The aggregated portfolio-level expected exposure across counterparties is given by

$$\text{Aggregated } \mathrm{PEE}_{\mathrm{t}} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \sum_{\mathrm{k}=1}^{\mathrm{K}} (1 - R_i) \int_t^{T_i} dt \, \Pr(t = t_i^D) \, \mathrm{E}\left[\max\left(\sum_{k=1}^{K_i} E^{U}_{i,k,t+\Delta t}\,, 0\right) | t = t_i^D\right] \tag{10}$$

Where N is the total number of the counterparties and T_i is the longest maturity of transactions with counterparty i. R_i is the recovery rate for counterparty i.

Classifying the Modeler's Situation

We classify the modeler's situation into four distinct cases to guide the selection of an appropriate haircut model. These cases are structured as a 2×2 framework based on two dimensions, as shown in Table 2.

- Type of collateral: Whether the asset pledged is marketable (e.g., securities) or non-marketable (primarily credit claims).
- Data availability: Whether there is sufficient data to support model estimation or the environment is data-scarce.

Based on this classification:

- For marketable assets in a data-rich environment, the proposed model is the Duration Approximation Stressed Volatility (DASV) model.
- For marketable assets in a data-scarce environment, the proposed model is the Duration Approximation Cross-country Regression (DACR) model.
- For non-marketable assets (credit claims) in a data-rich environment, the proposed model is the Asymptotic Single Risk Factor with Adjustments (ASRFA) model.
- For non-marketable assets in a data-scarce environment, haircut estimation relies on either a cross-country regression model or Merton's (1974) structural credit risk model to compute default probabilities (or extrapolation method when the sample period is short).

Table 2. Model Choice Given Data Availability and Asset Type

	Data-rich environment	Data-scarce environment
Marketable assets	Duration-Approximation Stressed Volatility (DASV) Model	Duration-Approximation Cross-country Regression (DACR) Model
Non-marketable assets	Asymptotic Single Risk Factor with Adjustment (ASRFA) model	(1) Structural credit risk model (2) Cross-coutry model

Source: Author.

Setting Duration Buckets

It is a common practice among central banks to set duration buckets by balancing two key criteria: operational efficiency and accuracy. For operational efficiency, less granular buckets are preferred, as this simplifies management. However, for accuracy, more granular buckets are preferred to precisely match the risk of potential losses. Balancing these criteria helps central banks optimize their duration bucket setting effectively. Note that in terms of conservativeness, assigning a duration number equal to the upper bound of each duration bucket is better. For example, consider the duration bucket from one year to two years. Assigning a two-year duration to this duration bucket is a more conservative approach than using one year, as it helps avoid underestimating duration risk.

Duration buckets should reflect the maturity structure of government bonds issued in each country. For instance, if a client country's government frequently issues 30-year bonds, it is more appropriate to create a dedicated bucket for those rather than grouping them with, say, 20-year bonds.

Marketable Assets in a Data-Rich Environment

Overview of DASV Model

We begin by introducing a model for setting haircuts on marketable assets in a data-rich environment. Specifically, we propose the Duration Approximation with Stressed Volatility (DASV) model, designed to determine haircuts for fixed-income instruments, with a particular focus on government bonds across different maturities. The DASV model offers two key advantages: simplicity and modularity. First, its formula is straightforward and easy to implement. Second, the modular structure allows each component to be analyzed and validated independently. Notably, the DASV model can generate haircut rates based on both Value-at-Risk (VaR) and Expected Shortfall (ES).

Duration Approximation

The duration-approximation formula estimates the potential losses from the bond price changes using a pre-estimated effective duration T, as described below. h_a for a bond with duration T under a stressed change in bond yield $\Delta y_{\alpha,t}$ is given by: α

$$h_a = T\Delta y_{\alpha,T}\ , \tag{11}$$

Where α is the percentile of VaR or ES.

Computing the duration for the fixed-coupon bonds is not difficult. The first method is taking numerical differentiation of the Net Present Value (NPV) of the bond with respect to the interest rate or bond yield.[10] The second method is applying an analytical formula such as modified duration. The third method is using the remaining maturity, which provides a conservative estimate in the case of coupon bonds.[11]

$$T = -\frac{1}{P}\frac{dP}{dy}. \tag{12}$$

The stressed bond yield change is computed using three inputs: (i) stressed volatility; (ii) time to liquidation; (iii) the multiplier for converting volatility to VaR or ES. The stressed bond yield change is computed using one of two equations presented below. The stressed volatility with pre-determined risk tolerance parameter α is denoted with σ_α. Models for estimating stressed volatility are discussed in the next subsection. The time to liquidation is denoted with Δt. In what follows, we set the time to liquidation equal to 10 business days, following the Basel framework ($\Delta t = 10$) in the case of liquid collateral. However, it can be longer if the securities of interest are more illiquid, as discussed later. For analytical tractability, the normal distribution is employed.[12] The multipliers for VaR and ES can be computed analytically under the assumption of the normal distribution.

$$\text{VaR: } \Delta y_{\alpha,T} \ = \ \Phi^{-1}(\alpha) \cdot \sigma_\alpha \cdot \sqrt{\Delta t} \tag{13}$$

$$\text{ES: } \Delta y_{\alpha,T} \ = \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha} \cdot \sigma_\alpha \cdot \sqrt{\Delta t} \tag{14}$$

Duration-based approximation is conservative in general. The duration is the first order derivative of the bond price with respect to the interest rate. The second order derivative called convexity is typically positive except specific types of bonds such as mortgage-backed securities. Since the convexity is positive, taking the convexity into account leads to less conservative estimate for haircuts. Therefore, duration-based approximation is easy to explain and communicate. It is noteworthy that the duration-based formula can be too

[10] For example, the duration can be numerically computed as: $T = -\frac{1}{P}\frac{P(y+\delta y)-P(y-\delta y)}{2\delta y}$

[11] The duration for fixed coupon bond is shorter than the remaining maturity when the principal is paid back.

[12] It is possible to use a fat-tailed distribution such as Student t distribution although the estimation becomes more difficult. As a robustness check, we estimated EGARCH with t distribution. The result shows that the estimated degree of the freedom is quite large, indicating the EGARCH model with a normal distribution is acceptable.

conservative when the shock is very large. In this case we need to use the nonlinear formula which is more accurate.[13]

The duration for floating coupon bonds can be approximated by the next coupon payment date. This can be understood by considering that floating coupon bonds effectively represent an investment strategy of rolling over short-term bonds. Consequently, the duration is closely linked to the coupon payment date. Additionally, it is noteworthy that higher interest rates decrease the future value of cash flows (discount effect) while simultaneously increasing future coupon payments linked to those interest rates (cash flow effect).

Stressing Volatility

Two approaches for estimating the stressed volatility are explained: Historical volatility approach and GARCH-type models.[14] While various approaches exist, we focus on two approaches for expository simplicity. Specifically, we explain the EGARCH model as an example of GARCH-type models.

The historical volatility approach is simple but produces a noisy series of the volatility. The historical volatility is computed as the standard deviation of d-days bond yield changes $\Delta y_t = y_t - y_{t-1}$ over a rolling window d. y_t is the bond yield. In our empirical application, d is set equal to the same number as the time to liquidation which is 10 business days. This approach is easy to implement but the time series of historical volatility tends to be very noisy and prone to outliers.

$$\sigma_t^{hist} = Stdev(\Delta y_t, \Delta y_{t-1}, \cdots, \Delta y_{t-d+1}) \tag{15}$$

The EGARCH model is a simple time-varying volatility model that is relatively easy to estimate. There are four parameters to be estimated: auto-coefficient of the volatility ρ, the mean-reverting level of the volatility $\bar{\sigma}$, volatility of volatility β, and γ is the asymmetry effect which is designed to capture the feedback effect from the bond yield to its volatility.[15] ϵ_t is generated from the standard normal distribution. The remaining model parameters are estimated based on maximum likelihood estimation.[16]

$$\Delta y_t = \sigma_t \cdot \epsilon_t \tag{16}$$

$$log\, \sigma_t^2 - log\, \bar{\sigma}^2 = \rho(log\, \sigma_{t-1}^2 - log\, \bar{\sigma}^2) + \nu\left(|\epsilon_{t-1}| - \sqrt{\frac{2}{\pi}}\right) + \gamma\, \epsilon_{t-1} \tag{17}$$

Imposing smoothness for the relationship between volatility and maturity may be necessary, guided by expert judgement. If the estimated parameter set for a target maturity appears unreasonable, the stressed volatility for

[13] The nonlinear formula is written as : $\Delta y_{\alpha,T} = 1 - \exp\left(-\Phi^{-1}(\alpha) \cdot \sigma_\alpha \cdot \sqrt{\Delta t}\right)$ for VaR .

[14] There are certainly other approaches. The simplest approach is to use historical VaR but this approach is completely forward-looking. Another possible approach is EWMA (Exponentially Weighted Moving Average) method proposed by RiskMetrics. EWMA can be interpreted as a simplified version of GARCH model. Please refer to Jorion (2007) for more details.

[15] Unlike the original GARCH model, there is no restriction on these parameters. But, if the auto-coefficient is equal or higher than 1, it means that the volatility does not converge to the mean-revering level.

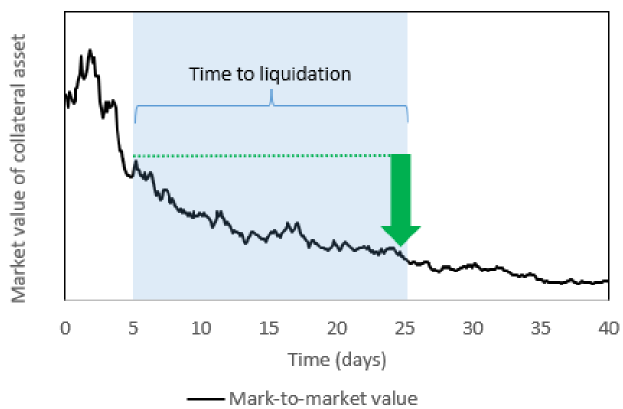
[16] To reduce the number of parameters to be estimated, we set the initial volatility equal to the average volatility. Also, to ensure the robustness of the estimation, we may need to assume the mean-reverting volatility level $\bar{\sigma}$ equal to the standard deviation of the bond yield changes during the entire estimation period but this assumption is not strictly necessary.

that maturity can be extrapolated from the volatilities for other maturities. In such cases, non-estimated volatilities should be extrapolated and floored with the minimum value among estimated volatilities to ensure consistency and avoid underestimation.

Time to Liquidation

The time to liquidation represents the number of business days required to liquidate a position. Longer liquidation periods imply greater uncertainty in price movements and, consequently, higher risk. Figure 5 provides an intuitive illustration of this relationship. In the figure, the counterparty defaults on the fifth business day. The time to liquidation is set at 20 business days. One approach to determining the liquidation period is to apply 10 business days for liquid assets and 20 business days for illiquid assets, following with Basel framework. [17] However, this value can be adjusted based on liquidity-related data and institutional conditions specific to each country.

Figure 5. Understanding the Relation Between Time to Liquidation and Market Risk



Source: Author.

Illiquidity adjustment can be incorporated by extending the time to liquidation. In general, this parameter is difficult to quantify, because what we aim to estimate is the time it would take to liquidate positions under stress conditions given a bank failure, which cannot be directly observed in normal circumstances.

Ensuring Consistency in Haircuts Models for Relevant Markets

In this subsection, we examine a case involving government bonds that are traded but less liquid than the primary benchmark bonds. Specifically, we treat domestic-currency bonds as less liquid assets and Eurobonds as liquid assets. To ensure internal consistency between the haircuts applied to these two types of bonds, it is important to establish a relationship between them. Information from the more liquid Eurobonds can be used to inform haircut levels for the less liquid domestic-currency bonds. We extend the DASV model by introducing

[17] Our situation is, of course, not exactly the same as the setting discussed in Basel Committee on Banking Supervision (2023). Therefore, the 10 business days for liquid collateral and 20 business days for illiquid assets should be regarded as a reference.

two additional parameters: spread volatility and an illiquidity adjustment for time-to-liquidation. The conventional model is described in the previous subsection. Our approach proceeds as follows:

First, we compute theoretical yield of the domestic-currency bond by using the no-arbitrage relationship between these two bonds. We then incorporate the effect of illiquidity of domestic-currency-denominated bonds relative to the foreign-currency-denominated bonds.

The formula for the stressed yield of the relatively illiquid domestic bond resembles that of the liquid Eurobond, but includes two additional components. In the equation below, σ_{spr} is the volatility of the spread of domestic-currency bond yield minus Eurobond yield. λ is the illiquidity adjustment on time to liquidation. If the domestic-currency bonds are less liquid than the Eurobonds, then $\lambda > 1$.

$$\text{VaR: } \Delta y_{\alpha,T} = \Phi^{-1}(\alpha) \cdot \left(\sigma_{\alpha} + \sigma_{spr}\right) \cdot \sqrt{\lambda \Delta t} \tag{18}$$

Spread volatility is computed as follows. Under the fixed FX regime pegged to US dollar, if there is no friction, the domestic bond yield should be equal to the US bond yields due to the no-arbitrage (i.e., the spread is zero). If there is a useful empirical proxy for the domestic-currency bond yields, we need to use short-term rates.

It is worth noting that the same methodology can be applied to determine haircuts for corporate bonds with varying credit ratings. In this context, we first calculate the volatility of credit spreads for corporate bonds corresponding to each credit rating. We then apply the formula outlined above to derive the appropriate haircut for each rating category.

Combining with Credit Rating Data

Extending the DASV model to incorporate credit rating would be promising and useful but it presents several practical challenges, especially in the context of sovereign credit risk in developing countries or low-income countries. Note that bond prices generally reflect several components: the risk-free rate, term premium, credit risk premium, and liquidity premium. The DASV model captures these various sources of risk via the bond yield volatility.[18] In the following section, we focus specifically on the methodology that integrates credit ratings with market data.

The first challenge is double-counting risk, where sovereign default risk inferred from sovereign credit rating may overlap with market-implied credit spread because market prices may already reflect real-time risk perceptions even before sovereign credit rating changes. Ensuring no double-counting of credit risk—between market data and ratings—without overcomplicating the model is a delicate balance. If sovereign credit risk that is not reflected in market prices is deemed important, an additional adjustment of haircuts could be considered; however, such an adjustment would require expert judgement.

The second challenge is data gaps, as many government bonds in developing countries lack up-to-date or reliable ratings—yet the absence of ratings does not equate to the absence of risk. Hence, we need another

[18] It is possible to develop a more sophisticated approach that separately models the risk-free rate, default risk, and liquidity components. However, such an approach requires multiple data sources to accurately identify and isolate these different risk factors. Since the objective of this paper is to determine appropriate haircuts rather than to decompose bond yields into their individual components, we consider the current approach to be suitable and operationally feasible.

satellite model to generate sovereign credit rating and corresponding additional risk premium while ensuring the no-double counting.

The third challenge concerns market stability. Abrupt rating changes—especially when delayed—can disrupt markets, particularly if ratings are central to haircut models.[20] This is related to the importance of mitigating procyclicality.

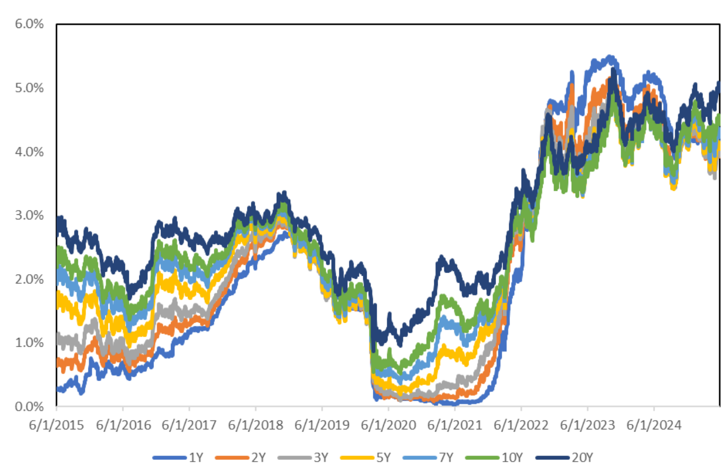
The fourth challenge relates to the credibility of the extended model. If authorities rely on ratings they do not fully endorse, it may raise concerns about the legitimacy and acceptance of haircut decisions based on the model. This introduces an operational risk that must be carefully managed.

In summary, while integrating credit ratings with market-based data offers a conceptually sound approach to capturing sovereign credit risk, it presents several modeling and implementation challenges. It should be noted, however, that this is conceptually distinct from the practice of central banks, which, for example, set haircut schedules according to credit ratings on corporate bonds. Since there should be a credit-risk consistent relationship between credit ratings and credit spreads, the proposed approach does not contradict such practices.

Application to Real-World Data

For illustrative purposes, we apply the DASV model to the U.S. Treasury bond yields. The historical yield curve data spans from June 1, 2005, to May 28, 2025, with daily frequency. Figure 6 presents the time series of yields across seven maturities, ranging from one year to twenty years. The dataset includes several stress periods, such as the COVID-19 crisis and the market turmoil in March 2023, allowing us to benchmark haircuts against historically severe yield shocks.

Figure 6. Historical Data of the US Treasury Bond Yields



Source: Federal Reserve Bank of St. Louis (FRED).

[20] See Broto and Molina (2016) for more detailed discussions on the asymmetric response of sovereign ratings to fundamentals and also the previous studies on the time lag in agencies' response to domestic economic variables.

We estimate EGARCH model for these bond yields. Table 3 reports the estimated parameters. We make two observations. First, the autoregressive coefficient is close to one, indicating that the volatility is mean reverting albeit slowly. Second, volatility of volatility tends to decrease with maturity except 20-year maturity, meaning that longer-term bond yields are less fat tailed. This pattern is also evident in Figure 6.

Table 3. EGARCH Model Parameter Estimates

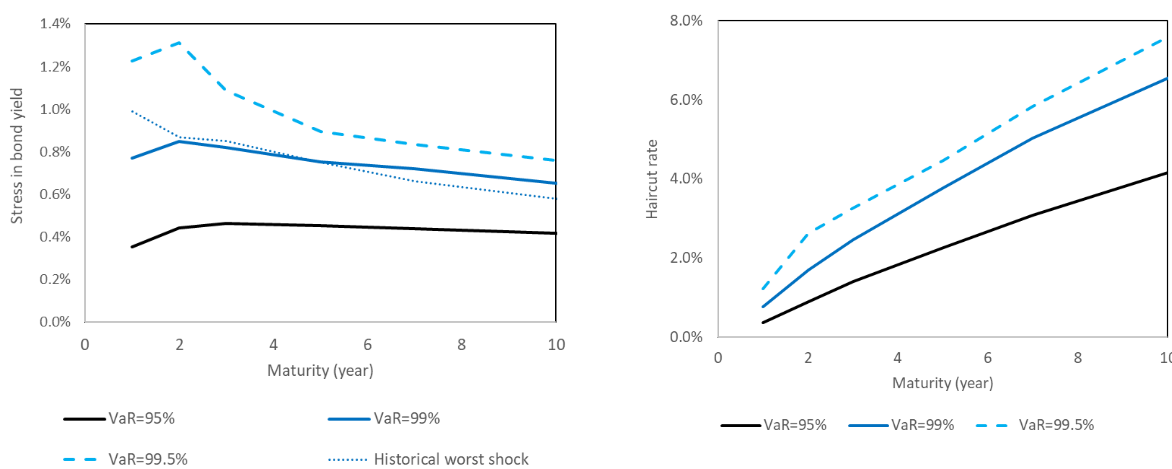
Maturity	1Y	2Y	3Y	5Y	7Y	10Y	20Y
Average vol	0.04%	0.05%	0.06%	0.06%	0.06%	0.05%	0.05%
Autocoefficient	0.98	0.99	0.99	0.99	0.99	0.99	0.98
Vol of vol	0.28	0.20	0.18	0.14	0.11	0.12	0.13
Asymmetry effect	-0.01	-0.01	-0.01	0.00	0.01	0.00	-0.01

Source: Author.

Figure 7-A shows the stressed yield shocks under different VaR percentiles, alongside historical worst shocks. The historical worst shocks are defined as the maximum of 10-day yield changes during the 2015/6-2025/5 period. These historical worst shocks are compared with model-generated stressed shocks under different levels of risk tolerance.

Figure 7. DASV Model with VaR

A. Stressed Yield Shocks **B. Haircuts**



Source: Author.

Figure 7-A indicates that the historical worst shocks are roughly similar to the 99 percentile VaR of the yield shocks based on the estimated EGARCH model. It is recommended to o use 99 percentile or an even higher percentile to ensure that haircuts are sufficiently conservative compared to the historical worst events.

Figure 7-B shows that haircuts increase with duration as longer-term bonds entail a higher duration risk. Also, a higher VaR percentile leads to higher haircuts as a higher VaR percentile makes the central bank more conservative.

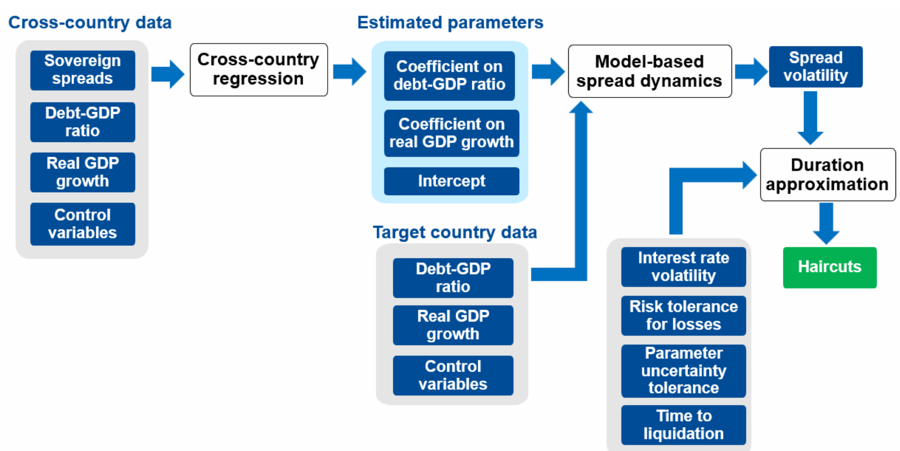
Marketable Assets in a Data-Limited Environment

Overview of DACR Model

In this section, we introduce the Duration Approximation with Cross-country Regression (DACR) model, designed to systematically set haircuts that help central banks mitigate potential losses arising from changes in interest rates and sovereign spreads. The DACR model combines a duration approximation formula with a cross-country regression framework. It addresses key challenges—such as the absence of sovereign credit ratings and secondary markets—by generating hypothetical sovereign spreads.

The DACR model is both simple and modular, enabling conservative loss estimation and future integration of actual market data. Adjustments for parameter uncertainty further enhance its robustness against model risk. Key modeling assumptions include spread and interest rate volatility, time to liquidation, and duration calculation. Methodologically, the model uses J.P. Morgan EMBIG spreads as input data and applies a cross-country regression approach to estimate hypothetical spreads. Figure 8 shows the overview of the DACR model.

Figure 8. Overview of the DACR Model



Source: Author.

Our focus in this section is on a cross-country regression model used to construct hypothetical sovereign spreads because the duration approximation formula was already explained in the previous section.

A key strength of the DACR model is its applicability in data-limited environments, where establishing risk-mitigating measures for government securities is particularly challenging. In many such countries, essential data sources required by standard methodologies are unavailable. These countries often lack sovereign credit ratings from international agencies and do not have a functioning secondary market for government securities. These gaps make it difficult to apply conventional haircut models directly. The DACR model addresses these limitations by offering a practical and adaptable framework for estimating risk in the absence of comprehensive market data.

Constructing Sovereign Spreads with Cross-Country Regression

We develop a cross-country regression model to construct hypothetical sovereign spreads for those countries where neither sovereign credit rating nor the secondary market of sovereign bonds is available. Specifically, we formulate the cross-country regression model where sovereign spreads is a linear function of macro variables. The regression coefficients are estimated using data from countries where both sovereign spreads and macroeconomic indicators are available. Once estimated, the model enables the construction of hypothetical sovereign spreads for data-scarce countries using their available macroeconomic variables.

In the cross-country regression models, key macroeconomic variables are the debt-to-GDP ratio and the real GDP growth. Two specifications are considered. In the first specification, only debt-to-GDP ratio and the real GDP growth are set as regressors as in Equation (19). In the second specification, other macro variables are included as control variables as in Equation (20). Note that $Debt_{i,t}$, $GDP_{i,t}$, and $Z_{i,t}$ are the debt-to-GDP ratio, the real GDP growth, and a vector of the control variables for the i-th country, respectively. $Z_{i,t}$ consists of trade openness, GDP per capita, FX depreciation, and CPI inflation ($Z_{i,t} = \left(Trade_{i,t}, PerGDP_{i,t}, FX_{i,t}, \pi_{\ i,t}\right)'$). The economic intuition is that higher debt-to-GDP ratio and more negative real GDP growth are associated with larger sovereign spreads ($\beta_{Debt,t} > 0$ and $\beta_{GDP,t} < 0$).

$$Spread_{i,t} = \alpha_{i,t} + \beta_{Debt,t} Debt_{i,t} + \beta_{GDP,t} GDP_{i,t} + \epsilon_{i,t} \tag{19}$$

$$Spread_{i,t} = \alpha_{i,t} + \beta_{Debt,t} Debt_{i,t} + \beta_{GDP,t} GDP_{i,t} + \beta_z Z_{i,t} + \epsilon_{i,t} \tag{20}$$

To ensure the stability of the key parameters, the five-year averages of the estimated coefficients are used as inputs of the haircut model. This approach mitigates short-term instability and provides a more reliable basis for constructing hypothetical spreads. The model's robustness is enhanced by this averaging technique. This five-year average method is chosen as a balanced approach to ensure the stability of the key parameters while allowing the estimated cross-country regression model to reflect the most recent global macroeconomic environment. It should be noted, however, that whether a five-year average is optimal depends on the judgement of the model developer.

In addition, adjustments for parameter uncertainty are incorporated to be robust against the model risk of the cross-country regression model. The cross-country model may be subject to misspecification or the parameter estimates errors due to limitations in the number of countries or the length of the time series used in the analysis. These adjustments are essential for maintaining the reliability and credibility of the model under such constraints.

The adjustment for parameter uncertainty makes the model-based sovereign spread changes more dynamically, resulting in a higher spread volatility. Specifically, the coefficient on the debt-GDP ratio could be adjusted to become more positive while the coefficient on the debt-GDP ratio could be adjusted to become more negative. The standard deviations of the key coefficients, σ_{Debt} for the debt-to-GDP ratio and σ_{GDP} for the real GDP growth, are used to account for the uncertainty that the coefficients may not be accurate. In Equations (21) and (22) below, $\overline{\beta_{Debt}}$ and $\overline{\beta_{GDP}}$ are the five-year average of the coefficients on the debt-to-GDP ratio and the real GDP growth, respectively. σ_{Debt} and σ_{GDP} are the standard deviations of the coefficients on the debt-to-GDP ratio and the real GDP growth, respectively.[21] θ is the risk tolerance for the parameter uncertainty which is conceptually distinct from the risk tolerance related to bond price fluctuations α. The coefficients on control variables could also be adjusted similarly to increase the spread volatility in a conservative manner.

$$\beta_{Debt}^{adj} = \overline{\beta_{Debt}} + \Phi^{-1}(\theta) \cdot \sigma_{Debt} \tag{21}$$

$$\beta_{GDP}^{adj} = \overline{\beta_{GDP}} - \Phi^{-1}(\theta) \cdot \sigma_{GDP} \tag{22}$$

The risk tolerance for the parameter uncertainty adjustment needs to be higher if the model is less frequently re-evaluated. In general, the frequency of the updating the haircuts depends on how much new data has been accumulated and the operational cost involved. However, additional consideration needs to be made for the cross-country regression model. As the model is less frequently re-evaluated, there is a higher parameter uncertainty, indicating the risk tolerance parameter needs to be higher.[22]

The estimated cross-country regression model can produce the time series of hypothetical sovereign spreads for any countries if the macro variables used as regressors are available. Suppose that the first specification of the cross-regression model (Equation (19)) is established. In the equation below, $\bar{\alpha}$ is the five-year average of the intercept. β_{Debt}^{adj} and β_{GDP}^{adj} are explained above. Then, by inputting the time series of the debt-to-GDP ratio and the real GDP growth of the data-scarce country, the model-based sovereign spreads are produced.

$$Spread_{target,t} = \bar{\alpha} + \beta_{Debt}^{adj} Debt_{target,t} + \beta_{GDP}^{adj} GDP_{target,t} \tag{23}$$

The model-based sovereign spreads need to be assessed and compared with other relevant indicators if available. As explained above, several modeling assumptions are made for constructing the model-based sovereign spreads because of data-limited environment. Therefore, the user needs to review whether the sovereign spreads are reasonable or not, based on the knowledge of the macroeconomy of the country.

The time-average volatility of the spread changes is computed as a key input for haircut model. Our assumption is to use five-year data to compute the volatility. With the time series of the model-based spreads, we can easily compute the standard deviation of the spread changes over the last five years. It is noteworthy that the intercept $\bar{\alpha}$ does not impact the computation of the spread volatility, which is often highly sensitive to the model specification and thus difficult to rely on.

Similarly to the coefficients of the cross-country model, the spread volatility is computed from the five-year model-based sovereign spreads as a balanced approach to make sure the stability of the key parameter while

[21] These standard deviations are obtained as the outputs of the cross-country regression.

[22] For example, θ can be set equal to $\theta = 1 - \frac{1}{1+N}$ where N means the model is updated once in N years, then annual revaluation ($N = 1$) leads to $\theta = 0.5$, which means no adjustment for parameter uncertainty. If the frequency is 4 years ($N = 4$), then $\theta = 1 - 1/(1 + 4)$=0.8.

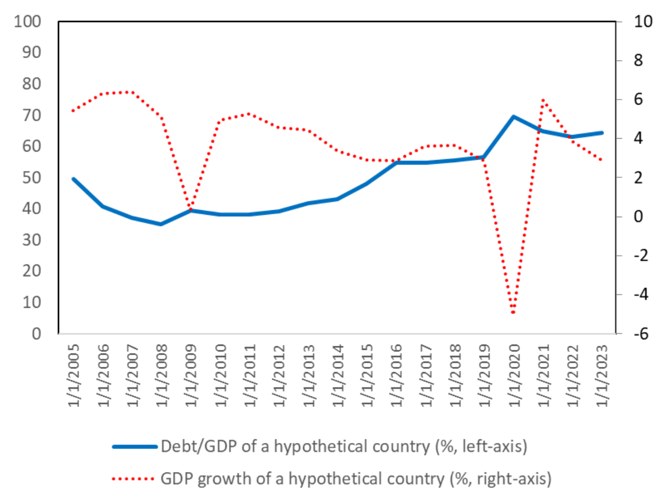
allowing the model-based sovereign spreads to reflect the most recent macroeconomic environment in the country. It is important to note, however, that the choice of a five-year average reflects the judgment of the model developer, and may not necessarily represent an optimal timeframe in all contexts.

It is worth noting that, unless country-specific factors change over time, they do not affect haircuts. This is because what matters is the volatility of the spread, not its level.

Application to a Hypothetical Country

For an illustrative purpose, we apply the DACR model to a hypothetical country where the historical data of macroeconomic variables is artificially constructed. The historical data of debt to GDP ratio and real GDP growth rates are shown in Figure 9.[23] It shows that the debt to GDP ratio has been increasing while the real GDP growth rate dramatically declined in 2008 and 2020.

Figure 9. Applying DACR Model to a Hypothetical Country

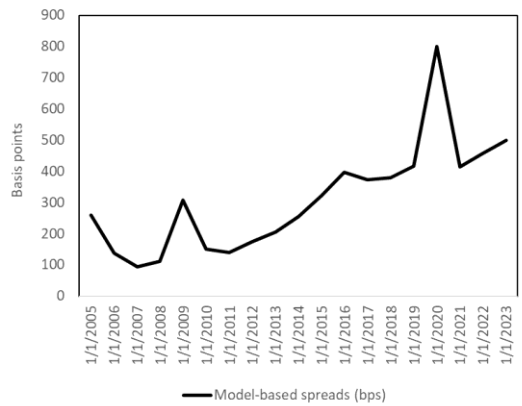


Source: Author.

Model-based sovereign spreads for a hypothetical country are derived from the estimated cross-country regression model. These spreads fluctuated over time, peaking in 2020 primarily due to a sharp contraction in real GDP, as illustrated in Figure 10. Between 2019 and 2023, the average spread volatility was 274 basis points. The most significant change occurred between 2019 and 2020, reaching 384 basis points—approximately the 92nd percentile. It is important to emphasize that haircut levels are driven by the volatility of spreads, not their absolute levels.

[23] To make the hypothetical data realistic, both debt-to-GDP ratio and real GDP growth are constructed by taking the cross-country average of 40 developing and low-income countries.

Figure 10: Model-based Sovereign Spreads



Source: Author.

To illustrate the impact of varying risk tolerance parameters on haircuts, we compute haircuts using the duration-based approximation formula and nonlinear formula with estimated inputs. In this context, the spread volatility is set at 0.0274, as previously discussed, while the risk-free rate volatility is assumed to be 0.01. The time to liquidation is assumed to be one month, represented as 1/12 in annualized terms. This assumption aligns with the Basel regulation, which defines the time to liquidation for illiquid assets as 20 business days.

$$h_\alpha = T \cdot \Phi^{-1}(\alpha) \cdot \left(\sigma_{IR} + \sigma_{Spr}\right) \cdot \sqrt{\Delta t} = T \cdot \Phi^{-1}(\alpha) \cdot (0.01 + 0.0274)\sqrt{1/12} \tag{24}$$

$$h_\alpha^{nonlinear} = 1 - \exp(-T \cdot \Phi^{-1}(\alpha) \cdot (0.01 + 0.0274)\sqrt{1/12}) \tag{25}$$

Table 4 shows haircut schedule under different VaR percentiles applied to spread volatility. We make two observations: First, a higher VaR percentile leads to higher haircuts. Second, duration-based approximation tends to produce higher haircuts compared to the more accurate nonlinear formula. While this reflects the conservative nature of the duration approximation—which can be seen as a strength—it is advisable to use the nonlinear formula in cases where greater accuracy is required, when resulting haircuts are too high.

Table 4. Haircut Schedule under Different VaR Percentiles for Spreads: Duration-based Formula vs. Nonlinear Formula

		Haircuts under selected percentile					
		Duration-based approximation			Nonlinear formula		
Duration range	Assigned duration	90%	95%	99%	90%	95%	99%
(0,0.5]	0.5	0.7%	0.9%	1.3%	0.7%	0.9%	1.2%
(0.5-1]	1	1.4%	1.8%	2.5%	1.4%	1.8%	2.5%
(1-3]	3	4.1%	5.3%	7.5%	4.1%	5.2%	7.3%
(3-5]	5	6.9%	8.9%	12.5%	6.7%	8.5%	11.8%
(5-7]	7	9.7%	12.4%	17.6%	9.2%	11.7%	16.1%
(7-10]	10	13.8%	17.7%	25.1%	12.9%	16.3%	22.2%
>10	20	27.7%	35.5%	50.2%	24.2%	29.9%	39.5%

Source: Author.

While the selection of this percentile ultimately rests with the central bank, it is desirable to adopt a conservative approach by using a VaR percentile that corresponds to, or exceeds, the severity of the worst historical event. The duration range presented serves solely illustrative purposes. The VaR percentile for parameter uncertainty is set at the 50th percentile in the table above.

Non-Marketable Assets in a Data-Rich Environment

Overview of ASRFA Model

We employ Asymptotic Single Risk Factor with Adjustment (ASRFA) model for computing the stressed PD with historical data. The original ARSF model is introduced by Vasicek (1991). Then it was adopted in Internal Rating Based (IRB) approach in Basel II regulation. The theoretical foundation is provided by Gordy (2003). Using the ASRF model, we can obtain stressed PD as VaR of PD or ES of PD given a pre-specified percentile.

The ASRFA model offers two practical advantages. First, it is easy to implement because closed-form formula for PD is available as we discuss below. Second, the ASRF model is simple and there are only three input parameters: PD, risk tolerance (alpha), and correlation parameter. Note that time horizon is determined by the how long it takes for a central bank to liquidate the loan portfolio pledged as collateral. The risk tolerance parameters such as percentile of VaR and adjustment of correlation should be based on policy decision.

There are two main assumptions in the ASRF model: single factor and perfect granularity. The first main assumption is single factor: a single macroeconomic factor drives default risks for loans to all sectors and with all maturities. Yet, it is relatively easy to empirically test this assumption. The second main assumption is perfect granularity: idiosyncratic risk can be fully diversified in the portfolio with an infinite number of small loans. When the perfect granularity does not hold, we need to apply the granularity adjustment, for example, as proposed by Gordy and Lütkebohmert (2013). It is noteworthy that Tarashev and Zhu (2008) reported that violations of these two key modeling assumptions have a limited impact on VaR, especially for large, well-diversified portfolios using Moody's data for US and western European countries.

There are a few more assumptions made in the ASRF model. The third assumption is normal distribution for both the macroeconomic factor and the idiosyncratic factor driving the credit quality. Replacing the normal distribution with more sophisticated distributions such as Student's t distribution is easy. Yet, it requires a larger data to allow users to estimate additional parameters. The fourth assumption is the independence between LGD and PD. In general, LGD tends to be high when PD is high. Hence, ignoring such a positive correlation leads to an underestimation of the haircut. In this case, we need to apply the adjustment to account for non-zero correlation between LGD and PD as proposed in the previous studies such as Barbagli and Vrins (2023).

Description of Original ASRF Model

In this section, we give an intuitive introduction of the ASRF model. Consider the credit quality of the i-th firm X_i is driven by two factors: macroeconomic factor M_t and idiosyncratic factor which is firm-specific.

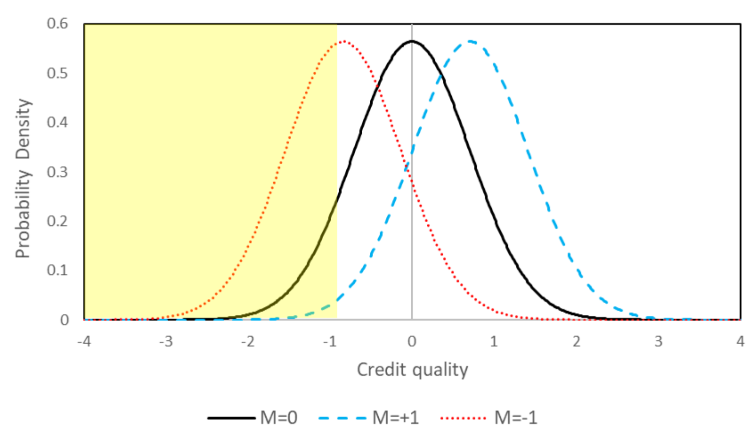
$$X_{i,t} = \sqrt{\rho_i} M_t + \sqrt{1-\rho_i} \epsilon_{i,t} \tag{26}$$

where M and ϵ are independently sampled from the standard normal distribution. ρ_i is correlation parameter which has two interpretations: sensitivity of credit quality to macroeconomy. We assume that the correlation parameter in the same industry means $\rho_i = \rho_j = \rho$. Hence, we obtain means $\rho_i = \rho_j = \rho$.

The default occurs when the total asset value falls below the default level in the ASRF model, which is set equal to zero. The ASRF model is essentially a stylized version of Merton (1974)'s structural credit risk model, which we discuss in the next section. Unlike the Merton model, which models total asset value as a geometric Brownian motion, the ASRF model simplifies asset price dynamics by directly modeling the change in asset value from time t=0 to the maturity.

The probability of the default changes depending on the macroeconomic environment proxied by the single factor. Figure 11 shows how the distribution of the credit quality changes depending on the macroeconomic factor under a hypothetical parameter ($\rho = 0.5$). The firm defaults when its credit quality falls below the default threshold, as highlighted in yellow. When $M_t = 0$, the center of the distribution does not move from the origin. When $M_t = 1$, the distribution is shifted toward right and there is a smaller probability of the default. By contrast, when $M_t = -1$, the distribution is shifted toward left and there is a larger probability of the default.

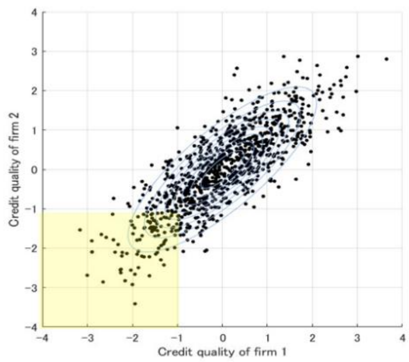
Figure 11. Conditional Distribution under ASRF Model

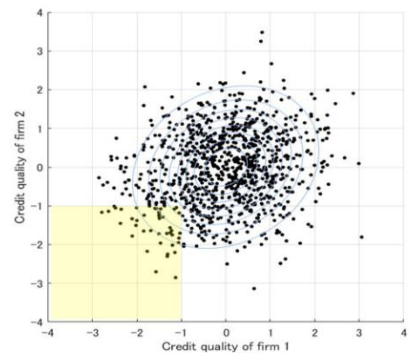


Source: Author.

Figure 12 shows the credit quality of two hypothetical firms under two settings: correlation parameter $\rho = 0.8$ (left) and $\rho = 0.2$ (right). The joint default region is shaded in yellow. The contour of the probability density function is indicated in blue. Comparing these two figures, we can see that there are more joint defaults with a higher correlation. In other words, we can estimate the correlation parameter ρ from the joint defaults among the firms within the same characteristics (e.g., same industry).

Figure 12-a. High Correlation $\rho = 0.8$ **Figure 12b. Low Correlation $\rho = 0.2$**





Source: Author.

The key advantage of the ASRF model is that a closed-form approximation to the density function of losses on a loan portfolio is available. The formula for VaR of the PD is given by

$$PD_{\alpha} = \Phi\left(\frac{\Phi^{-1}(\overline{PD}) - \sqrt{\rho}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right) \tag{27}$$

where Φ is the cumulative distribution function of the standard normal distribution. $\overline{PD}$ is the unconditional PD. α is the percentile for the VaR and represents the risk tolerance (e.g., 0.99). The accuracy of the approximation increases when the number of exposures grows and the largest exposure weight declines to zero. Similarly, the formula for the ES of PD is given by

$$ES_{\alpha} = \frac{1}{1-\alpha} \cdot \Phi_2\left(\Phi^{-1}(\overline{PD}), \Phi^{-1}(1-\alpha), \sqrt{\rho}\right) \tag{28}$$

where Φ_2 is the cumulative distribution function of the bivariate standard normal distribution. For derivation of these two formulae, see Hibbeln (2010).

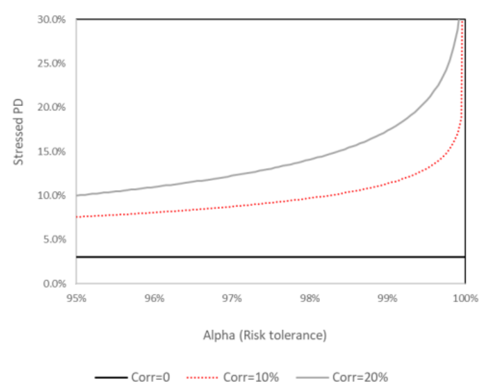
The time horizon of the stressed PD should be consistent with the time-to-liquidation. In the ASRF model, there is no explicit modeling of time dimension of the PD but implicitly modeled in the definition of PD. For example, consider that we use the monthly data of realized default rates as an empirical proxy for the PD, but the time it takes for the central bank to sell the loan portfolio in the market or to surviving bank, after the counterparty default, is three months. Then we need to convert monthly PD to quarterly PD. If there is additional uncertainty in the time to liquidate the collateral, it should be also taken into consideration.

Adjusting Parameters in ASRF Model

It is important to make the ASRF model robust against parameter uncertainty from the model risk management perspective. The ASRF model has four parameters: unconditional PD, unconditional LGD, the percentile, and the correlation. The first two parameters are proxied by the historical averages. The third parameter represents the central bank's risk tolerance and thus should be based on the central bank's decision. What remains is the correlation ρ. Unlike the unconditional PD and the unconditional LGD, the correlation parameter ρ is unobservable and needs to be estimated from realized joint default data or as the sensitivity of firms' credit quality to a systematic factor using time series of the macroeconomic variables and the PD. Therefore, the parameter uncertainty risk is severe for the correlation.

The stressed PD is sensitive to the estimate of the correlation parameter ρ. Figure 13 shows how the alpha-percentile VaR of PD increases with the correlation parameter ρ. The figure is for illustrative purpose and the unconditional PD is assumed to be equal to three percent.

Figure 13. VaR of PD as a Function of the Correlation Parameter



Source: Author.

To make the stress PD robust to the key parameter uncertainty, we may need to adjust the correlation parameter estimate. Specifically, we propose the adjustment below.

$$\rho^* = \hat{\rho} + M_\rho \sigma_\rho \tag{29}$$

where $\hat{\rho}$ is an estimate of the correlation parameter. $\sigma_\rho = std(\hat{\rho}\,)$ is the standard deviation of the correlation parameter which measures the parameter uncertainty.[32] M_ρ is a multiplier to account for the parameter uncertainty. This parameter M_ρ can be set equal to $M_\rho = \Phi^{-1}(\alpha)$ or determined based on the central bank's risk tolerance for the parameter uncertainty. With the parameter uncertainty adjustment for $\hat{\rho}$, the stressed PD is computed with this adjusted correlation ρ^*.

$$PD_\alpha = \Phi\left(\frac{\Phi^{-1}(\overline{PD}) - \sqrt{\rho^*}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho^*}}\right) \tag{30}$$

We may consider the adjustment to make the PD to be a monotonously increasing function of the maturity if the empirical relation is not reasonable due to the data quality issue. The previous empirical studies often find that

[32] One approach to obtain the standard deviation of the correlation parameter is as follows: Randomly select a sufficiently large number of loans from each category (e.g., sector). Estimate the correlation parameter for each subsample. Repeat steps 1 and 2 a large number of times to generate a distribution of correlation estimates. Compute the standard deviation of the resulting correlation estimates.

the default probability increases monotonously with maturity within each sector. If the data for a specific country shows that non-monotonous relationship between the PD and maturity, we may need to consider such a non-monotonous relationship arises due to data limitation. Then, we may apply the monotonicity adjustment. The simplest way is to apply the max operator as defined below.

$$h^*_{m+1} = \max(h_{m+1}, h^*_m) \tag{31}$$

where h_m is the haircut for the m-th maturity bucket. h^*_m is the adjusted haircut.

Additional adjustments may be considered when necessary: First, a granularity adjustment is required when the assumption of perfect granularity does not hold. Second, adjustments to the uncertainty of LGD and PD may be needed when data quality is inadequate. Third, any non-modeled but potentially material risk factors should be reassessed to support further model refinement.

Non-Marketable Assets in a Data-Limited Environment

In this section, we discuss haircut models in situations where data is limited – particularly when realized default rates are not available. Although data limitation can arise in various forms, we categorize and examine three representative cases.

Case I: Balance Sheet Data is Available

When firm-level balance sheet data is available, it becomes possible to construct a proxy for PD.

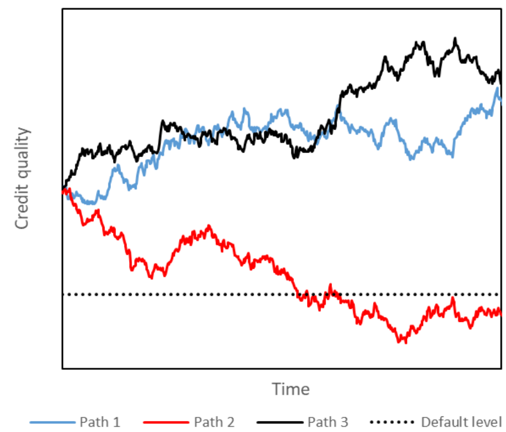
To understand this, consider the structural credit risk model developed by Merton (1974). In Merton (1974) model, the default occurs when the total value of a firm's assets falls below its total liabilities at the maturity of its debt obligations, as illustrated in Figure 14. A key input to the model is the volatility of the firm's total assets or equity. This can be estimated using historical data on asset or equity values. Based on these inputs, the PD can be computed as follows:

$$PD = \Phi\left(\frac{\log\left(\frac{X_0}{D}\right) + (\mu_X - 0.5\sigma_X^2)T}{\sigma_X\sqrt{T}}\right) \tag{32}$$

Where the initial total asset value X_0 and the total liabilities D are observable from the financial statement of the issuer. The asset growth rate μ_X and the asset volatility σ_X are estimated based on the historical data of the balance sheet.

As our goal is to obtain stress PDs, we can take either following two approaches. The first method is to stress parameters by increasing the total asset volatility σ_X and/or decreasing the asset growth rate under certain assumptions. The second method is to use this estimated PD as unconditional PD in ASRF model.

Figure 14. Default in Structural Credit Risk Model



Source: Author.

Note that we can also use the Black and Cox (1976) model where the default occurs any time before the maturity of the debt.

Case II: Balance Sheet Data is Not Available

This case represents the most challenging case. A practical and feasible solution is to employ a cross-country regression model for industry-level PD, as briefly discussed in the previous section. Specifically, we propose the following cross-country model for the sector-level credit spreads $Spread_{i,j,t}$.

In this cross-country sector-level regression model, explanatory variables include the country-level macroeconomic variables $Macro_{i,t}$ and sector-level variables $Sector_{i,j,t}$ as well as control variables $Z_{i,j,t}$.

$$Spread_{i,j,t} = \alpha_{i,t} + \beta_{M,i,t} Macro_{i,t} + \beta_{S,i,j,t} Sector_{i,j,t} + \beta_z Z_{i,j,t} + \epsilon_{i,t} \quad (33)$$

Once the cross-country model is estimated, we can use target country's data and obtain model-based spreads for each sector. Then, we can compute PD given an assumption of LGD.

Case III: Short Sample Periods Missing Crisis Events

When the available data period is too short—such as: (i) when it only includes tranquil periods and exclude any episodes of financial stress; or (ii) when the PD has been increasing but has not yet exhibited any downward trend—it becomes necessary to extrapolate the PD using a quantitative satellite model.[34]

One possible approach to estimate stressed probability of default (PD) in this case is as follows: First, apply a logistic function—or a similar method—to extrapolate the available data. This allows for the estimation of a steady-state PD. Then, adjust the steady-state PD under certain assumptions to derive the stressed PD. As

[34]According to FSB (2015), "the maximum price decline used to derive the applicable haircut should be calculated using a long time series of price data that covers at least one stress period."

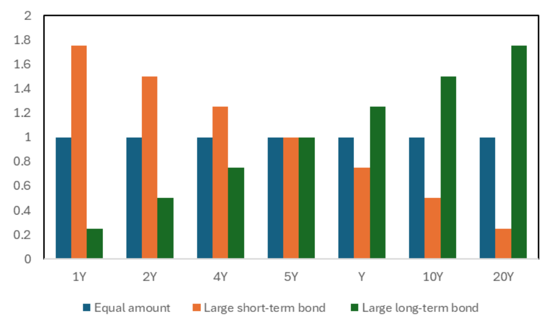
data accumulates over time, and the sample period eventually includes stress periods, the reliance of such satellite models would no longer be required.

Simulating Counterparty Risk Exposure with ML

Three Cases of Portfolios of Bonds Posted as Collateral

Let us consider three compositions of portfolio of bonds posted as collateral: (i) an equal allocation of bonds across maturities; (ii) a larger allocation of short-term bonds; (iii) a larger allocation of long-term bonds. Figure 15 illustrates the distribution of the bond amounts under each of these three different compositions.

Figure 15. Three Hypothetical Compositions of Bond Portfolio



Source: Author.

Given that collateral practices vary from one country to another, the settings above serve as illustrative examples.

Simulating Counterparty Risk Exposure with Nelson-Siegel Model

We conduct Monte Carlo simulation analyses to see the impact of yield curve volatility on the expected uncollateralized exposure of these bond portfolios. We employ a conventional Nelson-Siegel model to simulate yield curves.

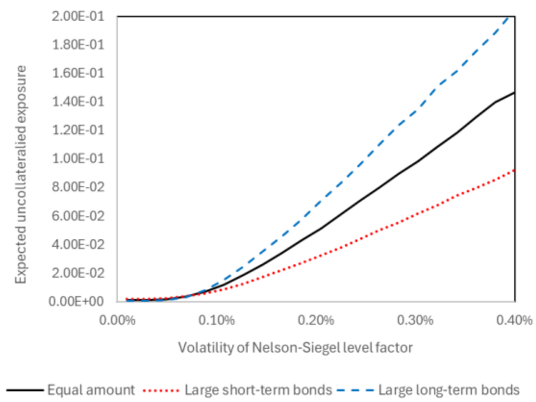
The Nelson-Siegel (NS) model is a widely adopted functional form for interpolating yields across different maturities. Its appeal lies in its parsimony—it effectively captures a variety of yield curve shapes using just three factors and a single parameter. This makes it a flexible yet efficient tool for modeling term structures. The NS model is mathematically expressed as follows.

$$y_{model,t}(\tau,\lambda) = L_t + \frac{1-\exp(-\lambda\tau)}{\lambda\tau} S_t + \left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)\right) C_t, \tag{34}$$

In what follows, we assume that the level and slope factors shocks are generated from the normal distribution and the curvature factor is kept constant. We can easily extend this model to incorporate macro-financial variables so that the NS factors are driven by those macro-financial variables in a similar manner to Diebold et al. (2006).

Figure 16 shows the relationship between the volatility of Nelson-Siegel level factor and the expected uncollateralized exposure. We make two observations. First, the expected exposure increases with the level factor volatility. Second, the expected exposure increases more strongly with the level factor volatility under the case where a larger amount of long-term bonds are posted as collateral. This is not surprising because long-term bonds are more impacted by parallel shifts of yield curves induced by the Nelson-Siegel level factor shocks.

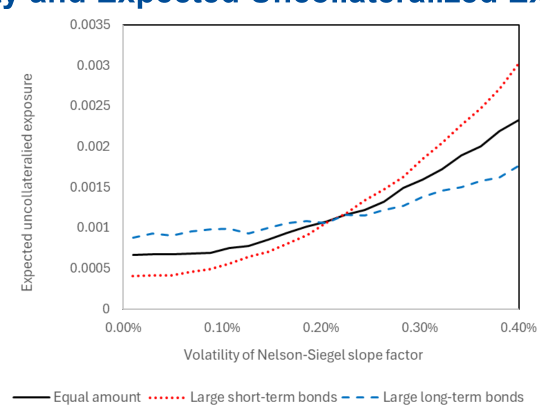
Figure 16. Relation Between Level Factor Volatility and Expected Uncollateralized Exposure



Source: Author.

Figure 17 shows the relationship between the volatility of Nelson-Siegel slope factor and the expected uncollateralized exposure. We make two observations. First, the expected exposure increases with the slope factor volatility. Second, the expected exposure increases more strongly with the slope factor volatility under the case where a larger amount of short-term bonds are posted as collateral. This is because short-term bonds are more impacted by flattening or steepening of yield curves induced by the Nelson-Siegel slope factor shock.

Figure 17. Relation Between Slope Factor Volatility and Expected Uncollateralized Exposure



Source: Author.

Simulate Counterparty Risk Exposure with VAE

We employ a Variational Autoencoder (VAE) to generate stress shocks of the bond yields. VAEs have been successfully used as interpretable generative neural networks across various real-world applications. One key advantage of VAE is their relatively high interpretability compared to other ML techniques.

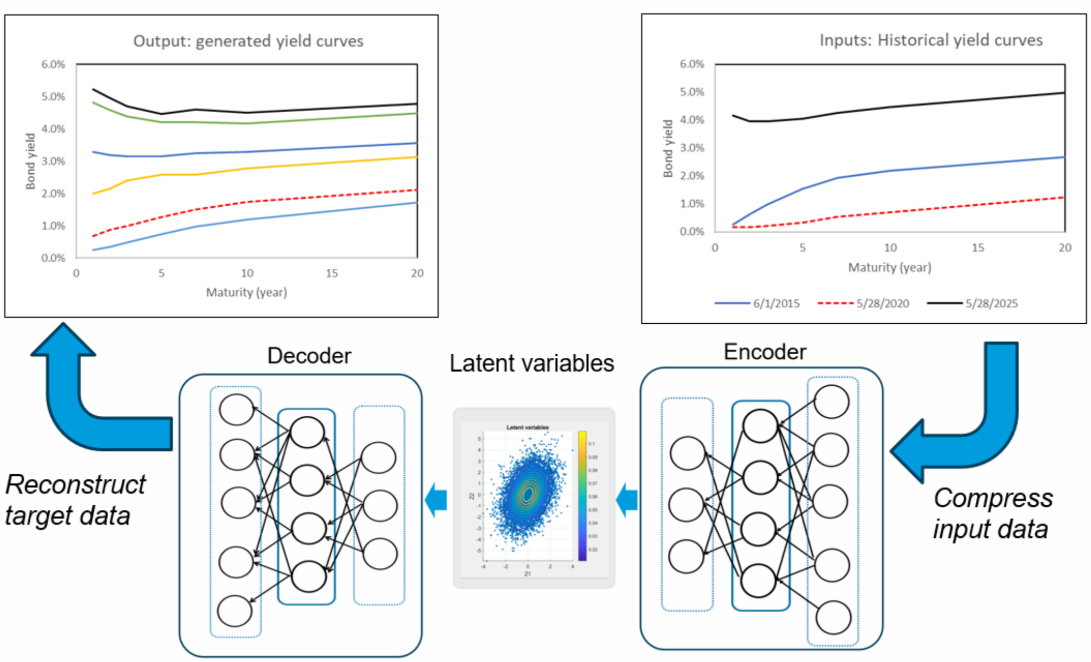
Figure 18 illustrates the architecture of the VAE. The historical yield curves are fed into the Encoder as input. Then Encoder extracts key factors into the latent stochastic variables. These stochastic variables are passed to the Decoder, which re-constructs the yield curves to capture realistic yield curves. Due to the stochastic latent variables, VAEs are well suited for generating simulated yield curves.

$$p(z)\text{: the multivariate normal distribution for the latent variables } z \tag{35}$$

$$p^*(x|z)\text{: The conditional distribution generating yield curve } x \text{ given latent variables } z \tag{36}$$

Roughly speaking, from the point of view of financial econometricians, VAEs can be considered as a nonlinear Principal Component Analysis where common factors are stochastic.

Figure 18. Architecture of Variational Autoencoders



Source: Author.

The parameter values and configuration of the VAE are presented in Table 5.

Table 5. Parameter Values and Configuration of VAE

Description	Value
Number of input features (yield curve maturities)	7
Dimension of the latent space (z_1, z_2)	2
Number of units in the encoder's hidden layer	16
Number of units in the decoder's hidden layer	16
Number of training epochs	100
Number of samples per mini-batch	64
Learning rate (suggested to reduce for stability)	0.001
Standardization method for input data	Z-score ((x - μ)/σ)
Clipping range for logvar to ensure numerical stability	[-10, 10]
Activation function for hidden layers	ReLU
Combination of reconstruction loss (MSE) and KL divergence	MSE + Kullback–Leibler divergence
Output dimension matches input feature count	7
Range used to vary latent variables for interpretability visualization	[-3, 3]

Source: Author.

We apply VAE for the historical data of the US Treasury yield curves and simulate 10000 paths of yield curves. Figures 19-a and 19-b show simulated distribution of one-year and 20-year bond yields, respectively. These figures indicate that VAE can describe non-normal distribution. Note that this is for illustrative purposes. In our empirical applications below, our focus is changes in yield curve, not level of yield curve.

Figure 19-a: Simulated Distribution of One-year Bond

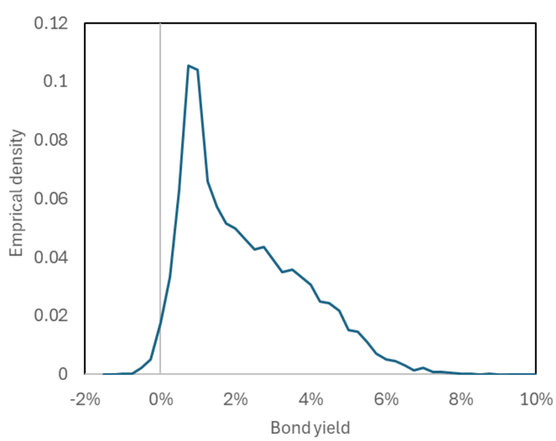
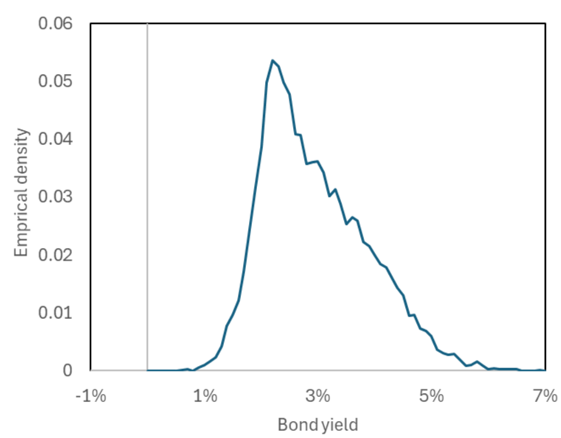


Figure 19-b: Simulated Distribution of 20-year Bond



Source: Author.

As mentioned above, the strength of VAEs is interpretability. Figure 20 shows how each latent factor impacts the shape of yield curve. The first latent variable functions primarily as the level factor but also as the slope factor to a certain extent. The second latent variable functions primarily as the slope factor but also as the curvature factor. Different from the Nelson Siegel model where the factors have distinct roles, level, slope, and curvature, the latent variables in the VAE show roles overlapping roles.

Figure 20-a: The Impact of the First Late Variable on Yield Curve

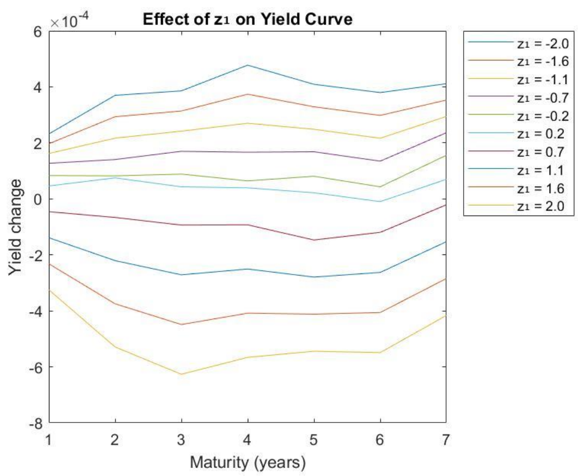
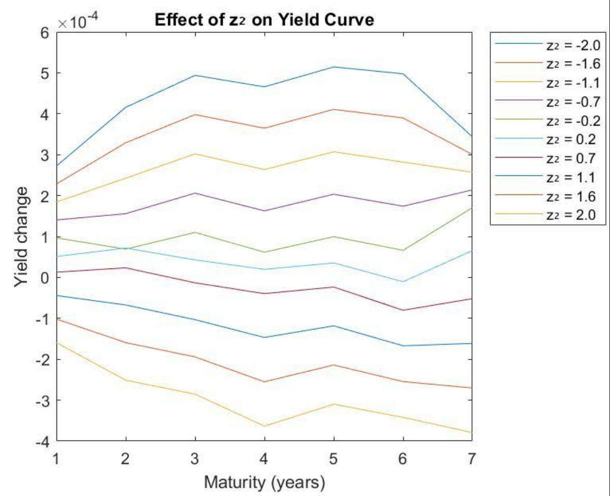


Figure 20-b: The Impact of the Second Late Variable on Yield Curve

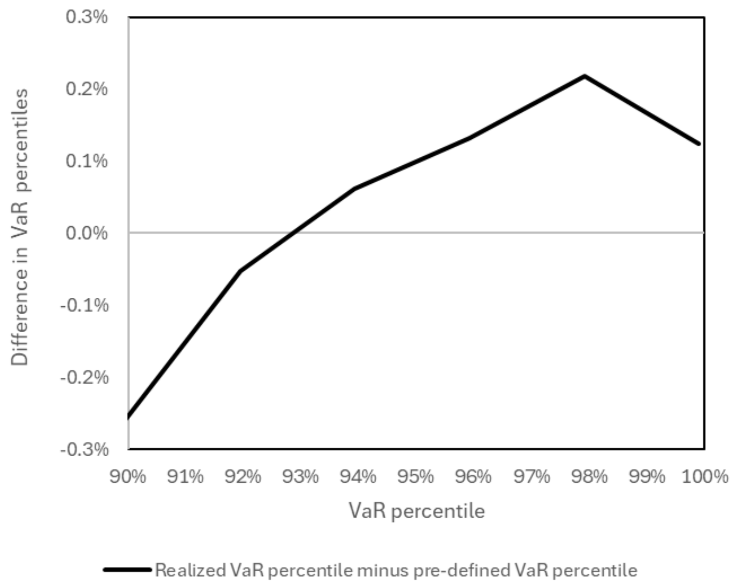


Source: Author.

We compute realized VaR breaches under VAE given each pre-defined VaR percentile used as an input to determine haircuts. Then, we compute the difference between the realized VaR percentile and pre-defined VaR. If the difference is positive, the VaR percentile realized under shocks generated by the VAE is higher than the percentile that would be expected if the haircut model were correct.

Figure 21 shows that The realized VAR percentiles under the VAE are slightly higher beyond 93 percentile, indicating more VaR breaches occur than what is expected and thus the underestimation of the haircuts. Yet, it is noteworthy that the difference is very small.

Figure 21. VaR Percentile for Setting Haircuts and VaR Percentile under VAE-generated Shocks



Source: Author.

The result suggests that the DASV model described above provides sufficiently conservative haircuts to the bonds and protects the central bank from potential losses even under simulated yield curves based on VAE.

In the analysis above, we employed a standard Variational Autoencoder (VAE). However, given that financial time series are known to exhibit fat tails, it can be beneficial to replace the Gaussian distribution typically assumed for the latent variables with a Student's t-distribution. A t-distribution-based VAE was proposed by Takahashi et al., (2018), though its application to haircut stress testing remains an area for future research.

In this section, we showed while using Machine Learning techniques directly for setting may be challenging due to concerns around interpretability and accountability, they can still be useful for model validation purposes. For instance, one possible application is to incorporate the gap in VaR percentile exemplified in the figure above as add-on.

Conclusion

In this paper, we proposed a suite of quantitative models tailored to the nature of eligible assets—marketable or non-marketable—and the availability of relevant data. One of our contributions is to develop a novel model applicable under data-limited environment. These models promote internal consistency, reduce subjectivity, and enhance public accountability. Key principles guiding haircut calibration include non-procyclicality, data-drivenness, conservatism, and the elimination of arbitrage opportunities.

In addition to haircut setting, we emphasize the importance of monitoring residual uncollateralized exposures, which can pose hidden risks during stress periods. We demonstrate how such exposures can be quantified using both traditional financial models and advanced techniques like Machine Learning.

Specifically, we apply Variational Autoencoders (VAEs) to simulate stress scenarios for bond yields and validate haircut models, illustrating their potential to enhance risk assessment.

Overall, this paper provides a robust framework for central banks to strengthen their collateral and counterparty risk management practices, contributing to financial stability and effective liquidity provision.

As a direction for future research, we aim to enhance the cross-country model using AI. Specifically, we plan to write a separate working paper on a “Super Economist” model, which leverages a Large Language Model (LLM) to transform Article IV reports into forward-looking quantitative indicators that can be included as a regressor in the cross-country model for constructing sovereign bond spreads for countries with limited data availability.

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