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Informality and Shock Propagation in an Open Economy

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ABSTRACT: The informal sector accounts for a large fraction of the economy and labor force in many emerging market and developing economies. This paper develops a dynamic stochastic general equilibrium model of a small open economy with an informal sector. Nominal price and wage rigidities are present in the formal sector, while prices and wages are flexible in the informal sector. Production of traded goods rely more on formal inputs (which can be produced at home or imported) while non-traded goods rely more on informal inputs. We show that, despite its costs, the informal sector can provide a flexible margin of adjustment in labor and product markets which helps buffer the impact of domestic and external shocks.

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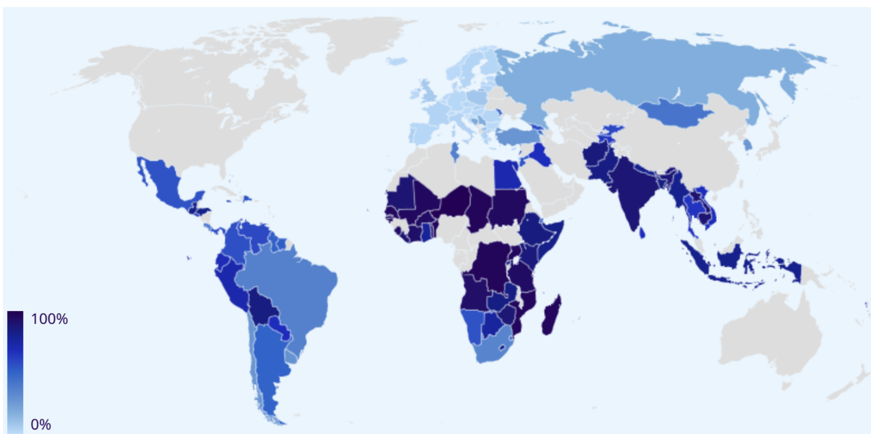
1 Introduction

The informal sector accounts for a large fraction of the economy and labor force in many emerging market and developing economies.[1] According to the International Labour Organization, in several Sub-Saharan African economies such as Niger, Burundi, Chad, and the Democratic Republic of the Congo, informality rates exceed 90%. This pattern is also prevalent across parts of South Asia (e.g., India and Bangladesh) and Latin America (e.g., Bolivia and Guatemala), where rates remain well above 80%. In contrast, high-income economies, particularly in Europe, report informality rates below 5%, with countries like Switzerland, Austria, and Belgium seeing less than 1.5% of their labor force in informal employment.

Figure 1: Informal Employment Rate by Country



Notes: This figure is from the International Labour Organization and presents the informal employment rate by country. Boundaries shown do not imply endorsement or acceptance.

While informality is associated with several structural weaknesses – such as lower productivity, lower growth in equilibrium, a weaker tax base, job and income insecurity, and limited access to social protection (International Monetary Fund, 2021), the informal sector can provide a flexible margin of adjustment in labor and product markets when countries are faced with economic shocks. Informal employment tends to be countercyclical (Fernández and Meza, 2015), expanding during downturns as formal employment contracts and firms reduce their workforce. Additionally, the informal sector demonstrates greater wage and price flexibility in both its labor and product markets (Agudelo and Sala, 2017; Ahmed et al.,

[1] We define informality as economic activity that falls outside the regulated economy and tax system (International Monetary Fund, 2021).

2014), enabling faster adjustments to changing economic conditions.

In this paper, we develop a dynamic stochastic general equilibrium model of a small open economy with an informal sector to analyze the role informality plays in the propagation of domestic and foreign shocks. In our model, nominal price and wage rigidities are present in the formal sector, while prices and wages are flexible in the informal sector. Price stickiness in the goods market comes from Calvo price adjustment for intermediate goods. Similarly, wage stickiness in the formal sector arises from an ex-ante wage bargaining arrangement conducted by unions. Production of traded goods relies more on formal intermediate goods (which can be produced at home or imported) while non-traded goods relies more on informal intermediate goods. The informal sector has lower productivity and is characterized by tax evasion; thus, a larger informal sector results in reduced tax revenue, decreased public investment, and lower overall output.

The model reveals that the informal sector can help buffer the impact of domestic and external shocks. Since formal input prices and wages are sticky and do not immediately adjust downwards in response to a negative shock, production shifts to informal inputs, consumption shifts to non-traded goods, and labor shifts to the informal sector. This results in a smaller impact on output, employment and exports, and lower inflation with informality than without.

We calibrate the model to the Bolivian economy, where the informal sector is large at 85% of employment and 68% of GDP, and simulate the effects of a range of macroeconomic shocks. We compare the responses of the full model—including an informal sector—with those of a counterfactual specification without informality. The results suggest that informality in Bolivia plays a significant shock-absorbing role: it dampens the adverse effects of negative supply-side shocks—such as productivity declines or increases in import prices—on output and inflation. In the case of a positive external demand shock, informality amplifies the gains in output while mitigating inflationary pressures. Moreover, it cushions the contractionary effects on output from a decline in domestic demand, such as a sharp fiscal consolidation. The inverse effects also hold symmetrically in response to opposite shocks.

This paper builds on the economic literature studying informality. Early theoretical work on informality builds on the Harris and Todaro (1970) framework, modeling segmented labor markets with dual wage equilibria. Extensions such as Brueckner et al. (1999) introduce endogenous wage setting through land markets, while more recent studies incorporate search and matching frictions (e.g., Boeri and Garibaldi, 2005; Fugazza and Jacques, 2003; Badaoui et al., 2006), allowing for flows between formal, informal, and unemployed states. These models often emphasize the intersectoral labor margin, particularly for workers (Albrecht et al., 2009; Zenou, 2008; Satchi and Temple, 2009), and analyze how institutions and fiscal policy influence informality. The closest papers to ours are Ospina (2023), Alberola and Urrutia (2020), and Castillo and Montoro (2010) who study the interaction between informality and monetary policy in closed economy models. Our paper focuses on how informality affects the propagation of macroeconomic shocks in an open economy set up. Other papers that explore similar questions abstract from price rigidities and monetary policy (Yépez, 2019; Fiess et al., 2010).

The rest of the paper is organized as follows. Section 2 presents the dynamic stochastic general equilibrium model of a small open economy with informality. Section 3 presents the results. We conclude in Section 4.

2 Model

Our model builds on Gonzalez et al. (2022). There is a continuum of workers who decide to supply their time to the formal or informal labor market. There are tradable good and non-tradable goods, produced by perfectly competitive firms that source a continuum of composite intermediate goods. Domestic intermediate goods are composed of formal and informal inputs from monopolistically competitive domestic suppliers. Price stickiness in the goods market comes from Calvo price adjustment for intermediate goods. Similarly, wage stickiness in the formal sector arises from an ex-ante wage bargaining arrangement conducted by unions, which can result in underemployment.

The government generates revenue through taxes on consumption, labor, and capital, as well as from commodity-related income and transfers from the central bank's quasi-fiscal activities (such as seigniorage). Public spending includes household transfers, public investment, government consumption, and interest payments on sovereign debt. The government can borrow from financial markets, with both the central bank and the private sector participating as buyers of government securities.

The model features key elements of neo-Keynesian economics. First, rigidities in nominal prices and wages allow aggregate demand to influence output and employment, as workers may not be able to supply their desired labor at the current wage. Second, it distinguishes between Ricardian households and those with limited liquidity, "hand-to-mouth" households. Lastly, investment decisions face adjustment costs, which dampen the immediate crowding-out effects of public spending on private investment.

2.1 Households

There are two types of representative households in the economy, indexed by $i = 1, 2$. The first type of representative household ($i = 1$) has access to credit markets and owns physical capital. They use this access and adjust their holdings of physical capital to smooth their consumption. The second type of representative household ($i = 2$) does not have access to credit markets and can only consume each period according to their disposable income. These households maximize their lifetime utility.

Each representative household consists of a continuum of workers, each with a different labor variety j, these labor types are uniformly distributed across household types. Workers can decide to supply their time to the formal labor market, the informal labor market, or both. Time supplied to the formal market results in their labor variety remaining differentiated from other workers' labor varieties (e.g., $N^F_{i,j,t}$). In contrast, time supplied to the informal labor market results in their labor variety being perceived as homogeneous to the variety of other workers in the informal market $N^I_{i,j,t} = N^I_{i,t}$.

Workers in the formal sector pay payroll taxes. The representative household wage W^F_t is the average of the wages of the differentiated labor varieties of the workers in the household (e.g., $W^F_t = \int W^F_{j,t} dj$). The wage of each variety, $W^F_{j,t}$, is set in a non-competitive labor market by a continuum of labor unions indexed by j. In the informal sector, households do not pay payroll taxes, and their wage, W^I_t, is determined competitively.

The total labor effort of the representative households is a composite of their labor effort in the formal labor market, $N^F_{i,j,t} = \int N^F_{j,t} dj$, and in the informal labor market, $N^I_{i,t}$. Work in these two labor markets and their corresponding types of jobs is not fully substitutable:

$$N_{i,t} = \left[\left(\iota^N\right)^{\frac{1}{\rho^N}} \left(N^F_{i,t}\right)^{\frac{\rho^N - 1}{\rho^N}} + \left(1 - \iota^N\right)^{\frac{1}{\rho^N}} \left(N^I_{i,t}\right)^{\frac{\rho^N - 1}{\rho^N}} \right]^{\frac{\rho^N}{\rho^N - 1}}$$

Preferences are common to all households and are represented in each period by the following utility function:

$$u\left(c_t, N_t, \frac{M_t}{P_t}\right) = z_{i,t}^u ln(c_t - \varsigma c_{t-1}) - \gamma_N \frac{1}{1+\sigma_N} N_t^{1+\sigma_N} + \gamma_M \frac{1}{1-\sigma_M}\left(\frac{M_t}{P_t}\right)^{1-\sigma_M}$$

where c_t denotes consumption, N_t labor, and $\frac{M_t}{P_t}$ the stock of real money balances, with P_t denoting the nominal price of consumption goods, which is the numeraire in this economy. The parameters in the utility function are ς (the parameter controlling the importance of the habits in consumption), σ_N (the inverse of the Frisch elasticity), σ_M (the elasticity of money demand), and γ_N and γ_M (the weights in the preferences of labor and real money balances, respectively). z_{it}^u is the preference shock which follows and ARMA process.

The budget constraint for the first type of households is given by

$$\begin{aligned}(1+\tau_t^c)P_t c_{1,t} + P_t^I I_t + M_{1,t} + B_t^h + S_t B_t^{h,*} &= \left[\left(1-\tau_t^k\right) R_t^k + \tau_t^k \delta Q_{t-1}\right] K_{t-1} \\ &+ \left(1-\tau_t^w\right) \int W_{jt}^F N_{1,j,t}^F dj + W_t^I N_{1,t}^I + \frac{\gamma^O}{1-\nu} S_t \bar{O}_t P_t^{O^*} \\ &+ R_{t-1} B_{t-1}^h + S_t R_{t-1}^* B_{t-1}^{h,*} + T_{1,t} + \xi_t + M_{1,t-1}\end{aligned}$$

where P_t^I is the nominal price of investment goods, I_t is investment in physical capital goods, B_t^h is a nominal government bond that pays a risk-free nominal interest rate R_t, $B_t^{h,*}$ is a nominal bond denominated in foreign currency that pays a risk-free nominal interest rate R_t^*, S_t is the nominal exchange rate defined as domestic currency per unit of foreign currency, R_t^k is the nominal return rate on physical capital, Q_t is the nominal price of a unit of installed capital, K_t is the physical capital available at time t, P_t^O is the external nominal price of commodity goods, $\bar{O}_t$ is the flow of commodity exports. τ_t^c,τ_t^w, and τ_t^k are consumption taxes, payroll taxes, and capital income taxes, respectively, $T_{i,t}$ are government transfers in nominal terms, and ξ_t are the firms' profits in nominal terms. δ is a parameter that determines the depreciation rate of physical capital, γ^O is a parameter that governs the share of revenues coming from the commodity export sector that households receive, and $\frac{\gamma^O}{1-\nu}$ is the per-capita share of the revenues coming from the commodity export sector for type $i = 1$.

Rapid changes in investment are costly, and this cost is given by the following function:

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{a}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

Considering the adjustment costs of investment, capital evolves according to the following law of motion:

$$K_t = (1-\delta) K_{t-1} + z_t^k I_t \left(1 - f\left(\frac{I_t}{I_{t-1}}\right)\right)$$

where z_t^k is an investment-specific exogenous shock that follows an ARMA process.

The budget constraint for the second type of households is given by:

$$(1+\tau_t^c) P_t c_{2,t} + M_{2,t} = (1-\tau_t^w) \int W_{j,t}^F N_{2,j,t}^F dj + W_t^I N_{2,t}^I + T_{2,t} + M_{2,t-1}$$

2.2 Labor Markets and Wage Setting

The formal labor market is dominated by a continuum of labor unions, which make formal labor supply decisions, $N_{i,j,t}^F$, on behalf of the households and set the nominal formal wage, $W_{j,t}^F$. Due to its structure, the formal labor market is characterized by nominal wage rigidity.

Once the labor supply to the formal labor market is determined by the unions, workers in the representative households decide how much of their remaining time to supply to the informal labor market, $N_{i,t}^I$, where they take the informal nominal wage, W_t^I, as given, and which is determined competitively.

Workers from both types of households ($i = 1, 2$) are randomly assigned to the unions, ensuring that all unions have an identical representation of workers from both households types as in the overall population.

Labor unions maximize their profits, subject to two constraints. The first is a resource constraint:

$$\overline{N} \geq N_{j,t}^F = \Omega \int N_{j,1,t}^F dj + (1-\Omega) \int N_{j,2,t}^F dj$$

where $\overline{N}$ corresponds to the maximum available time to dedicate to work for any labor type, Ω is the share in the population of households of type $i = 1$. The constraint limits the total available labor of union j. The second constraint is the demand for formal labor for workers in the union j from the firms:

$$N_{j,t}^F = \left(\frac{W_{j,t}^F}{W_t^F}\right)^{-\epsilon^w} N_t^{F,d}$$

where ϵ^w denotes the elasticity of substitution across formal labor types represented by each union, $N_t^{F,d}$ is the aggregate demand for formal labor, $W_{j,t}^F$ is the nominal formal wage at time t for workers in union j, and W_t^F is the aggregate nominal wage for formal jobs.

Every union sets its wage according to Calvo's staggered wages setting. In each period, there is a probability θ^w that the union will not be able to set a new wage. Consequently, wages are only updated in periods when the union receives a random signal, which occurs with a probability $1-\theta^w$. When the union can adjust its wage, it does so by maximizing the average lifetime utility of the representative households, which will depend not only on the marginal utility of wealth of the workers, but also on their labor supply in the informal labor market. This optimization is subject to the two constraints:

$$\max_{W_{j,t}^F} \mathbb{E}_t \sum_{s=0}^{\infty} (\theta^w \beta)^s \left\{ (1-\nu)\, u\left(c_{1,t+s}, N_{1,j,t+s}^F, m_{1,t+s}\right) + \nu u\left(c_{2,t+s}, N_{2,j,t+s}^F, m_{2,t+s}\right) \right\}$$

The optimization by the unions correspond to the optimization for labor type j of a weighted average of the lifetime utility of labor type j as part of both types households, $i = 1, 2$, as labor types are uniformly distributed across household types. Therefore, the demand for labor of type j is spread uniformly across households. When unions are not able to adjust wages optimally, they adjust them accordingly to the indexation rule:

$$W_{j,t}^F = W_{j,t-1}^F g^z \pi_{t-1}^{\chi^w} \bar{\pi}^{(1-\chi^w)}$$

where π_{t-1} denotes the consumer price inflation of the previous period, and $\bar{\pi}$ is the inflation target. The indexation rule implies that nominal wages are indexed to a weighted average of past inflation and the inflation target and long-run productivity growth g^z. χ^w is the wage indexation parameter. If $\chi^w = 1$, there is full indexation to past inflation.

The unions' optimization problem can be written (in real terms) as maximizing the lifetime utility of its union members, $\mathcal{U}$ with respect to $w_{j,t}^F$, and subject to their aggregate budget constraint.

The union j's period utility is given by:

$$\mathcal{U}_{|} = (1-\nu)\, u\left(c_{j,1,t+s}, N_{1,t+s}, \frac{M_{1,t+s}}{P_{t+s}}\right) + \nu u\left(c_{j,2,t+s}, N_{2,t+s}, \frac{M_{2,t+s}}{P_{t+s}}\right)$$

2.3 Final Sector Firms

There are different types of firms in the economy. Firms producing final goods and services, firms producing investment goods, and monopolistic competitive firms producing intermediate goods.

2.3.1 Producers of Final Consumption Goods

These firms are competitive and produce final consumption goods c_t combining final tradable c_t^T and non tradable goods c_t^{NT} according to the following technology:

$$c_t = \left[(\iota^c)^{\frac{1}{\rho^c}} \left(c_t^T \right)^{\frac{\rho^c - 1}{\rho^c}} + (1 - \iota^c)^{\frac{1}{\rho^c}} \left(c_t^{NT} \right)^{\frac{\rho^c - 1}{\rho^c}} \right]^{\frac{\rho^c}{\rho^c - 1}}$$

where ρ^c is the elasticity of substitution between tradable and non tradable goods, and ι^c is the share of tradable goods in the production technology of final consumption goods. These firms take as given the nominal prices of their goods, and the tradable and non tradable goods, P_t, P_t^T and P_t^{NT}, respectively, and minimize their production cost, subject to the previous production technology. Their cost function is given by

$$P_t c_t = P_t^T c_t^T + P_t^{NT} c_t^{NT}$$

Prices and inflation are given by

$$P_t = \left[\iota^c \left(P_t^T \right)^{1-\rho^c} + (1 - \iota^c) \left(P_t^{NT} \right)^{1-\rho^c} \right]^{\frac{1}{1-\rho^c}}$$

$$\pi_t = \left[\iota^c \left(\pi_t^T p_{t-1}^T \right)^{1-\rho^c} + (1 - \iota^c) \left(\pi_t^{NT} p_{t-1}^{NT} \right)^{1-\rho^c} \right]^{\frac{1}{1-\rho^c}}$$

where π_t^T and π_t^{NT} are the inflation rates in the tradable and non tradable sectors and p_t^T and p_t^{NT} are relative prices of the tradable and non tradable goods in terms of consumption goods.

2.3.2 Producers of Composite Tradable and Non-Tradable Consumption Goods

Tradable and non-tradable consumption goods are also composite goods, produced by competitive firms taking as given their prices and the prices of their respective inputs.

Tradable goods, c_t^T, are produced combining exportable, c_t^X, and importable goods, c_t^M according to the following technology.

$$c_t^T = \left[\left(\iota^T\right)^{\frac{1}{\rho^T}} \left(c_t^X\right)^{\frac{\rho^T-1}{\rho^T}} + \left(1-\iota^T\right)^{\frac{1}{\rho^T}} \left(c_t^M\right)^{\frac{\rho^T-1}{\rho^T}} \right]^{\frac{\rho^T}{\rho^T-1}}$$

where ρ^T is the elasticity of substitution between exportable and importable goods, and ι^T is the share of exportable goods in the production technology of the tradable consumption goods.

Non-tradable composite goods, c_t^{NT}, are produced competitively by aggregating S types of services, $c_t^{S_i}$ for $i = 1, \ldots, S$, according to the following production function:

$$c_t^{NT} = \left(\sum_{i=1}^{S} \left(\iota_i^{NT}\right)^{\frac{1}{\rho^{NT}}} \left(c_t^{S_i}\right)^{\frac{\rho^{NT}-1}{\rho^{NT}}} \right)^{\frac{\rho^{NT}}{\rho^{NT}-1}}$$

where ρ^{NT} is the elasticity of substitution between the different types of services, and ι_i^{NT} is the share parameter for service type i in the production of the non-tradable consumption good, with $\sum_{i=1}^{N} \iota_i^{NT} = 1$.

The producers minimize their cost functions, which are given by:

$$P_t^T c_t^T = P_t^X c_t^X + P_t^M c_t^M$$

$$P_t^{NT} c_t^{NT} = \sum_i P_t^{S_i} c_t^{S_i}$$

P_t^X is the price of exportable goods, and P_t^M is the price of the importable goods. While exportable goods are produced domestically for domestic and the foreign good markets, importable goods are bought from producers abroad. Services goods are produced domestically for the domestic market only. Producers of non-tradable goods take the price of their goods P_t^{NT} and the prices of their inputs ($P_t^{S_i}$) as given.

The price indexes are given by

$$P_t^T = \left[\iota^T \left(P_t^X\right)^{1-\rho^T} + \left(1-\iota^T\right)\left(P_t^M\right)^{1-\rho^T}\right]^{\frac{1}{1-\rho^T}}$$

$$P_t^{NT} = \left(\sum_{i=1}^{N} \iota_i^{NT} \left(P_t^{S_i}\right)^{1-\rho^{NT}}\right)^{\frac{1}{1-\rho^{NT}}}$$

This can be rearranged and written as

$$\pi_t^T = \left[\iota^T \left(\pi_t^X \frac{p_{t-1}^X}{p_{t-1}^T}\right)^{1-\rho^T} + \left(1-\iota^T\right)\left(\pi_t^M \frac{p_{t-1}^M}{p_{t-1}^T}\right)^{1-\rho^T}\right]^{\frac{1}{1-\rho^T}}$$

$$\pi_t^{NT} = \left(\sum_{i=1}^{N} \iota_i^{NT} \left(\pi_t^{S_i} \frac{p_{t-1}^{S_i}}{p_{t-1}^{NT}}\right)^{1-\rho^{NT}}\right)^{\frac{1}{1-\rho^{NT}}}$$

where π_t^X, π_t^M, and $\pi_t^{S_i}$ are the inflation rates in the exportable, importable, and services sectors and p_t^X, p_t^M, and $p_t^{S_i}$ are the corresponding relative prices.

2.3.3 Producers of Investment Goods

Investment goods, I_t, are also composite goods. They are produced combining exportable, I_t^X, and imported goods, I_t^M according to the following technology.

$$I_t = \left[\left(\iota^I\right)^{\frac{1}{\rho^I}} \left(I_t^X\right)^{\frac{\rho^I-1}{\rho^I}} + \left(1-\iota^I\right)^{\frac{1}{\rho^I}} \left(I_t^M\right)^{\frac{\rho^I-1}{\rho^I}}\right]^{\frac{\rho^I}{\rho^I-1}}$$

where ρ^I is the elasticity of substitution between exportable goods I_t^X and importable goods I_t^M, and ι^I is the share of the exportable in the production technology of the investment goods.

The producers of composite investment goods are also competitive firms that take as given the price of exportable P_t^X and imported goods P_t^M, when minimizing their cost function, which is given by:

$$P_t^I I_t = P_t^X I_t^X + P_t^M I_t^M$$

Prices and inflation is given by

$$P_t^I = \left[\iota^I \left(P_t^X\right)^{1-\rho^I} + \left(1-\iota^I\right)\left(P_t^M\right)^{1-\rho^I}\right]^{\frac{1}{1-\rho^I}}$$

$$\pi_t^I = \left[\iota^I \left(\pi_t^X \frac{p_{t-1}^X}{p_{t-1}^I}\right)^{1-\rho^I} + \left(1-\iota^I\right)\left(\pi_t^M \frac{p_{t-1}^M}{p_{t-1}^I}\right)^{1-\rho^I}\right]^{\frac{1}{1-\rho^I}}$$

where π_t^I is the inflation rate of the investment goods and p_t^I the relative price of the investment goods.

2.3.4 Producers of Final Goods and Services

Final goods and services Y_t^a for $a \in \{X, S_i\}$ are produced by competitive firms combining formal $Y_t^{a,F}$ and informal composite intermediate inputs $Y_t^{a,I}$, according to the following production technology:

$$Y_t^a = \left((\kappa^a)^{\frac{1}{\varepsilon^a}} \left(Y_t^{a,F}\right)^{\frac{\varepsilon^a-1}{\varepsilon^a}} + (1-\kappa^a)^{\frac{1}{\varepsilon^a}} \left(Y_t^{a,I}\right)^{\frac{\varepsilon^a-1}{\varepsilon^a}}\right)^{\frac{\varepsilon^a}{\varepsilon^a-1}}$$

where $Y_t^{a,F}$ and $Y_t^{a,I}$ are composite intermediate goods that combine a continuum of intermediate goods of two different types, formal and informal inputs, respectively. These intermediate goods produced by intermediate firms are indexed by (a, l^F) and (a, l^I):

$$Y_t^{a,F} = \left(\int_0^1 \left(Y_t^{a,l^F}\right)^{\frac{\epsilon^{a,F}-1}{\epsilon^{a,F}}} dl^{a,F}\right)^{\frac{\epsilon^{a,F}}{\epsilon^{a,F}-1}}$$

$$Y_t^{a,I} = \left(\int_0^1 \left(Y_t^{a,l^I}\right)^{\frac{\epsilon^{a,I}-1}{\epsilon^{a,I}}} dl^{a,F}\right)^{\frac{\epsilon^{a,I}}{\epsilon^{a,I}-1}}$$

ε^a is the elasticity of substitution in sector a between composite intermediate goods $Y_t^{a,F}$ and $Y_t^{a,I}$, which are produced by formal and informal intermediate firms respectively, $\epsilon^{a,F}$ is the elasticity of substitution between intermediate goods Y_t^{a,l^F} produced by formal firms, and $\epsilon^{a,I}$ is the elasticity of substitution between intermediate goods Y_t^{a,l^I} produced by informal firms. Producers take the prices of their inputs as given and choose the quantities of intermediate goods that minimize their costs.

$$P_t^a Y_t^a = \int_0^1 P_t^{a,l^F} Y_t^{a,l^F} dl^{a,F} + \int_0^1 P_t^{a,l^I} Y_t^{a,l^I} dl^{a,I}$$

The price index is given by

$$P_t^a = \left[\kappa^a \left(P_t^{a,F}\right)^{1-\varepsilon^a} + (1-\kappa^a)\left(P_t^{a,I}\right)^{1-\varepsilon^a}\right]^{\frac{1}{1-\varepsilon^a}}$$

which implies

$$\pi_t^a = \left[\kappa^a \left(\pi_t^{a,F} \frac{p_{t-1}^{a,F}}{p_{t-1}^a}\right)^{1-\varepsilon^a} + (1-\kappa^a)\left(\pi_t^{a,I} \frac{p_{t-1}^{a,I}}{p_{t-1}^a}\right)^{1-\varepsilon^a}\right]^{\frac{1}{1-\varepsilon^a}}$$

where $\pi_t^{a,F}$ and $\pi_t^{a,I}$ are the inflation rates in the formal and informal intermediate goods sectors $Y_t^{a,F}$ and $Y_t^{a,I}$ that are used for the production of the final good or service Y_t^a. In turn, the demand for intermediate varieties (a, l^F) and (a, l^I) are given by

$$\begin{aligned} Y_t^{a,l^F} &= \left(\frac{p_t^{a,l^F}}{p_t^{a,F}}\right)^{-\epsilon^{a,l^F}} Y_t^{a,F} \\ Y_t^{a,l^I} &= \left(\frac{p_t^{a,l^I}}{p_t^{a,I}}\right)^{-\epsilon^{a,l^I}} Y_t^{a,I} \end{aligned}$$

where $p_t^{a,l^F} = \frac{P_t^{a,l^F}}{P_t^c}$, and $p_t^{a,l^I} = \frac{P_t^{a,l^I}}{P_t^c}$. The price indexes for the composite intermediate goods in the sector are given by:

$$\begin{aligned} P_t^{a,F} &= \left(\int_0^1 \left(P_t^{a,l^F}\right)^{1-\epsilon^{a,F}} dl^{a,F}\right)^{\frac{1}{1-\epsilon^{a,F}}} \\ P_t^{a,I} &= \left(\int_0^1 \left(P_t^{a,l^I}\right)^{1-\epsilon^{a,I}} dl^{a,I}\right)^{\frac{1}{1-\epsilon^{a,I}}} \end{aligned}$$

Demand for the final goods and services are given by

$$Y_t^X = c_t^X + I_t^X + G_t^X + I_t^{g,X} + c_t^{X^*}$$

$$Y_t^{S_i} = c_t^{S_i} + G_t^{S_i}$$

where $c_t^{X^*}$ is the foreign demand for domestic output (exports).

2.4 Intermediate Sector Firms

Intermediate goods $Y_t^{a,l^{ii}}$ for $a \in \{X, S_i\}$ and $ii = F, I$ are produced by a continuum of monopolistic firms. The firms are indexed by l^F, in the case of formal intermediate goods firms, and l^I in the case of informal intermediate goods firms. These firms face the demand from the final firms in the sectors $a \in \{X, S_i\}$, and $ii = F, I$ below

$$Y_t^{a,l^{ii}} = \left(\frac{p_t^{a,l^{ii}}}{p_t^{a,ii}} \right)^{-\epsilon^{a,l^{ii}}} Y_t^{a,ii}$$

Intermediate sector firms use capital and labor to produce $Y_t^{a,l^{ii}}$. There are a couple of important differences between formal and informal firms: Formal producers only use labor hired formally from the unions; informal firms hired their labor in the informal labor market, and if they are caught doing so, they face a penalty imposed by the government. Additionally, total factor productivity in the informal sector is a fraction ϑ, such that $0 \leq \vartheta \leq 1$ of the total factor productivity in the formal sector.

The production function of the intermediate firms is given by

$$Y_t^{a,l^{ii}} = z_t^{a,ii} \left(Z_t^{a,ii}\right)^{1-\alpha^{a,ii}-\alpha^{a,g,ii}} \left(K_{t-1}^{a,l^{ii}}\right)^{\alpha^{a,ii}} \left(N_t^{a,l^{ii}}\right)^{1-\alpha^{a,ii}} \left(K_{t-1}^{g}\right)^{\alpha^{a,g,ii}}$$

where $\alpha^{a,ii}$ is the capital share of output in the intermediate goods firms producing for sector $a \in \{X, S_i\}$ respectively, $Z_t^{a,ii}$ is a permanent productivity shock such that

$$\frac{Z_{t+1}^{a,ii}}{Z_t^{a,ii}} = g_t^Z$$
$$g_t^Z = \left(1-\varrho^{g^Z}\right)\bar{g}^Z + \varrho^{g^Z} g_{t-1}^Z + \varepsilon_t^Z$$

where $z_t^{a,ii}$ is a transitory productivity shock, g_t^Z is a transitory shock to the growth rate of productivity, and

$$Z_t^{a,I} = \vartheta Z_t^{a,F}$$

K_{t-1}^g is public capital. Note that each intermediate sector firm $l^{a,ii}$, $ii = F, I$ has access to the same public capital stock, which grows along the balanced growth path.

2.4.1 Cost Minimization Problem

The main aspect that differentiates the firms corresponds to the labor input that they use. As in Schmitt-Grohe and Uribe (2007), we assume that the labor input used by firm $l^{a,ii}$ is a composite made of a continuum of differentiated labor services. This differentiation in labor services implies that the labor input of formal intermediate firms, N_t^{a,l^F}, is provided as follows

$$N_t^{a,l^F} = \left(\int_0^1 \left(N_{j,t}^{a,l^F}\right)^{\frac{\epsilon^w-1}{\epsilon^w}} dj^F\right)^{\frac{\epsilon^w}{\epsilon^w-1}}$$

However, while formal workers are able to provide differentiated labor services, workers hired informally are not able to differentiate their labor varieties from that of other workers, resulting in perfect substitution among informally provided labor services. This perfect substitutability of labor hired informally implies that the labor input of informal intermediate firms, N_t^{a,l^I}, is provided as follows

$$N_t^{a,l^I} = \int_0^1 N_{j,t}^{a,l^I} dj^I$$

Formal and informal intermediate firms select the optimal combination of labor varieties to obtain their labor input by minimizing their labor costs, giving the technology defining their composite labor input.

Specifically, their cost minimization problem to select labor varieties is given by :

$$\min_{N_{j,t}^{a,l^{ii}}} \quad W_t^{ii} N_t^{a,l^{ii}} = \int_0^1 W_{j,t}^{ii} N_{j,t}^{a,l^{ii}} dj^{ii}$$

subject to the definition of the composite labor input for firm $l^{a,ii}$.

The solution to the formal sector firms cost minimization problem aimed to select their labor input implies that the optimal demand for labor services j by firm $l^{a,F}$ is

$$N_{j,t}^{a,l^F} = \left(\frac{w_{j,t}^F}{w_t^F} \right)^{-\epsilon^w} N_t^{a,l^F}$$

where w_t^F is the real wage index (notice that formal intermediate firms in all sectors combine labor varieties in the same way to create one unit of the composite labor input N_t^{a,l^F}, and therefore the nominal wage index is identical across formal intermediate firms in all sectors).

$$w_t^F = \left(\int_0^1 \left(w_{j,t}^F\right)^{1-\epsilon^w} dj^F \right)^{\frac{1}{1-\epsilon^w}}$$

The total demand for labor services j in sector a is

$$\begin{aligned} N_{j,t}^{a,F} &= \int_0^1 N_{j,t}^{a,l^F} dl^{a,F} \\ &= \int_0^1 \left(\frac{w_{j,t}^F}{w_t^F} \right)^{-\epsilon^w} N_t^{a,l^F} dl^{a,F} \\ &= \left(\frac{w_{j,t}^F}{w_t^F} \right)^{-\epsilon^w} \int_0^1 N_t^{a,l^F} dl^{a,F} \\ &= \left(\frac{w_{j,t}^F}{w_t^F} \right)^{-\epsilon^w} N_t^{a,F} \end{aligned}$$

and the total demand for labor services j by all intermediate firms hiring in the formal labor sector (which is the demand for services j faced by the households in the economy) is given by:

$$N_t^{F,d} = N_t^{X,F} + \sum_i^S N_t^{S_i,F}$$

The solution to the informal sector firms cost minimization problem aimed to select their labor input implies that optimal demand for labor services j by firm $l^{a,I}$, $N^{a,l^I}_{j,t}$, is such that $w^I_{j,t} = w^I_t$. Since the wages are the same, the supply of labor to the informal sector a firms is also the same for all types j, therefore

$$N^{a,l^I}_{j,t} = \int_0^1 N^{a,l^I}_{j,t} dj^I = N^{a,I}_t$$

Total demand for informal labor (the one faced by the households) is defined as:

$$N^{I,d}_t = N^{X,I}_t + \sum_i^S N^{S_i,I}_t$$

Once the firms have chosen the composition of their labor inputs, they need to determine their demand for labor and capital. The firms choose $N^{a,l^{ii}}_t$ and $K^{a,l^{ii}}_{t-1}$ to minimize their costs $Co^{a,l^{ii}}_t$

$$\min_{N^{a,l^{ii}}_t, K^{a,l^{ii}}_{t-1}} \quad cost^{a,l^{ii}}_t = mc^{a,l^{ii}}_t y^{a,l^{ii}}_t = w^{ii}_t N^{a,l^{ii}}_t + r^k_t K^{a,l^{ii}}_{t-1} \left(1 + f\left(\frac{K^{a,ii}_{t-1}}{N^{a,ii}_t} - \frac{K^{a,ii}_{t-2}}{N^{a,ii}_{t-1}}\right)\right)$$

subject to their production technology.

Intermediate firms are subject to adjustments costs in their capital demand, which depend on the evolution of per-worker capital in their sector (e.g. $\frac{K^{a,ii}_{t-1}}{N^{a,ii}_t}$). The firms' adjustment cost function represents costs for firms $l^{a,F}$ to increase their demand for capital if the capital per-worker level in their sector is increasing rapidly through time.

2.4.2 Firms Optimization

Intermediate firms produce differentiated intermediate goods, and therefore are able to choose the price of their good, $P^{a,l^{ii}}_t$, to maximize their profits, but these firms cannot optimally update their price in every period, instead in each period a share $(1-\theta^{ii})$ of the firms will adjust its prices optimally, while the remaining share θ^{ii} will adjust their prices following a simple indexation rule given by:

$$P^{a,l^{ii}}_t = P^{a,l^{ii}}_{t-1} \pi^{lii}_{t-1} \overline{\pi}^{(1-lii)}$$

where lii is a parameter that controls the degree of price indexation in the formal and informal sectors. If $lii = 1$, there is full indexation to past inflation, if $lii = 0$, price changes follow the inflation target.

Consequently, the firms will choose their price knowing that with a positive probability, this price might not be optimally adjusted in the near future. Because the informal sector firms are producing hiring labor informally, and workers don't report their working income and therefore are able to avoid income taxes, once an informal firm produces Y_t^{a,l^I}, it faces a probability Ξ^I of being caught breaking labor rules, and upon that finding a share ψ^I of its revenues is confiscated. The probability of being caught is the same for all sectors a. In nominal terms, the profits of the intermediate firms in period t are $\Pi_t^{a,l^{ii}}$, $ii = F, I$ which are given by:

$$\begin{aligned}
\Pi_t^{a,l^F} &= P_t^{a,l^F} Y_t^{a,l^F} - MC_t^{a,F} Y_t^{a,l^F} \\
\Pi_t^{a,l^I} &= \left[\left(1 - \Xi^I\right) + \Xi^I \left(1 - \psi^I\right)\right] P_t^{a,l^I} Y_t^{a,l^I} - MC_t^{a,I} Y_t^{a,l^I} \\
&= \left(1 - \Xi^I \psi^I\right) P_t^{a,l^I} Y_t^{a,l^I} - MC_t^{a,I} Y_t^{a,l^I}
\end{aligned}$$

where $P_t^{a,l^{ii}}$ might be an optimal price $P_t^{a,l^{ii},*}$ or a price updated following the simple indexation rule previously defined.

Firms in the intermediate sector maximize their value $V_t^{a,l^{ii}}$ which corresponds to the discounted sum of their profits and is given (in real terms and adjusted by productivity growth) by

$$\max_{P_t^{a,l^{ii}}} \quad V_t^{a,l^{ii}} = \sum_{s=0}^{\infty} \left(\beta\theta^{ii}\right)^s \mathcal{E}_t \frac{\lambda_{1,t+s} Z_{t+s}^{a,iii}}{\lambda_{1,t} Z_t^{a,ii}} \left(\left(1 - \Xi^{iii}\psi^{ii}\right) p_{t+s}^{a,l^{ii}} Y_{t+s}^{a,l^{ii}} - mc_{t+s}^{a,ii} Y_{t+s}^{a,l^{ii}}\right)$$

subject to the demand for their goods

$$Y_t^{a,l^{ii}} = \left(\frac{p_t^{a,l^{ii}}}{p_t^{a,ii}}\right)^{-\epsilon^{a,l^{ii}}} Y_t^{a,ii,d}$$

and the simple indexation rule for those periods in which the firms cannot optimally adjust their prices

$$P_t^{a,l^{ii}} = P_{t-1}^{a,l^{ii}} \pi_{t-1}^{lii} \overline{\pi}^{(1-lii)}$$

The simple indexation rule implies that in the periods in which the firm cannot adjust its price, the price in period $t+s$ can be expressed as

$$P_{t+s}^{a,l^{ii}} = P_t^{a,l^{ii}} X_{t+s}^{a,ii}$$

where

$$X_{t+s}^{a,ii} = \prod_{j=1}^{s} \left[\pi_{t+j-1}^{lii} \overline{\pi}^{(1-lii)} \right]$$

As firms in the formal sector don't break any rules $\Xi^F = 0$, and $\psi^F = 0$.

2.5 Monetary Policy

The model allows for two monetary policy regimes. An inflation targeting regime with flexible exchange rates and a peg regime. In the inflation targeting regime, the central bank controls the short-term nominal interest rate and sets it following a rule that responds to deviations of inflation from the target. In particular, the monetary policy rule is:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\varrho^R} \left(\left(\frac{\pi_t^c}{\bar{\pi}^c} \right)^{\varphi^\pi} \right)^{1-\varrho^R} \mathbf{exp}\left(z_t^m \right)$$

where ϱ^R is the smoothing parameter, φ^π measures the sensibility of the policy rule to deviations of inflation from the target, and z_t^m is the monetary policy shock. This shock is exogenous and follows an ARMA model.

In the peg regime, the nominal devaluation rate is constant

$$d_t = \frac{S_t}{S_{t-1}} = \bar{d}$$

To completely characterize the policy regime, we write down the balance sheet of the central bank. The bank issues money M_t, holds foreign reserves $B_t^{cb,*}$, and net domestic assets comprising government and central bank bonds, B_t^{cb}. Hence, the balance sheet of the central bank is

$$M_t = B_t^{cb} + S_t B_t^{cb,*}$$

The central bank flow of funds is

$$M_t - M_{t-1} + R_{t-1}B_{t-1}^{cb} + R_{t-1}^{*}S_t B_{t-1}^{cb,*} = B_t^{cb} + S_t B_t^{cb,*} + P_t^c qfb_t$$

Accordingly, the quasi-fiscal balance, qfb_t, is a function of the return on external and domestic assets, the domestic inflation rate, and the real exchange rate.

2.6 Fiscal Policy

The government collects taxes on consumption, capital, and labor, receives the quasi-fiscal balance from the central bank and revenues from the commodity sector. It issues public debt to finance its overall balance. The central bank holds a fraction ς^g of the government debt and households the remaining part. The government spends on consumption, investment, transfers to households, and interest payments on its debt.

The government budget constraint is

$$S_t P_t^{O^*}\left(1-\gamma^O\right)\bar{O}_t + \Theta_t^C + \Theta_t^N + \Theta_t^K + B_t + P_t^c qfb_t = P_t^X\left(G_t^X + I_t^{g,X}\right) + P_t^M\left(G_t^M + I_t^{g,M}\right)$$
$$+\sum_i^S P_t^{S_i}\left(G_t^{S_i} + I_t^{g,S_i}\right) + R_{t-1}B_{t-1} + T_t$$

Consumption tax revenues in period t are given by $\Theta_t^C = P_t^c \tau_t^c C_t$, with $C_t = \int_j \left(c_{1,j,t} + c_{2,j,t}\right) dj$. Payroll taxes in period t are given by $\Theta_t^N = \tau_t^w \int_j W_{j,t}^F N_{j,t}^F dj$, with formal labor in a given union j defined as $N_{j,t}^F = \left(N_{1,t}^F + N_{2,t}^F\right)$. Capital tax revenues are given by $\Theta_t^K = \tau_t^k\left(R_t^k - \delta Q_{t-1}\right)K_{t-1}$. G_t^a and $I_t^{g,a}$ for $a \in \{X, M, S_i\}$ are government expenditures in consumption and investment goods.

Transfers are set optimally to maximize the following government objective:

$$U_t^{g,T} = U\left(c_t, N_t, m_t\right) + \omega^{g,1}\left(b_t - \bar{b}\right)^2 + \omega^{g,2}\left(tr_t - \overline{tr}\right)^2$$

$U\left(c_t, N_t, m_t\right)$ is a weighted average of the two types of households. The terms $\omega^{g,1}\left(b_t - \bar{b}\right)^2$

and $(tr_t - \bar{tr})^2$ capture respectively the costs of changing the indebtedness of the government and of adjusting the level of transfers (due to inability or unwillingness to make this adjustment abruptly). $\bar{b}$ and $\overline{tr}$ correspond to the real level of public debt and transfers in steady-state. Low $\omega^{g,1}$ and $\omega^{g,2}$ make the adjustments less costly.

Public investment is used to build public capital that enters with a lag in the production function of the intermediate good producers. Public capital is accumulated according to the following equation:

$$K_t^g = (1-\delta)\,K_{t-1}^g + z_t^{I,g} A_{t-L}^g$$

where A_{t-L} denotes the authorized budget for government investment in period $t-L$. Government investment implemented in t is:

$$I_t^g = \sum_{h=0}^{L-1} \vartheta_h A_{t-h}^g$$

with $\sum_{h=0}^{L-1} \vartheta_h = 1$. This specification of the investment process assumes that it takes time to build public investment and there are lags between the announcement of public investment and its implementation. $z_t^{I,g}$ is a productivity shock in public investment.

2.7 External Sector and Current Account

The external interest rate is the sum of an external risk free rate $\bar{R}_t^*$ and an endogenous risk premium. That is:

$$R_t^* = \bar{R}_t^* - \Omega_u^* \left(\mathbf{exp} \left(s_t \frac{nfa_t}{GDP_t} - \bar{s} \frac{\overline{nfa}}{\overline{GDP}} \right) - 1 \right)$$

The country risk premium is a negative function of the ratio of net foreign assets to GDP nfa_t, and Ω_u^* is the elasticity of the country risk to the net foreign asset-to-GDP ratio, nfa_t, which is defined as $\left(nfa_t = b_t^{cb,*} + b_t^{h,*} \right)$.

We define gross domestic product, GDP_t, as follows:

$$GDP_t = p_t^X Y_t^X + \sum_i^S p_t^{S_i} Y_t^{S_i} + s_t p_t^{O^*} O_t$$

Non-commodity exports are modeled as

$$C_t^{*,X} = \left(\frac{p_t^X}{s_t}\right)^{-\epsilon^*} C_t^*$$

where C_t^* is proportional to the external output and ϵ^* is the elasticity of exports to the exchange rate. The balance of payments equation is found by aggregating the households budget constraint, the government budget constraint, and the balance sheet of the central bank.

$$S_t \left(B_t^{h,*} + B_t^{cb,*} - R_{t-1}^* B_{t-1}^{h,*} - R_{t-1}^* B_{t-1}^{cb,*}\right) = P_t^X C_t^{*,X} + S_t \bar{O}_t P_t^{O^*} - P_t^M \left(C_t^M + I_t^M + G_t^M + I_t^{g,M}\right)$$

The country trade balance tb_t in real terms is given by

$$tb_t = exports_t - imports_t = \left[p_t^X C_t^{*,X} + s_t p_t^{O^*} \bar{O}_t\right] - \left[s_t \left(C_t^M + I_t^M + G_t^M + I_t^{g,M}\right)\right]$$

3 Results

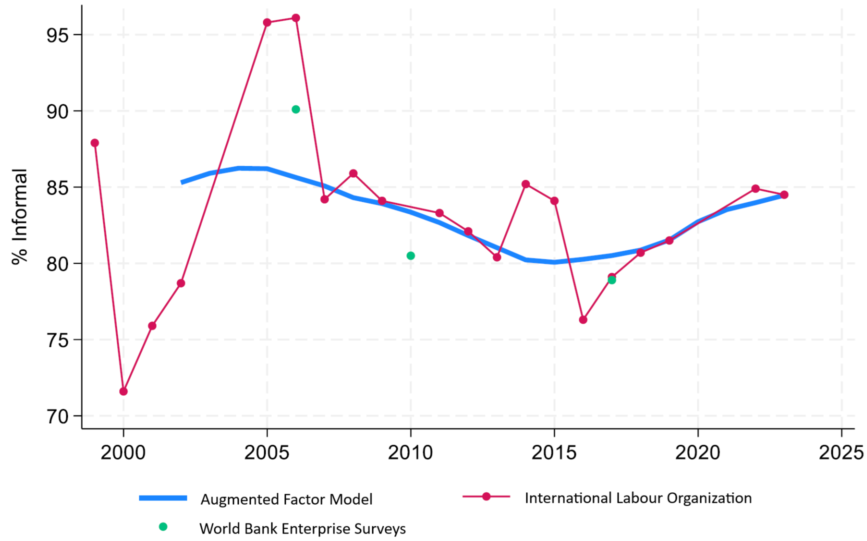
This section presents the results of four shock scenarios: a productivity shock, a fiscal consolidation, a foreign demand shock, and an exchange rate devaluation. We compare the responses of the full model—including an informal sector—with those of a model without informality. The inverse effects hold symmetrically in response to opposite shocks.

The model is calibrated for Bolivia using the past twenty years of Bolivian macroeconomic data to capture the main structural parameters of the economy, following Gonzalez et al. (2022).[2] The initial state of the model, calculated to reflect Bolivia's situation at the end of 2024, incorporates an external deficit of 5.0 percent of GDP,[3] a fiscal deficit of 10.3 percent of GDP, and foreign reserves at 4.1 percent of GDP. We combine World Bank Enterprise Survey and International Labor Organization data to calibrate the relative size of the informal sector.

[2] See Table A1 for a summary.

[3] Includes both the current account deficit and net errors and omissions.

Figure 2: Informal Employment Share in Bolivia

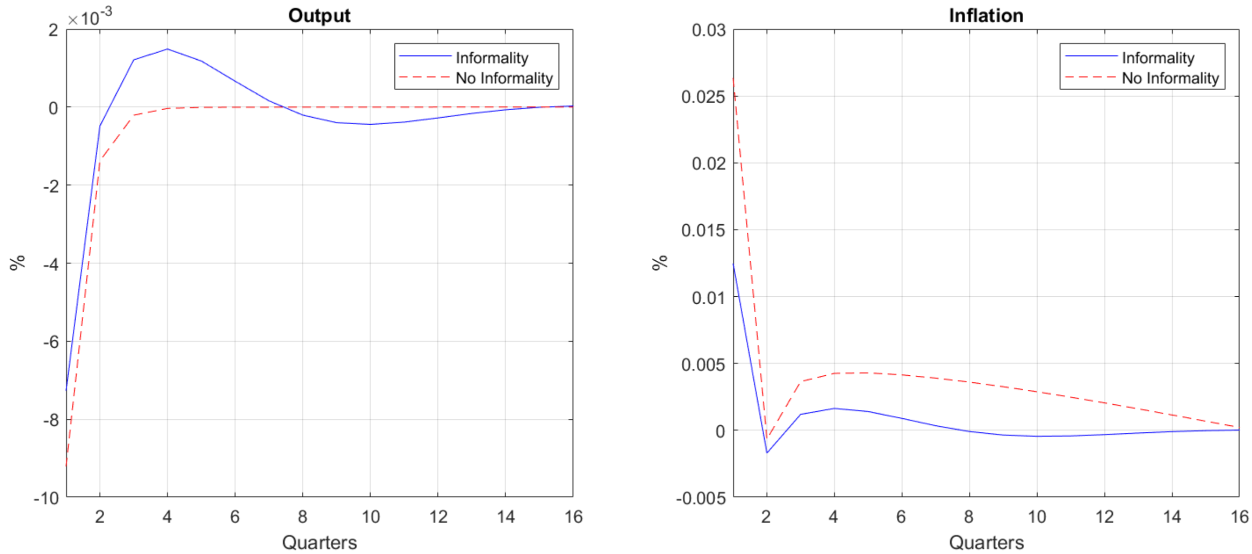


Notes: This figure presents estimates of the informal employment rate for Bolivia from the International Labour Organization, the World Bank Enterprise Surveys, and an augmented factor model based on Yao 2024.

Following the International Labor Organization, we assume that 85 percent of employment is informal. Further, we use an augmented factor model (following Yao, 2024) with World Bank Enterprise Survey data from 2006, 2010, and 2017 and estimate that the informal sector makes 68 percent of GDP as of end-2024.[4] In Figure 2, we validate the accuracy of our augmented factor model estimates by showing that its estimated informal employment shares over time in Bolivia match that of the International Labor Organization. We split the services sector by two types: basic and technical services. Basic services are more reliant on informal inputs, while technical services are more reliant on formal inputs. By sector, informality makes up 34 percent of technical services value added, 79 percent of basic services value added, and 75 percent of exportable goods value added. We assume that the elasticity of substitution between formal and informal inputs is higher for exportable goods and basic services ($\epsilon^{X,F} = \epsilon^{basic,F} = 5$), than for technical services ($\epsilon^{techincal,F} = 2$). In the model without informality, we assume that all labor is formal (or $\iota^N = 0$ and $N_{i,t} = N_{i,t}^F$) and all production relies solely on formal inputs (or $\kappa^a = 1$ and $Y_t^{a,I} = 0$). All other calibrated parameters and variables are held fixed.

[4]See Annex A.2 for details.

Figure 3: Impulse Response of Output and Inflation to a 1% Negative Productivity Shock



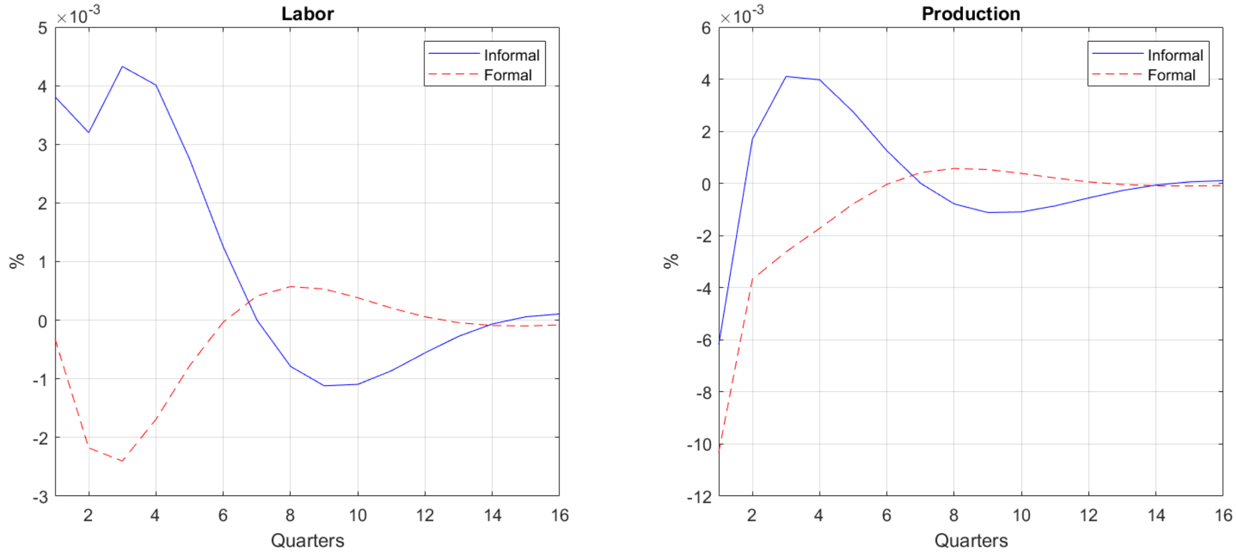
Notes: This figure presents the impulse response functions of output and inflation to a uniform 1% negative productivity shock to all sectors. The impulse response from the full model with informality is presented in blue and that from a model without informality is presented in red.

3.1 Negative Productivity Shock

We find that informality reduces the negative output and inflationary impact of negative supply shocks. Figure 3 presents the impulse responses of output and inflation to a uniform 1% negative productivity shock across all sectors, comparing the full model with informality to a model without. The initial output decline is smaller in the model with informality, and output subsequently overshoots to partially recover early losses—an effect not present in the model without informality. Inflation also responds more mildly and returns more quickly to baseline in the model with informality, whereas inflation remains elevated for longer in the model without informality.

Because formal input prices and wages are sticky and do not adjust downward in response to the negative productivity shock, production reallocates toward informal inputs, while consumption shifts toward non-traded goods. As shown in Figure 4, this leads to a reallocation of labor and production toward the informal sector: informal labor increases significantly in the initial quarters, while formal labor contracts. Similarly, informal production falls by less than formal production when the shock hits and then rises temporarily following as firms

Figure 4: Impulse Response of Informal vs Formal Labor and Production to a 1% Negative Productivity Shock



Notes: This figure presents the impulse response functions of labor and production to a uniform 1% negative productivity shock to all sectors. The impulse response for formal labor/production is presented in red and informal labor/production is presented in blue.

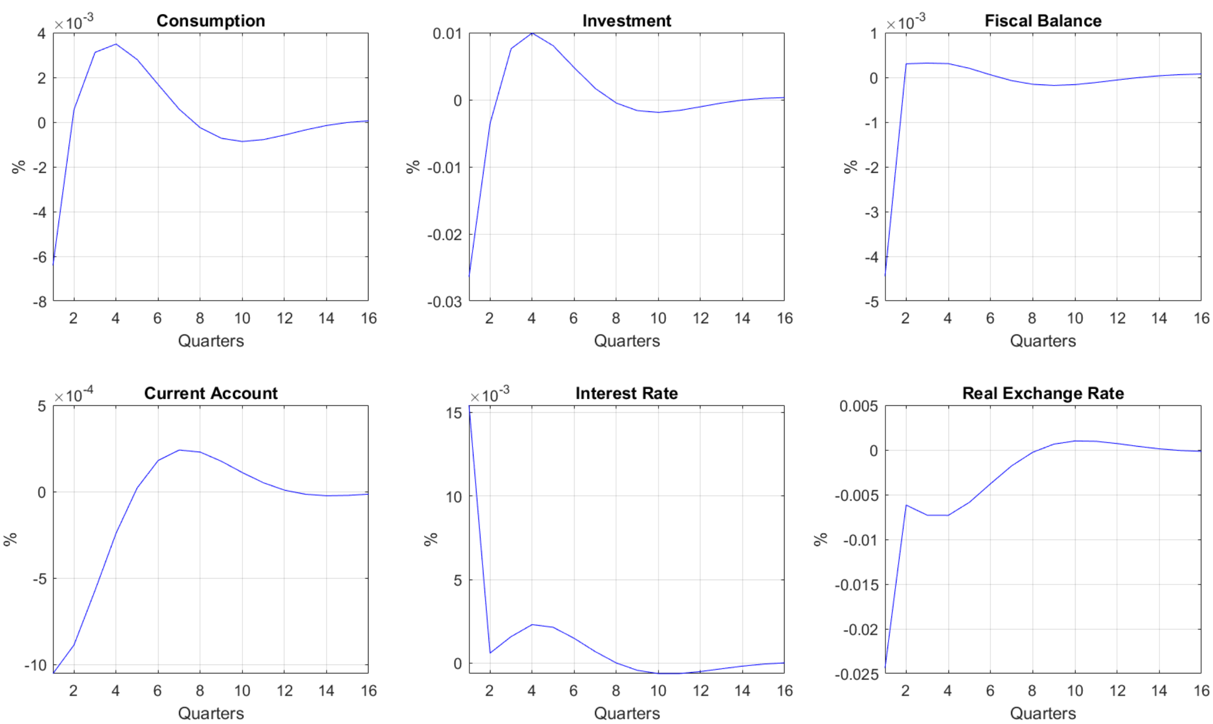
substitute away from costlier formal inputs, cushioning the overall output and inflation impact. In Figure 5, we report the impulse response functions for other variables in the model for completion.

3.2 Fiscal Consolidation

We find that informality buffers the negative impact on output from a contraction in domestic demand. Figure 6 presents the impact of a fiscal contraction via a 1 percent reduction in government consumption on output and inflation,[5] comparing the full model with informality to a model without. The simulations show that the negative impact on output is significantly smaller with informality. However, the presence of informality amplifies the deflationary response: inflation initially falls more sharply and takes longer to stabilize, with some output cost in the longer term.

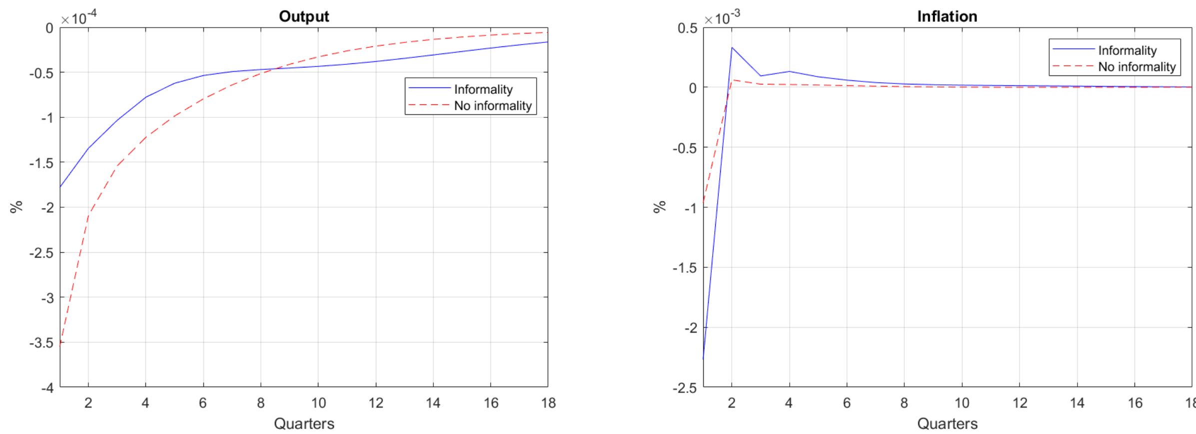
[5]In Figure A1, we report the impact of a 1% increase in the income tax on labor.

Figure 5: Impulse Response to a 1% Negative Productivity Shock: Other Variables



Notes: This figure presents the impulse response functions of consumption, investment, fiscal balance, current account, domestic interest rate, and real exchange rate to a uniform 1% negative productivity shock to all sectors.

Figure 6: Impulse Response of Output and Inflation to a 1% Reduction in Government Consumption



Notes: This figure presents the impulse response functions of output and inflation to a 1% reduction in government consumption. The impulse response from the full model with informality is presented in blue and that from a model without informality is presented in red.

The price of informal inputs decreases relative to that of the formal sector due to nominal rigidities. As a result, tradable production shifts to cheaper informal inputs, consumption shifts to cheaper non-traded goods, and labor shifts to the informal sector. Figure 7 presents the impulse responses of informal versus formal labor and production.[6]

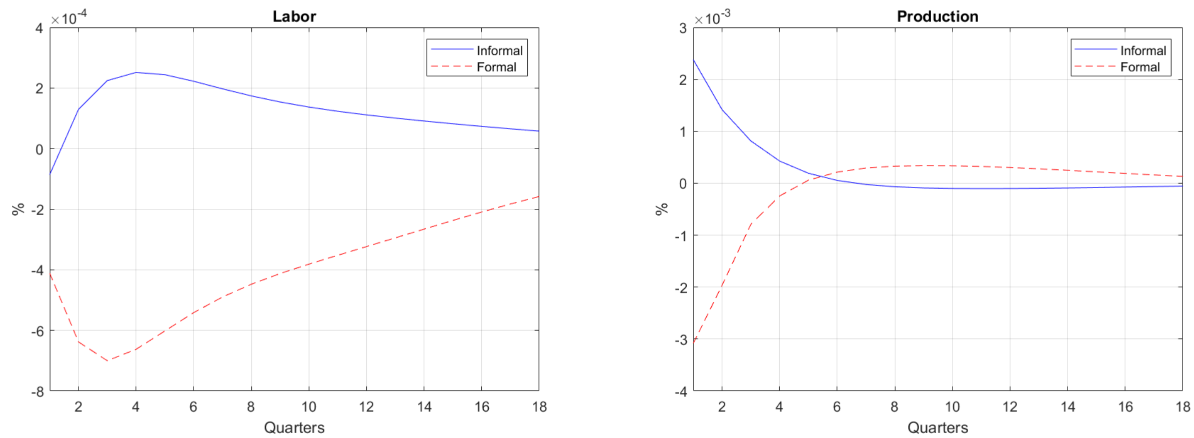
3.3 Foreign Demand Shock

We find that informality bolsters the gains in output from an increase in foreign demand, while softening the impact on inflation. Figure 8 presents the impact of an increase in foreign demand on output and inflation,[7] comparing the full model with informality to a model without. The simulations show that the positive impact on output is slightly larger with informality. At the same time, the inflationary impact is slightly smaller with informality. Since formal input prices and wages are sticky, formal sector production and employment expands more slowly in response to an increase in demand. The informal sector expands more quickly as its wages and prices adjust flexibly. Since formal and informal inputs are

[6]We report the impulse response functions for other variables in the model in Figure A2b.

[7]We report the impulse response functions for other variables in the model in Figure A2c.

Figure 7: Impulse Response of Informal vs Formal Labor and Production to a 1% Reduction in Government Consumption



Notes: This figure presents the impulse response functions of labor and production to a a 1% reduction in government consumption. The impulse response for formal labor/production is presented in red and informal labor/production is presented in blue.

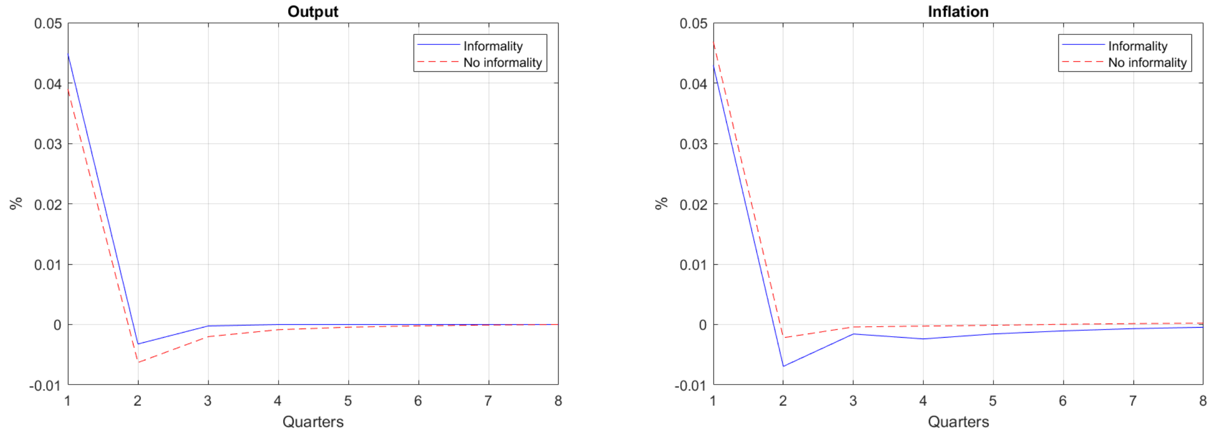
substitutable in the production of tradables, output in the tradable sector expands more with a smaller increase in price. Additionally, domestic consumption shifts to non-tradable goods as the price of tradables increases, also since that tradable and non-tradable goods are substitutable in consumption, further boosting output and softening the upward pressure on prices.

3.4 Exchange Rate Devaluation

A devaluation of the exchange rate raises the price of imports but increases foreign demand for exports. However, Bolivia's exports are mostly commodities which are relatively inelastic to the exchange rate.[8] We find that informality reduces the negative output and inflationary impact of a devaluation from increased import prices. Specifically, we consider a 30% devaluation of the exchange rate and present the impulse response functions of output and inflation in Figure 9. We find that GDP contracts by about 0.4% in the first year relative to the baseline in the model with informality, while the negative impact is larger at 0.5% in the model without informality. We find that inflation spikes by 8 percentage points in the

[8]In the case where exports are more elastic to the exchange rate, a devaluation would boost exports and hence output.

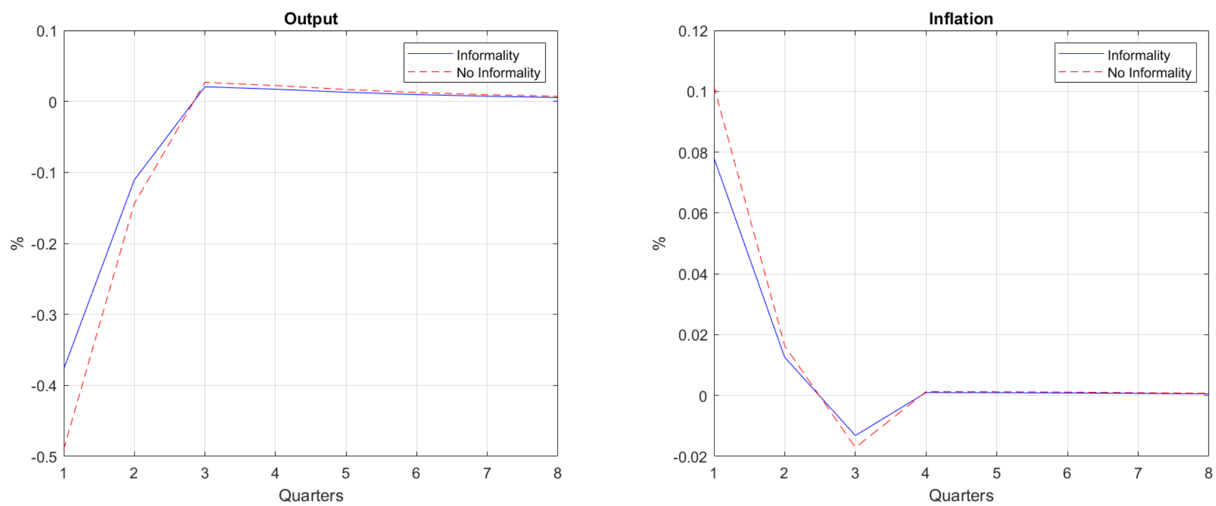
Figure 8: Impulse Response of Output and Inflation to a 1% Increase in Foreign Demand



Notes: This figure presents the impulse response functions of output and inflation to a 1% increase in foreign demand. The impulse response from the full model with informality is presented in blue and that from a model without informality is presented in red.

first quarter following devaluation in the model with informality, while the inflation impact is 10 percentage points in the model without informality.[9]

Figure 9: Impulse Response of Output and Inflation to a 30% Exchange Rate Devaluation



Notes: This figure presents the impulse response functions of output and inflation to a 30% exchange rate devaluation. The impulse response from the full model with informality is presented in blue and that from a model without informality is presented in red.

[9]We report the impulse response functions for other variables in the model in Figure A2d.

4 Conclusion

The informal sector accounts for a large fraction of the labor force in many emerging market and developing economies. This paper develops a dynamic stochastic general equilibrium model of a small open economy with an informal sector. Simulation results suggest that informality can play a significant shock-absorbing role. Informality dampens the adverse effects of negative supply-side shocks—such as productivity declines or increases in import prices—on output and inflation. In the case of a positive external demand shock, informality amplifies the gains in output while mitigating inflationary pressures. Moreover, it cushions the contractionary effects on output from a decline in domestic demand, such as a sharp fiscal consolidation.

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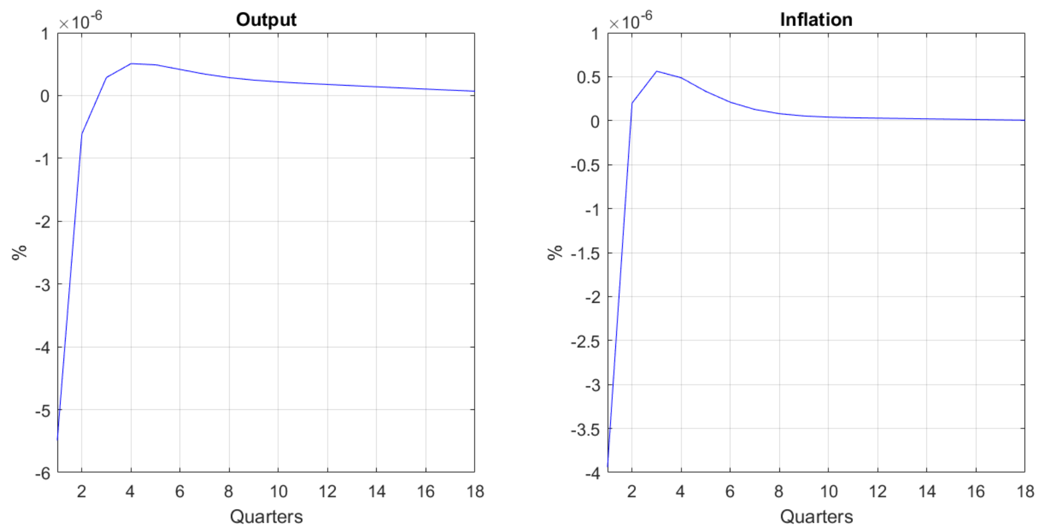
Appendix

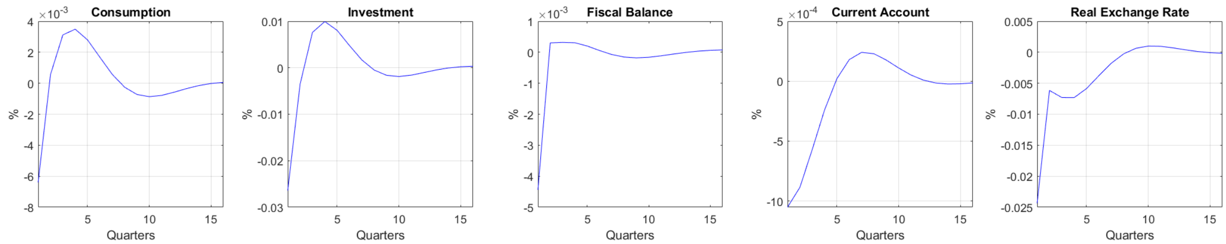
A.1 Appendix Figures and Tables

Figure A1: Impulse Response of Output and Inflation to 1% increase in the Income Tax on Labor

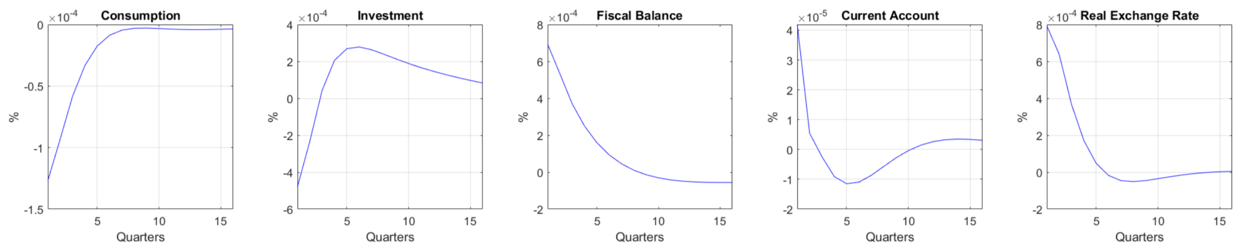


Notes: This figure presents the impulse response functions of output and inflation to a 1% increase in the income tax on labor.

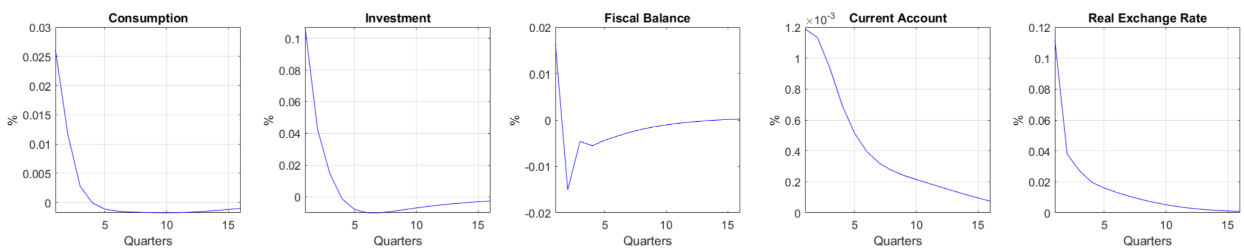
Figure A2: Impulse Response of Other Variables



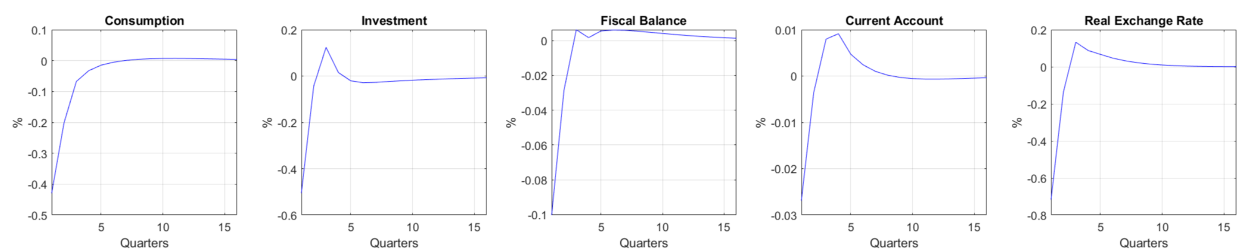
(a) 1% Negative Productivity Shock



(b) 1% Reduction in Government Consumption



(c) 1% Increase in Foreign Demand



(d) 30% Exchange Rate Devaluation

Notes: This figure presents the impulse response functions of consumption, investment, fiscal balance, current account, and real exchange rate to shocks as specified by the panel titles.

Table A1: Key Model Parameters

Parameter	**Value**
External deficit	5.0% of GDP[10]
Fiscal deficit	10.3% of GDP
Foreign reserves	4.1% of GDP
Share of informal employment	85%
Share of informal GDP	68%
Informal share of value added, technical services	34%
Informal share of value added, basic services	79%
Informal share of value added, exportable goods	75%
Elasticities of substitution	
Formal vs. informal inputs, exportable goods	$\epsilon^{X,F} = 5$
Formal vs. informal inputs, basic services	$\epsilon^{basic,F} = 5$
Formal vs. informal inputs, technical services	$\epsilon^{technical,F} = 2$

A.2 Informality in Bolivia

The informal sector accounts for a large fraction of economy in Bolivia, as in many other emerging market economies. Micro data on informality is only available infrequently, making it difficult to assess the current size and dynamics of the informal economy. The most recent World Bank Enterprise Survey, measuring informality among firms in Bolivia, was conducted in 2017. Recent International Labor Organization data points to 85 percent of employment being in the informal economy.

For our analysis, we use an augmented factor model based on Yao (2024) that estimates factors that link the causes and indicators of the informal economy (Table A2). The model establishes a predictive relationship between the estimated factors and survey estimates across countries to estimate informality in Bolivia over time. It also allows for decomposition of the estimated degree of informality into contributions by factors and by projected indicators.[11] World Bank Enterprise Surveys were collected in Bolivia in 2006, 2010 and 2017 report firm level data on informality. These survey data points discipline the predictions by the augmented factor model by setting the model average in these years as the average of these data points.

The model estimates that 84 percent of firms operate informally, accounting for 68 percent of GDP. Informality is higher in the services sector and firms with lower value-added are more likely to be informal. Informality makes up 34 percent of technical services value added, 79 percent of basic services value added, and 75 percent of exportable goods value added (Figure A3a).

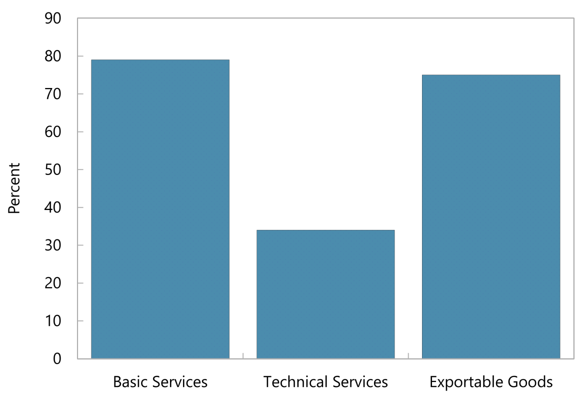
The model estimates that the informality has increased since 2016 (Figure A3b). The share of informal firms rose from around 80 percent in 2016 to 84 percent in 2023. The model reveals that this increase was driven by a decline in export sectors, slower per capita income growth, higher unemployment, and a larger footprint of the state. Likewise, the share of informal employment rose from 76 percent in 2016 to 85 percent in 2023.

[11] The MIMIC model is a special case of the augmented factor model. Its latent variable is the first principal component of the augmented factor model under strong assumptions about the indicators and Yao (2024) shows that it is not a useful predictor of the degree of informality in survey data.

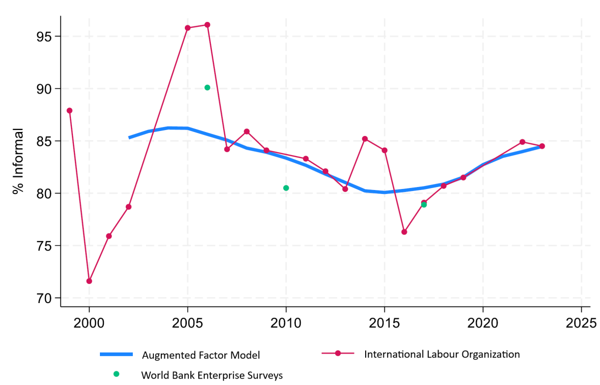
Table A2: Causes and Indicators of Augmented Factor Model

Variable	**Source**
Causes	
PPP GDP per capita, unemployment rate	World Development Indicators
Rule of law, control of corruption, government effectiveness, voice and accountability, regulatory quality, political stability and absence of violence/terrorism	Worldwide Governance Indicators
Trade openness, tax-to-GDP ratio, government consumption-to-GDP	World Economic Outlook
Indicators	
PPP GDP per capita	World Development Indicators
Currency in circulation	International Financial Statistics
Labor participation rate (aged 15-64)	World Development Indicators
Electricity consumption	World Development Indicators

Figure A3: Agumented Factor Model Estimates of Informality in Bolivia



(a) Informal Value-Added Share by Sector



(b) Informal Employment Share in Bolivia

Notes: This figure presents estimates of the informal value added and employeemnt shares for Bolivia from an augmented factor model based on Yao 2024.

A.3 Additional Model Details

A.3.1 Households First Order Conditions

As informal labor markets are non-unionized, households also choose on their own their supply of hours in the informal labor market.

The first order conditions for the households $i = 1$ are given by:

$$
\begin{aligned}
z_t^u \frac{1}{c_{1,t} - \varsigma c_{1,t-1}} &- \beta \mathbb{E}_t z_{t+1}^u \varsigma \frac{1}{c_{1,t+1} - \varsigma c_{1,t}} = \Lambda_{1,t} \left(1 + \tau_t^c\right) P_t \\
\Lambda_{1,t} &= \beta \mathbb{E}_t \Lambda_{1,t+1} R_t \\
\Lambda_{1,t} S_t &= \beta \mathbb{E}_t \Lambda_{1,t+1} S_{t+1} R_t^* \\
\Lambda_{1,t} P_t^I &= \Upsilon_t z_t^I \left(1 - f\left(\frac{I_t}{I_{t-1}}\right) - f'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) + \beta \mathbb{E}_t \Upsilon_{t+1} z_{t+1}^I \left(f' \frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2 \\
\Upsilon_t &= \beta \Upsilon_{t+1} (1-\delta) + \beta \mathbb{E}_t \Lambda_{1,t+1} \left(\left(1 - \tau_{t+1}^k\right) R_{t+1}^k + \tau_{t+1}^k \delta Q_t\right) \\
\gamma^M \left(\frac{M_{1,t}}{P_t^c}\right)^{-\sigma^M} &= \left(\Lambda_{1,t} - \beta \mathbb{E}_t \Lambda_{1,t+1}\right) P_t
\end{aligned}
$$

Where $f'(\cdot)$ refers to the derivative of f with respect to $\frac{I_t}{I_{t-1}}$ which corresponds to $a\left(\frac{I_t}{I_{t-1}} - 1\right)$.

The additional, new first order condition is the one for the informal labor supply:

$$
\gamma^N N_{1,t}^{\sigma^N} \frac{\partial N_{1,t}}{\partial N_{1,t}^I} = \Lambda_{1,t} W_t^I - \tilde{\Upsilon}_{1,t}
$$

where

$$
\frac{\partial N_{1,t}}{\partial N_{1,t}^I} = \left(1 - \iota^N\right)^{\frac{1}{\rho^N}} \left(\frac{N_{1,t}^I}{N_{1,t}}\right)^{-\frac{1}{\rho^N}}
$$

and $\tilde{\Upsilon}$ is the Lagrange multiplier of the labor force constraint stating that labor supply must be less than the available household time endowment, e.g., $N_t \leq \overline{N}$.

In real terms these conditions are given by

$$
\begin{aligned}
z_t^u \frac{1}{c_{1,t} - \varsigma c_{1,t-1}} &- \beta \mathbb{E}_t z_{t+1}^u \varsigma \frac{1}{c_{1,t+1} - \varsigma c_{1,t}} = \lambda_{1,t} \left(1 + \tau_t^c\right) \\
\lambda_{1,t} &= \beta \mathbb{E}_t \lambda_{1,t+1} \frac{R_t}{\pi_{t+1}} \\
\lambda_{1,t} s_t &= \beta \mathbb{E}_t \lambda_{1,t+1} s_{t+1} \frac{R_t^*}{\pi_{t+1}^*} \\
\lambda_{1,t} p_t^I &= \mu_t z_t^I \left(1 - f\left(\frac{I_t}{I_{t-1}}\right) - f'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right) + \beta \mathbb{E}_t \mu_{t+1} z_{t+1}^I f'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \\
\mu_t &= \beta \mu_{t+1} \left(1 - \delta\right) + \beta \mathbb{E}_t \lambda_{1,t+1} \left[\left(1 - \tau_{t+1}^k\right) r_{t+1}^k + \tau_{t+1}^k \delta \frac{q_t}{\pi_{t+1}}\right] \\
\gamma^M \left(m_{1,t}\right)^{-\sigma^M} &= \lambda_{1,t} - \beta \mathbb{E}_t \lambda_{1,t+1} \frac{1}{\pi_{t+1}} \\
\lambda_{1,t} w_t^I - \tilde{\Upsilon}_{1,t} &= \gamma^N N_{1,t}^{\sigma^N} \left[\left(1 - \iota^N\right) \frac{N_{1,t}}{N_{1,t}^I}\right]^{\frac{1}{\rho^N}}
\end{aligned}
$$

The first order conditions for the households without access to credit markets are:

$$
\begin{aligned}
z_t^u \frac{1}{c_{2,t} - \varsigma c_{2,t-1}} &- \beta \mathbb{E}_t z_{t+1}^u \frac{1}{c_{2,t+1} - \varsigma c_{2,t}} = \Lambda_{2,t} \left(1 + \tau_t^c\right) P_t^c \\
\gamma^M \left(\frac{M_{2,t}}{P_t^c}\right)^{-\sigma^M} &= \left(\Lambda_{2,t} - \beta \mathbb{E}_t \Lambda_{2,t+1}\right) P_t^c
\end{aligned}
$$

and

$$
\gamma^N N_{2,t}^{\sigma^N} \frac{\partial N_{2,t}}{\partial N_{2,t}^I} = \Lambda_t W_t^I - \tilde{\Upsilon}_{2,t}
$$

where

$$
\frac{\partial N_{2,t}}{\partial N_{2,t}^I} = \left(1 - \iota\right)^{\frac{1}{\rho^N}} \left(\frac{N_{2,t}^I}{N_{2,t}}\right)^{-\frac{1}{\rho^N}}
$$

In real terms these conditions are

$$
\begin{aligned}
z_t^u \frac{1}{c_{2,t} - \varsigma c_{2,t-1}} &- \beta \mathbb{E}_t z_{t+1}^u \varsigma \frac{1}{c_{2,t+1} - \varsigma c_{2,t}} = \lambda_{2,t} \left(1 + \tau_t^c\right) \\
\gamma^M \left(m_{2,t}\right)^{-\sigma^M} &= \lambda_{2,t} - \beta \mathbb{E}_t \lambda_{2,t+1} \frac{1}{\pi_{t+1}} \\
\lambda_{2,t} w_t^I - \tilde{\Upsilon}_{2,t} &= \gamma^N N_{2,t}^{\sigma^N} \left[\left(1 - \iota^N\right) \frac{N_{2,t}}{N_{2,t}^I}\right]^{\frac{1}{\rho^N}}
\end{aligned}
$$

A.3.2 Unions

The optimization problem of the unions for labor type j -which corresponds to choosing the wage rate $W^F_{j,t}$, subject to the households preferences, budget constraints, the law of movement of physical capital, and the law of movement for the formal sector wages when the unions are not able to adjust the wage, and under the assumption that $N^I_{i,j,t} = N^I_{i,t} = 0$, and therefore $N_{i,t} = \int N^F_{i,j,t}$- is given by the following Lagrangian :

$$\begin{aligned}
\mathcal{L} = \mathcal{E}_0 \sum_{s=0}^{\infty} (\beta\theta^w)^s \Bigg\{ &(1-\nu)\Big(z^u_{t+s} \ln(c_{j,1,t+s} - \varsigma c_{j,1,t+s-1}) \\
&- \gamma_N \frac{1}{1+\sigma_N} N^{1+\sigma_N}_{j,1,t+s} + \gamma_M \frac{1}{1-\sigma_M} m^{1-\sigma_M}_{j,1,t+s} \Big) \\
&+ \nu \Big(z^u_{t+s} \ln(c_{j,2,t+s} - \varsigma c_{j,2,t+s-1}) \\
&- \gamma_N \frac{1}{1+\sigma_N} N^{1+\sigma_N}_{j,2,t+s} + \gamma_M \frac{1}{1-\sigma_M} m^{1-\sigma_M}_{j,2,t+s} \Big) \\
&+ (1-\nu)\lambda_{j,1,t+s} \Big[\Big(\Big(1 - \tau^k_{t+s}\Big) \frac{R^k_{t+s}}{\pi_{t+s-1}} + \tau^k_{t+s}\delta \frac{Q_{t+s-1}}{\pi_{t+s-1}} \Big) K_{j,t+s-1} \\
&+ (1-\tau^w_{t+s}) w^F_{j,t+s} N^F_{1,j,t+s} + w^I_{t+s} N^I_{j,1,t+s} + \frac{\gamma^O}{1-\omega} S_t \overline{O}_{t+s} P^{O^*}_{t+s} \\
&+ \frac{R_{t+s-1}}{\pi_{t+s-1}} b_{j,t+s-1} + s_{t+s} \frac{R^*_{t+s-1}}{\pi^*_{t+s-1}} b^*_{j,t+s-1} + tr_{j,1,t+s} \\
&+ \frac{\xi_{t+s}}{P_{t+s}} + \frac{m_{1,t+s-1}}{\pi_{t+s-1}} - (1+\tau^c_{t+s}) c_{1,t+s} \\
&- p^I_{t+s} I_{t+s} - m_{1,t+s} - b_{t+s} - s_{t+s} b^*_{t+s} \Big] \\
&+ \nu\lambda_{j,2,t+s} \Big[(1-\tau^w_{t+s}) w^F_{j,t+s} N^F_{j,2,t+s} + W^I_{j,t+s} N^I_{j,2,t+s} + tr_{j,2,t+s} \\
&+ \frac{m_{j,2,t+s-1}}{\pi_{t+s-1}} \Big] \\
&- (1+\tau^c_{t+s}) c_{j,2,t+s} - m_{2,t+s} \Bigg\}
\end{aligned}$$

the first order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c^{j}_{t+s}} = (1-\nu) * \left(z_{t+s} \frac{1}{c_{j,1,t+s} - \varsigma c_{j,1,t+s-1}} - \mathcal{E} z_{t+s+1} \frac{\beta \varsigma}{c_{j,1,t+s+1} - \varsigma c_{j,1,t+s}} \right) + \nu * \left(z_{t+s} \frac{1}{c_{j,1,t+s} - \varsigma c_{j,1,t+s-1}} - \mathcal{E} z_{t+s+1} \frac{\beta \varsigma}{c_{j,1,t+s+1} - \varsigma c_{j,1,t+s}} \right)$$

where from the point of view of the unions $\tilde{\iota} = 1$ and $N^{I}_{i,j,t} = 0$ which implies:

$$\begin{aligned} N_{i,j,t} &= \left[\left(\tilde{\iota}^{N}\right)^{\frac{1}{\rho^N}} \left(N^{F}_{i,j,t}\right)^{\frac{\rho^N-1}{\rho^N}} + \left(1-\tilde{\iota}^{N}\right)^{\frac{1}{\rho^N}} \left(N^{I}_{i,t}\right)^{\frac{\rho^N-1}{\rho^N}} \right]^{\frac{\rho^N}{\rho^N-1}} = \left(\iota^{N}\right)^{\frac{1}{\rho^N-1}} N^{F}_{i,j,t} = N^{F}_{i,j,t} \\ N^{F}_{j,t} &= N^{F}_{1,j,t} + N^{F}_{2,j,t} \\ N^{F}_{j,t} &= \left(\frac{W^{F}_{j,t}}{W^{F}_{t}} \right)^{-\epsilon^{w}} N^{F,d}_{t} \end{aligned}$$

Wages are given by the following in the periods in which unions are not able to optimally adjust them

$$W^{F}_{j,t} = W^{F}_{j,t-1} g^{z} \pi^{\chi^{w}}_{t-1} \bar{\pi}^{(1-\chi^{w})}$$

Therefore real wages are adjusted as:

$$w^{F}_{j,t} = w^{F}_{j,t-1} \frac{g^{z} \pi^{\chi^{w}}_{t-1} \bar{\pi}^{(1-\chi^{w})}}{\pi^{c}_{t-1}}$$

The first order conditions with respect to $c_{j,t}$ ($c_{j,t} = \nu c_{1,j,t} + (1-\nu) c_{2,j,t} = \nu c_{1,t} + (1-\nu) c_{2,t}$), $m_{j,t}$ ($m_{j,t} = \nu m_{1,j,t} + (1-\nu) m_{2,j,t} = \nu m_{1,t} + (1-\nu) m_{2,t}$), K_t, I_t, b_t, and b^{*}_{t} are given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_{j,t}} &= (\beta\theta^w)^t z_t^u \left(\frac{\nu}{c_{1,t}-\varsigma c_{1,t-1}} + \frac{1-\nu}{c_{2,t}-\varsigma c_{2,t-1}}\right) + (\beta\theta^w)^{t+1} \mathcal{E}_t z_{t+1}^u \varsigma \left(\frac{\nu}{c_{1,t+1}-\varsigma c_{1,t}} + \frac{1-\nu}{c_{2,t+1}-\varsigma c_{2,t}}\right) \\
&\quad - (\beta\theta^w)^t \tilde{\lambda}_{j,t} (1-\tau_t^c) = 0 \\
\frac{\partial \mathcal{L}}{\partial m_{j,t}} &= (\beta\theta^w)^t \gamma_M \left(\nu (m_{1,t})^{-\gamma_M} + (1-\nu)(m_{2,t})^{-\gamma_M}\right) - (\beta\theta^w)^t \tilde{\lambda}_{j,t} + (\beta\theta^w)^{t+1} \mathcal{E}_t \frac{\tilde{\lambda}_{j,t+1}}{\pi_t} = 0 \\
\frac{\partial \mathcal{L}}{\partial K_{j,t}} &= -(\beta\theta^w)^t \nu \tilde{\mu}_{j,t} + (\beta\theta^w)^{t+1} \nu \mathcal{E}_t \left(\tilde{\lambda}_{j,t+1}\left(\left(1-\tau_{t+1}^k\right) r_{t+1}^k + \tau_{t+1}^k \delta \frac{q_t}{\pi_t}\right) + \tilde{\mu}_{j,t+1}(1-\delta)\right) = 0 \\
\frac{\partial \mathcal{L}}{\partial I_{j,t}} &= (\beta\theta^w)^t \nu \left(-\tilde{\lambda}_{j,t} p_t^I + \tilde{\mu}_{j,t} z_t^I \left(1 - f\left(\frac{I_{j,t}}{I_{j,t-1}}\right) - I_{j,t} f'\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\frac{1}{I_{j,t-1}}\right)\right) \\
&\quad + (\beta\theta^w)^{t+1} \nu \mathcal{E}_t \tilde{\mu}_{j,t+1} z_{t+1}^I I_{j,t+1} f'\left(\frac{I_{j,t+1}}{I_{j,t}}\right)\frac{I_{j,t+1}}{(I_{j,t})^2} = 0 \\
\frac{\partial \mathcal{L}}{\partial b_t} &= -(\beta\theta^w)^t \nu \tilde{\lambda}_{j,t} + (\beta\theta^w)^{t+1} \nu \mathcal{E}_t \tilde{\lambda}_{j,t+1} \frac{R_t}{\pi_t} = 0 \\
\frac{\partial \mathcal{L}}{\partial b_t^*} &= -(\beta\theta^w)^t \nu \tilde{\lambda}_{j,t} s_t + (\beta\theta^w)^{t+1} \nu \mathcal{E}_t \tilde{\lambda}_{j,t+1} s_{t+1} \frac{R_t^*}{\pi_t^*} = 0
\end{aligned}$$

They imply

$$z_t^u \left(\frac{\nu}{c_{1,t}-\varsigma c_{1,t-1}} + \frac{1-\nu}{c_{2,t}-\varsigma c_{2,t-1}}\right) - \beta\theta^w \varsigma \mathcal{E}_t z_{t+1}^u \left(\frac{\nu}{c_{1,t+1}-\varsigma c_{1,t}} + \frac{1-\nu}{c_{2,t+1}-\varsigma c_{2,t}}\right) = \tilde{\lambda}_{j,t}(1-\tau_t^c)$$

$$\gamma_M \left(\nu (m_{1,t})^{-\gamma_M} + (1-\nu)(m_{2,t})^{-\gamma_M}\right) = \tilde{\lambda}_{j,t} - \beta\theta^w \mathcal{E}_t \frac{\tilde{\lambda}_{j,t+1}}{\pi_t}$$

$$\tilde{\mu}_{j,t} = \beta\theta^w \mathcal{E}_t \left(\tilde{\lambda}_{j,t+1}\left(\left(1-\tau_{t+1}^k\right) r_{t+1}^k + \tau_{t+1}^k \delta \frac{q_t}{\pi_t}\right) + \tilde{\mu}_{j,t+1}(1-\delta)\right)$$

$$\tilde{\lambda}_{j,t} p_t^I - \tilde{\mu}_{j,t} z_t^I \left(1 - f\left(\frac{I_{j,t}}{I_{j,t-1}}\right) - f'\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right) = \beta\theta^w \mathcal{E}_t \left(\tilde{\mu}_{j,t+1} z_{t+1}^I f'\left(\frac{I_{j,t+1}}{I_{j,t}}\right)\left(\frac{I_{j,t+1}}{I_{j,t}}\right)^2\right)$$

$$\tilde{\lambda}_{j,t} = \beta\theta^w \mathcal{E}_t \tilde{\lambda}_{j,t+1} \frac{R_t}{\pi_t}$$

$$\tilde{\lambda}_{j,t} s_t = \beta\theta^w \mathcal{E}_t \tilde{\lambda}_{j,t+1} s_{t+1} \frac{R_t^*}{\pi_t^*}$$

The first order condition for consumption implies that

$$z_t^u \left(\frac{\nu}{c_{1,t} - \varsigma c_{1,t-1}} + \frac{1-\nu}{c_{2,t} - \varsigma c_{2,t-1\beta\theta^w\varsigma}} \right) + \beta\theta^w \varsigma \mathcal{E}_t z_{t+1}^u \left(\frac{\nu}{c_{1,t+1} - \varsigma c_{1,t}} + \frac{1-\nu}{c_{2,t+1} - \varsigma c_{2,t}} \right) =$$
$$\nu \left(\frac{z_t^u}{c_{1,t} - \varsigma c_{1,t-1}} - \beta\theta^w \varsigma \mathcal{E}_t \frac{z_{t+1}^u}{c_{1,t+1} - \varsigma c_{1,t}} \right) + (1-\nu) \left(\frac{z_t^u}{c_{2,t} - \varsigma c_{2,t-1}} - \beta\theta^w \varsigma \mathcal{E}_t \frac{z_{t+1}^u}{c_{2,t+1} - \varsigma c_{2,t}} \right) =$$
$$\nu \left[U_c \left(c_{1,t} \right) \right] + (1-\nu) \left[U_c \left(c_{2,t} \right) \right] = \tilde{\lambda}_{j,t} \left(1 - \tau_t^c \right)$$

Therefore

$$\tilde{\lambda}_{j,t} = \frac{\nu \left[U_c \left(c_{1,t} \right) \right] + (1-\nu) \left[U_c \left(c_{2,t} \right) \right]}{\left(1 - \tau_t^c \right)}$$

The first order condition with respect to $w_{j,t}^F = \frac{W_{j,t}^F}{P_t}$ is given by

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w_{j,t}^F} &= (\beta\theta^w)^t \gamma_N \epsilon^w \left(\frac{w_{j,t}^F}{w_t^F}\right)^{-\epsilon^w-1} \frac{1}{w_t^F} N_t^{F,d} \left[\left(\frac{w_{j,t}^F}{w_t^F}\right)^{-\epsilon^w} N_t^{F,d}\right]^{\sigma_N} \\
&+\mathcal{E}_t (\beta\theta^w)^{t+1} \gamma_N \epsilon^w \left(\frac{w_{j,t+1}^F}{w_{t+1}^F}\right)^{-\epsilon^w-1} \frac{1}{w_{t+1}^F} N_{t+1}^{F,d} \left(\frac{g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}}{\pi_t^c}\right) \left[\left(\frac{w_{j,t+1}^F}{w_{t+1}^F}\right)^{-\epsilon^w} N_{t+1}^{F,d}\right]^{\sigma_N} \\
&+\mathcal{E}_t (\beta\theta^w)^{t+2} \gamma_N \epsilon^w \left(\frac{w_{j,t+2}^F}{w_{t+2}^F}\right)^{-\epsilon^w-1} \frac{1}{w_{t+2}^F} N_{t+2}^{F,d} \left(\frac{\left(g^z \bar{\pi}^{(1-\chi^w)}\right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w}}{\pi_{t+1}^c \pi_t^c}\right) \left[\left(\frac{w_{j,t+2}^F}{w_{t+2}^F}\right)^{-\epsilon^w} N_{t+2}^{F,d}\right]^{\sigma_N} \\
&+\cdots \\
&+ (\beta\theta^w)^t \tilde{\lambda}_{j,t} (1-\tau_t^w) \left(-w_{j,t}^F \epsilon^w \left(\frac{w_{j,t}^F}{w_t^F}\right)^{-\epsilon^w-1} \frac{1}{w_t^F} N_t^{F,d} + \left(\frac{w_{j,t}^F}{w_t^F}\right)^{-\epsilon^w} N_t^{F,d}\right) \\
&+ (\beta\theta^w)^{t+1} \mathcal{E}_t \tilde{\lambda}_{j,t+1} \left(1-\tau_{t+1}^w\right) \left(-w_{j,t+1}^F \epsilon^w \left(\frac{w_{j,t+1}^F}{w_{t+1}^F}\right)^{-\epsilon^w-1} \frac{1}{w_{t+1}^F} N_{t+1}^{F,d} \left(\frac{g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}}{\pi_t^c}\right)\right. \\
&\left. + \left(\frac{g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}}{\pi_t^c}\right) \left(\frac{w_{j,t+1}^F}{w_{t+1}^F}\right)^{-\epsilon^w} N_{t+1}^{F,d}\right) \\
&+ (\beta\theta^w)^{t+2} \mathcal{E}_t \tilde{\lambda}_{j,t+2} \left(1-\tau_{t+2}^w\right) \left(-w_{j,t+2}^F \epsilon^w \left(\frac{w_{j,t+2}^F}{w_{t+2}^F}\right)^{-\epsilon^w-1} \frac{1}{w_{t+2}^F} N_{t+2}^{F,d} \left(\frac{\left(g^z \bar{\pi}^{(1-\chi^w)}\right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w}}{\pi_{t+1}^c \pi_t^c}\right)\right. \\
&\left. + \left(\frac{\left(g^z \bar{\pi}^{(1-\chi^w)}\right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w}}{\pi_{t+1}^c \pi_t^c}\right) \left(\frac{w_{j,t+2}^F}{w_{t+2}^F}\right)^{-\epsilon^w} N_{t+2}^{F,d}\right) \\
&+\cdots \\
&= 0
\end{aligned}
$$

This condition implies

$$
\begin{aligned}
& \gamma_N \epsilon^w \frac{1}{w_{j,t}^F} \left[\left(\frac{w_{j,t}^F}{w_t^F} \right)^{-\epsilon^w} N_t^{F,d} \right]^{1+\sigma_N} \\
& + \left(\beta \theta^w \right) \gamma_N \epsilon^w \frac{1}{w_{j,t}^F} \mathcal{E}_t \left[\left(\frac{w_{j,t+1}^F}{w_{t+1}^F} \right)^{-\epsilon^w} N_{t+1}^{F,d} \right]^{1+\sigma_N} \\
& + \left(\beta \theta^w \right)^2 \gamma_N \epsilon^w \frac{1}{w_{j,t}^F} \mathcal{E}_t \left[\left(\frac{w_{j,t+2}^F}{w_{t+2}^F} \right)^{-\epsilon^w} N_{t+2}^{F,d} \right]^{1+\sigma_N} \\
& + \cdots \\
& = \left(\epsilon^w - 1 \right) \tilde{\lambda}_{j,t} \left(1 - \tau_t^w \right) \left(\frac{w_{j,t}^F}{w_t^F} \right)^{-\epsilon^w} N_t^{F,d} \\
& + \left(\epsilon^w - 1 \right) \left(\beta \theta^w \right) \mathcal{E}_t \tilde{\lambda}_{j,t+1} \left(1 - \tau_{t+1}^w \right) \left(\frac{w_{j,t+1}^F}{w_{t+1}^F} \right)^{-\epsilon^w} N_{t+1}^{F,d} \left(\frac{g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}}{\pi_t^c} \right) \\
& + \left(\epsilon^w - 1 \right) \left(\beta \theta^w \right)^2 \mathcal{E}_t \tilde{\lambda}_{j,t+2} \left(1 - \tau_{t+2}^w \right) \left(\frac{w_{j,t+2}^F}{w_{t+2}^F} \right)^{-\epsilon^w} N_{t+2}^{F,d} \left(\frac{\left(g^z \bar{\pi}^{(1-\chi^w)} \right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w}}{\pi_{t+1}^c \pi_t^c} \right) \\
& + \cdots \\
= \quad & 0
\end{aligned}
$$

When unions can't adjust optimally their wages those are $W_{j,t+s}^F = W_{j,t}^F \prod_{a=1}^{s-1} g^z \bar{\pi}^{(1-\chi^w)} \pi_{t+a-1}^{\chi^w}$. Therefore real wages are $w_{j,t+s}^F = w_{j,t}^F \prod_{a=1}^{s-1} \frac{g^z \bar{\pi}^{(1-\chi^w)} \pi_{t+a-1}^{\chi^w}}{\pi_{t+a-1}^c}$. Defining $X_{j,t,s}^F = \prod_{a=1}^{s-1} \frac{g^z \bar{\pi}^{(1-\chi^w)} \pi_{t+a-1}^{\chi^w}}{\pi_{t+a-1}^c}$, we can re-write those wages as $w_{j,t+s}^F = w_{j,t}^F X_{j,t,s}^F$, and the previous expression as

$$
\begin{aligned}
& \gamma_N \epsilon^w \frac{1}{w_{j,t}^F} \mathcal{E}_t \sum_{s=0}^{\infty} \left(\beta \theta^w \right)^s \left[\left(\frac{w_{j,t}^F X_{j,t,s}^F}{w_{t+s}^F} \right)^{-\epsilon^w} N_{t+s}^{F,d} \right]^{1+\sigma_N} \\
& = \left(\epsilon^w - 1 \right) \mathcal{E}_t \sum_{s=0}^{\infty} \left(\beta \theta^w \right)^s \tilde{\lambda}_{t+s} \left(1 - \tau_{t+s}^w \right) \left(\frac{w_{j,t}^F X_{j,t,s}^F}{w_{t+s}^F} \right)^{-\epsilon^w} N_{t+s}^{F,d} X_{j,t,s}^F
\end{aligned}
$$

Given that

$$U_{N^F_{t+s}}\left(N^F_{j,t+s}\right) = -\gamma_N\left(N^F_{j,t,s}\right)^{\sigma_N} = \left[\left(\frac{w^F_{j,t}X^F_{j,t,s}}{w^F_{t+s}}\right)^{-\epsilon^w} N^{F,d}_{t+s}\right]^{1+\sigma_N}$$

The previous expression can be re-written as

$$\begin{aligned}
&-\epsilon^w\mathcal{E}_t\sum_{s=0}^{\infty}(\beta\theta^w)^s\left[U_{N^F_{t+s}}\left(N^F_{j,t+s}\right)\right]\left[\left(\frac{X^F_{j,t,s}}{w^F_{t+s}}\right)^{-\epsilon^w} N^{F,d}_{t+s}\right]\\
&= \left(\epsilon^w-1\right)\mathcal{E}_t\sum_{s=0}^{\infty}(\beta\theta^w)^s\,\tilde{\lambda}_{j,t+s}\left(1-\tau^w_{t+s}\right)w^F_{j,t}X^F_{j,t,s}\left(\frac{X^F_{j,t,s}}{w^F_{t+s}}\right)^{-\epsilon^w} N^{F,d}_{t+s}
\end{aligned}$$

Which can be written as

$$\mathcal{E}_t\sum_{s=0}^{\infty}(\beta\theta^w)^s\left[U_{N^F_{t+s}}\left(N^F_{j,t+s}\right)\right]\left[\left(\frac{X^F_{j,t,s}}{w^F_{t+s}}\right)^{-\epsilon^w} N^{F,d}_{t+s}\right]\left(\frac{\epsilon^w}{\epsilon^w-1}-\frac{\tilde{\lambda}_{j,t+s}\left(1-\tau^w_{t+s}\right)w^F_{j,t}X^F_{j,t,s}}{\left[U_{N^F_{t+s}}\left(N^F_{j,t+s}\right)\right]}\right)=0$$

substituting in the previous expression $\tilde{\lambda}_{j,t+s}$ we get

$$\begin{aligned}
&\mathcal{E}_t\sum_{s=0}^{\infty}(\beta\theta^w)^s\left[U_{N^F_{t+s}}\left(N^F_{j,t+s}\right)\right]\left[\left(\frac{X^F_{j,t,s}}{w^F_{t+s}}\right)^{-\epsilon^w} N^{F,d}_{t+s}\right]\\
&*\left(\frac{\epsilon^w}{\epsilon^w-1}-\frac{\nu\left[U_c\left(c_{1,t+s}\right)\right]+(1-\nu)\left[U_c\left(c_{2,t+s}\right)\right]}{\left(1-\tau^c_{t+s}\right)}\frac{\left(1-\tau^w_{t+s}\right)w^F_{j,t}X^F_{j,t,s}}{\left[U_{N^F_{t+s}}\left(N^F_{j,t+s}\right)\right]}\right)=0
\end{aligned}$$

After further manipulation the expression can be re-written as:

$$\begin{aligned}
&\mathcal{E}_t\sum_{s=0}^{\infty}(\beta\theta^w)^s\left[U_{N^F_{t+s}}\left(N^F_{j,t+s}\right)\right]\left(X^F_{j,t,s}\right)^{-\epsilon^w}\left(w^F_{t+s}\right)^{\epsilon^w}N^{F,d}_{t+s}\left[\left(\frac{\nu}{MRS_{1,t+s}}+\frac{1-\nu}{MRS_{2,t+s}}\right)w^F_{j,t}X^F_{j,t,s}-\frac{\epsilon^w}{\epsilon^w-1}\right]\\
&=0
\end{aligned}$$

where

$$MRS_{i,t+s} = -\frac{U_{N^F_{t+s}}\left(N^F_{j,t,s}\right)}{U_{c^i_{t+s}}\left(c_{i,j,t,s}\right)}\frac{\left(1-\tau^c_{t+s}\right)}{\left(1-\tau^w_{t+s}\right)}$$

A.3.3 Labor Market First Order Conditions

The optimality condition for the unions that can adjust wages is given by:[12]

[12]Union's first order conditions with respect to consumption and labor would be given by

$$\begin{aligned}
(1-\nu)\left\{z^u_{t+s}\frac{1}{c_{j,1,t+s}-\varsigma c_{j,1,t+s-1}} - \beta\mathbb{E}_t z^u_{t+s+1}\frac{\varsigma}{c_{j,1,t+s+1}-\varsigma c_{j,1,t+s}}\right\} &+ \\
\nu\left\{z^u_{t+s}\frac{1}{c_{j,2,t+s}-\varsigma c_{j,2,t+s-1}} - \beta\mathbb{E}_t z^u_{t+1}\frac{\varsigma}{c_{j,2,t+s+1}-\varsigma c_{j,2,t+s}}\right\} &= \\
\left\{(1-\nu)\lambda_{j,1,t+s}+\nu\lambda_{j,2,t+s}\right\}\left(1+\tau^c_{t+s}\right) &= \\
(1-\nu)\frac{\partial u(\cdot)}{\partial c_{j,1,t}} + \nu\frac{\partial u(\cdot)}{\partial c_{j,2,t}} &= \\
\tilde{u}^U_c\left(c_{j,1,t}, c_{j,2,t}\right) & \\
(1-\nu)\left\{\gamma^N N^{\sigma^N}_{j,1,t+s}\left[\iota^N\frac{N_{j,1,t+s}}{N^F_{j,1,t+s}}\right]^{\frac{1}{\rho^N}}\right\} &+ \\
\nu\left\{\gamma^N N^{\sigma^N}_{j,2,t+s}\left[\iota^N\frac{N_{j,2,t+s}}{N^F_{j,2,t+s}}\right]^{\frac{1}{\rho^N}}\right\} &= \\
\left\{(1-\nu)\lambda_{j,1,t+s}+\nu\lambda_{j,2,t+s}\right\}\left(1-\tau^w_{t+s}\right)w^F_{j,t+s} &= \\
(1-\nu)\frac{\partial u(\cdot)}{\partial N^F_{j,1,t}} + \nu\frac{\partial u(\cdot)}{\partial N^F_{j,2,t}} &= \\
\tilde{u}^U_{N^F}\left(N^F_{j,1,t}, N^F_{j,2,t}\right) &
\end{aligned}$$

Unions choose their wage such that it is above the one that would be dictated by the first order conditions of workers of type j (e.g., $w^{F,foc}_{j,t+s}$)

$$w^F_{j,t+s} > w^{F,foc}_{j,t+s} = \frac{\left(1+\tau^c_{t+s}\right)}{\left(1-\tau^w_{t+s}\right)}\frac{\tilde{u}^U_{N^F}(N^F_{j,1,t}, N^F_{j,2,t})}{\tilde{u}^U_c(c_{j,1,t}, c_{j,2,t})} = \frac{1}{\widetilde{MRS}(c_{j,1,t}, c_{j,2,t}, N^F_{j,1,t}, N^F_{j,2,t}))}$$

$$\mathcal{E}_t \sum_{s=0}^{\infty} (\beta\theta^w)^s \qquad \Bigg\{ \tilde{u}^U_{N^F}(N^F_{j,1,t}, N^F_{j,2,t}) \frac{\partial N^F_{j,t+s}}{\partial w^F_{j,t+s}} \frac{\partial w^F_{j,t+s}}{\partial w^{F,*}_{j,t}}$$

$$+ \tilde{\lambda}^U_{j,t+s} \left(1 - \tau^w_{t+s}\right) \left[\frac{\partial w^F_{j,t+s}}{\partial w^{F,*}_{j,t}} N^F_{j,t+s} + w^F_{j,t+s} \frac{\partial N^F_{j,t+s}}{\partial w^F_{j,t+s}} \frac{\partial w^F_{j,t+s}}{\partial w^{F,*}_{j,t}} \right] \Bigg\} = 0$$

where:

$$\tilde{\lambda}^U_{j,t+s} = [(1-\nu)\lambda_{j,1,t+s} + \nu\lambda_{j,1,t+s}]$$

$$\tilde{u}^U_{N^F}(N^F_{j,1,t}, N^F_{j,2,t}) = \left[(1-\nu) \frac{\partial u(\cdot)}{\partial N^F_{j,1,t}} + \nu \frac{\partial u(\cdot)}{\partial N^F_{j,2,t}} \right] = -\gamma^N \left[(1-\nu) N_{j,1,t+s}^{\sigma^N + \frac{1}{\rho^N}} + \nu N_{j,2,t+s}^{\sigma^N + \frac{1}{\rho^N}} \right] \left(\frac{\iota^N}{N^F_{j,t+s}} \right)^{\frac{1}{\rho^N}}$$

As the nominal wage s periods after the last re-optimization is given by:

$$W^F_{j,t+s} = W^F_{j,t} \prod_{a=1}^{s-1} g^z \bar{\pi}^{(1-\chi^w)} \pi^{\chi^w}_{t+a-1}$$

and in real terms this wage is given by

$$w^F_{j,t+s} = w^F_{j,t+s} \tilde{X}^F_{j,t,s}$$

$$\tilde{X}^F_{j,t,s} = \prod_{a=1}^{s-1} \left(\frac{g^z \bar{\pi}^{(1-\chi^w)} \pi^{\chi^w}_{t+a-1}}{\pi^c_{t+a-1}} \right)$$

Given $N^F_{j,t+s} = \left(\frac{w^F_{j,t+s}}{w^F_{t+s}} \right)^{-\epsilon^w} h^{F,d}_{t+s}$:

$$\frac{\partial N^F_{j,t+s}}{\partial w^F_{j,t+s}} = -\epsilon^w \left(\frac{w^{F,*}_{j,t} \tilde{X}^F_{j,t,s}}{w^F_{t+s}} \right)^{-\epsilon^w - 1} \left(\frac{1}{w^F_{t+s}} \right) h^{F,d}_{t+s}$$

The previous first order condition can be re-written as:

$$\mathcal{E}_t \sum_{s=0}^{\infty} (\beta\theta^w)^s \left(\frac{\epsilon^w}{\epsilon^w - 1}\right) \gamma^N \left[(1-\nu) N_{j,1,t+s}^{\sigma^N + \frac{1}{\rho^N}} + \nu N_{j,2,t+s}^{\sigma^N + \frac{1}{\rho^N}}\right] \left(\frac{\iota^N}{N_{j,t+s}^F}\right)^{\frac{1}{\rho^N}} \left(\frac{w_{j,t}^{F,*} \tilde{X}_{j,t,s}^F}{W_{t+s}^F}\right)^{-\epsilon^w} h_{t+s}^{F,d}$$

$$= \quad \mathcal{E}_t \sum_{s=0}^{\infty} (\beta\theta^w)^s \tilde{\lambda}_{j,t+s}^U \left(1 - \tau_{t+s}^w\right) \tilde{X}_{j,t,s}^F \left(w_{j,t}^{F,*}\right)^{1-\epsilon^w} \left(\frac{\tilde{X}_{j,t,s}^F}{w_{t+s}^F}\right)^{-\epsilon^w} h_{t+s}^{F,d}$$

some further manipulation allows us to define the two sides of the union's first order condition in recursive terms as $f_t^{w,L}$ and $f_t^{w,R}$

$$\begin{aligned} f_t^{w,L} &= \gamma^N \left[(1-\nu) N_{j,1,t}^{\sigma^N + \frac{1}{\rho^N}} + \nu N_{j,2,t}^{\sigma^N + \frac{1}{\rho^N}}\right] \left(\frac{\iota^N}{N_{j,t}^F}\right)^{\frac{1}{\rho^N}} \left(w_t^F\right)^{\epsilon^w} \left(w_{j,t}^{F,*}\right)^{-\epsilon^w} h_t^{F,d} \\ &+ \beta\theta^w \left(\frac{w_{j,t}^{F,*}}{w_{j,t+1}^{F,*}}\right)^{-\epsilon^w} \left(\frac{\bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}}{\pi_{t+1}}\right)^{-\epsilon^w} f_{t+1}^{w,L} \\ f_t^{w,R} &= \left(\frac{\epsilon^w - 1}{\epsilon^w}\right) \tilde{\lambda}_{j,t}^U \left(1 - \tau_t^w\right) \left(w_{j,t}^{F,*}\right)^{1-\epsilon^w} \left(w_t^F\right)^{\epsilon^w} h_t^{F,d} \\ &+ \beta\theta^w \left(\frac{w_{j,t}^{F,*}}{w_{j,t+1}^{F,*}}\right)^{1-\epsilon^w} \left(\frac{\bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}}{\pi_{t+1}}\right)^{1-\epsilon^w} f_{t+1}^{w,R} \end{aligned}$$

If the wages were flexible the optimality condition would take the form of:

$$\left(\frac{\lambda_{t+s}^{\tilde{U}} \left(1 - \tau_t^w\right)}{\left[(1-\nu) N_{j,1,t}^{\sigma^N + \frac{1}{\rho^N}} + \nu N_{j,2,t}^{\sigma^N + \frac{1}{\rho^N}}\right] \left(\frac{\iota^N}{N_{j,t}^F}\right)^{\frac{1}{\rho^N}}}\right) w_{j,t}^{F,*} = \frac{\epsilon^w}{\epsilon^w - 1} > 1$$

There is always a markup between the union's "Marginal Rate of Substituion"(MRS) –which is a function of the weigthed marginal utility of formal labor, and marginal utility of wealth across types $i = 1, 2$ for labor type j– and the real wage, and both types of households will always be willing to supply more labor when the real wage increases.

Wages negotiated by the unions in each period are identical. Each period a share $(1 - \theta^w)$ of the labor type belonging to each union is able to negotiate its wage. Then we have the following equilibrium condition in the formal labor market

$$N_t^F = v_t^w N_t^{F,d}$$

where $v_t^w > 1$ measures the inefficiency created by the wage dispersion. Because $v_t^w > 1$, it implies that the labor supply is higher than what the firms use effectively in production.

$$v_t^w = \theta^w \left(\frac{w_{t-1}^F}{w_t^F} g^z \frac{\pi_{t-1}^{\chi^w} \overline{\pi}^{(1-\chi^w)}}{\pi_t} \right)^{-\epsilon^w} v_{t-1}^w + (1-\theta^w) \left(\frac{w_{j,t}^{F,*}}{w_t^F} \right)^{-\epsilon^w}$$

when wages are flexible the dispersion disappear and $v_t^w = 1$.

The aggregate real wage index evolves as in the following equation:

$$w_t^F = \left(\theta^w \left(w_{t-1}^F g^z \frac{\pi_{t-1}^{\chi^w} \overline{\pi}^{1-\chi^w}}{\pi_t} \right)^{1-\epsilon^w} + (1-\theta^w) \left(w_{j,t}^{F,*} \right)^{1-\epsilon^w} \right)^{\frac{1}{1-\epsilon^w}}$$

The overall wage index of the economy weights both formal and informal wages. It is given by:

$$w_t = \left(\iota^N \left(w_t^F \right)^{1-\rho^N} + \left(1-\iota^N \right) \left(w_t^I \right)^{1-\rho^N} \right)^{\frac{1}{1-\rho^N}}$$

A.3.4 Intermediate Formal Firms - Cost Minimization

We assume two types of services: S1 and S2.

The solution to cost minimization problem of the firms according to which they decide their demand for labor and capital is given by the following first order conditions:

$$
\begin{aligned}
w_t^F &= mc_t^{a,l^F}\left(1-\alpha^{a,F}\right)\frac{Y_t^{a,l^F}}{N_t^{a,l^F}} \\
r_t^{kF,a} &= mc_t^{a,l^F}\alpha^{a,F}\frac{Y_t^{a,l^F}}{K_{t-1}^{a,l^F}} \\
w_t^I &= mc_t^{a,l^I}\left(1-\alpha^{a,I}\right)\frac{Y_t^{a,l^I}}{N_t^{a,l^I}} \\
r_t^{kI,a} &= mc_t^{a,l^I}\alpha^{a,I}\frac{Y_t^{a,l^I}}{K_{t-1}^{a,l^I}}
\end{aligned}
$$

and $mc_t^{a,l^{ii}}$ corresponds to the real marginal cost of the firm $l^{a,ii}$, given by

$$
mc_t^{a,l^{ii}} = \frac{\left(\frac{r_t^{kii,a}}{\alpha^{a,ii}}\right)^{\alpha^{a,ii}}\left(\frac{w_t^{ii}}{1-\alpha^{a,ii}}\right)^{1-\alpha^{a,ii}}}{z_t^{a,ii}\left(Z_t^{a,ii}\right)^{1-\alpha^{a,ii}-\alpha^{a,g,ii}}\left(K_{t-1}^{g}\right)^{\alpha^{a,g,ii}}}
$$

This cost is identical across firms in the sector (a, ii). This marginal cost not only depends on the price of the factors $r_t^{kii,a}$, and w_t^{ii}, as in standard models, but also includes the costs of adjusting their capital.

The previous equations define factor demands of each sector (a, ii), which are implied by

$$
\begin{aligned}
\frac{w_t^F}{r_t^{kF,a}} &= \frac{1-\alpha^{a,F}}{\alpha^{a,F}}\frac{K_{t-1}^{a,l^F}}{N_t^{a,l^F}} = \frac{1-\alpha^{a,F}}{\alpha^{a,F}}\frac{K_{t-1}^{a,F}}{N_t^{a,F}} \\
\frac{w_t^I}{R_t^{kI,a}} &= \frac{1-\alpha^{a,I}}{\alpha^{a,F}}\frac{K_{t-1}^{a,l^I}}{N_t^{a,l^I}} = \frac{1-\alpha^{a,I}}{\alpha^{a,F}}\frac{K_{t-1}^{a,I}}{N_t^{a,I}}
\end{aligned}
$$

where

$$
r_t^{kii,a} = r_t^k * f\left(\frac{K_{t-1}^{a,ii}}{N_t^{a,ii}} - \frac{K_{t-2}^{a,ii}}{N_{t-1}^{a,ii}}\right)
$$

The corresponding demand for capital is given by

$$r_t^{kii} K_{t-1}^{d} = r_t^{kii,X} K_{t-1}^{X,iii,a,d} + r_t^{kii,S1} K_{t-1}^{S1,iii,a,d} + r_t^{kii,S2} K_{t-1}^{S2,iii,a,d}$$

Capital is sector-specific and it is defined as a composite factor:

$$\begin{aligned}
K_t &= \left(\zeta^{\frac{1}{\varsigma}} (K_t^F)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} (K_t^I)^{\frac{\varsigma-1}{\varsigma}}\right)^{\frac{\varsigma}{\varsigma-1}} \\
K_t^F &= \left((\chi^{xF})^{\frac{1}{\vartheta^F}} (K_t^{xF})^{\frac{\vartheta^F-1}{\vartheta^F}} + (\chi^{S1F})^{\frac{1}{\vartheta^F}} (K_t^{S1F})^{\frac{\vartheta^F-1}{\vartheta^F}} + (\chi^{S2F})^{\frac{1}{\vartheta^F}} (K_t^{S2F})^{\frac{\vartheta^F-1}{\vartheta^F}}\right)^{\frac{\vartheta^F}{\vartheta^F-1}} \\
K_t^I &= \left((\chi^{xI})^{\frac{1}{\vartheta^I}} (K_t^{xI})^{\frac{\vartheta^I-1}{\vartheta^I}} + (\chi^{S1I})^{\frac{1}{\vartheta^I}} (K_t^{S1I})^{\frac{\vartheta^I-1}{\vartheta^I}} + (\chi^{S2I})^{\frac{1}{\vartheta^I}} (K_t^{S2I})^{\frac{\vartheta^I-1}{\vartheta^I}}\right)^{\frac{\vartheta^I}{\vartheta^I-1}}
\end{aligned}$$

The previous optimization problem implies that

$$\begin{aligned}
\mathbb{E}[r_{t+1}^k] \left(\zeta \frac{K_t}{K_t^F}\right)^{\frac{1}{\varsigma}} \left(\chi^{xF} \frac{K_t^F}{K_t^{xF}}\right)^{\frac{1}{\vartheta^F}} &= \mathbb{E}[r_{t+1}^{k,xF}] \\
\mathbb{E}[r_{t+1}^k] \left(\zeta \frac{K_t}{K_t^I}\right)^{\frac{1}{\varsigma}} \left(\chi^{xI} \frac{K_t^I}{K_t^{xI}}\right)^{\frac{1}{\vartheta^I}} &= \mathbb{E}[r_{t+1}^{k,xI}] \\
\mathbb{E}[r_{t+1}^k] \left(\zeta \frac{K_t}{K_t^F}\right)^{\frac{1}{\varsigma}} \left(\chi^{S1F} \frac{K_t^F}{K_t^{S1F}}\right)^{\frac{1}{\vartheta^F}} &= \mathbb{E}[r_{t+1}^{k,S1F}] \\
\mathbb{E}[r_{t+1}^k] \left(\zeta \frac{K_t}{K_t^I}\right)^{\frac{1}{\varsigma}} \left(\chi^{S1I} \frac{K_t^I}{K_t^{xI}}\right)^{\frac{1}{\vartheta^I}} &= \mathbb{E}[r_{t+1}^{k,S1I}] \\
\mathbb{E}[r_{t+1}^k] \left(\zeta \frac{K_t}{K_t^F}\right)^{\frac{1}{\varsigma}} \left(\chi^{S2F} \frac{K_t^F}{K_t^{S2F}}\right)^{\frac{1}{\vartheta^F}} &= \mathbb{E}[r_{t+1}^{k,S2F}] \\
\mathbb{E}[r_{t+1}^k] \left(\zeta \frac{K_t}{K_t^I}\right)^{\frac{1}{\varsigma}} \left(\chi^{S2I} \frac{K_t^I}{K_t^{S2I}}\right)^{\frac{1}{\vartheta^I}} &= \mathbb{E}[r_{t+1}^{k,S2I}]
\end{aligned}$$

A.3.5 Intermediate Formal Firms - Optimization

Intermediate formal firms take as given the price of their inputs, and because they produce a differentiated intermediate good, they are able to choose the price of their good P_t^{a,l^F} subject to their production technology, the composite labor technology, the adjustment costs for labor, and the fact that the firms cannot optimally update those prices in every period. In each period a share $(1-\theta^H)$ of the firms will adjust its prices optimally, while the remaining share θ^H will adjust their prices following a simple rule. The firms profits in period t are Π_t^{a,l^F} which are given by:

$$\Pi_t^{a,l^F} = P_t^{a,l^F} Y_t^{a,l^F} - \lambda_{H,t}^{a,l^F} Y_t^{a,l^F}$$

where P_t^{a,l^F} might be an optimal price $P_t^{a,l^F,*}$ or a price updated following the simple indexation rule previously defined.

Firms in the intermediate formal sector maximize their value $V_{H,t}^{a,l^F}$ which corresponds to the discounted sum of their profits and is given (in nominal terms and adjusted by productivity growth) by

$$\max_{P_t^{a,l^F}} \quad V_{H,t}^{a,l^F} = \sum_{s=0}^{\infty} \left(\beta\theta^H\right)^s \mathcal{E}_t \frac{\Lambda_{1,t+s} P_{t+s}^c Z_{t+s}^a}{\Lambda_{1,t} P_t^c Z_t^a} \left(P_{t+s}^{a,l^F} Y_{t+s}^{a,l^F} - MC_{H,t+s}^{a,F} Y_{t+s}^{a,l^F}\right)$$

subject to the demand for their goods

$$Y_t^{a,l^F} = \left(\frac{P_t^{a,l^F}}{P_t^{a,F}}\right)^{-\epsilon^{a,F}} Y_t^{a,F}$$

and the simple indexation rule for those periods in which the firms cannot optimally adjust their prices

$$P_t^{a,l^F} = P_{t-1}^{a,l^F} \pi_{t-1}^{lH} \overline{\pi}^{(1-lH)}$$

The simple indexation rule implies that for those periods in which the firm is not able to adjust its price, the price in period $t+s$ can be expressed as

$$P_{t+s}^{a,l^F} = P_t^{a,l^F} X_{t+s}^H$$

where

$$X_{H,t+s} = \prod_{j=0}^{s-1} \left[\pi_{t+j}^{lH} \overline{\pi}^{(1-lH)} \right]$$

Consequently, the demand for goods in period $t+s$ when the firm has not being able to adjust its price is given by:

$$Y_{t+s}^{a,l^F} = \left(\frac{P_{t+s}^{a,l^F}}{P_{t+s}^{a,F}} \right)^{-\epsilon^{a,F}} Y_{t+s}^{a,F} = \left(\frac{P_{t}^{a,l^F}}{P_{t+s}^{a,F}} \right)^{-\epsilon^{a,F}} \left(X_{H,t+s} \right)^{-\epsilon^{a,F}} Y_{t+s}^{a,F}$$

After substituting for the expressions for prices and the demand for formal intermediate goods, the value of the firm can be re-written as:

$$\max_{P_t^{a,l^F}} \quad V_{H,t}^{a,l^F} = \sum_{s=0}^{\infty} \left(\beta \theta^H \right)^s \mathcal{E}_t \frac{\Lambda_{1,t+s} P_{t+s}^c Z_{t+s}^a}{\Lambda_{1,t} P_t^c Z_t^a} \left(P_t^{a,l^F} X_{H,t+s} - \lambda_{H,t+s}^{a,l^F} \right) \left(\frac{P_t^{a,l^F}}{P_{t+s}^{a,F}} \right)^{-\epsilon^{a,F}} \left(X_{H,t+s} \right)^{-\epsilon^{a,F}} Y_{t+s}^{a,F}$$

The first order condition for the maximization of the value of the firm with respect to P_t^{a,l^F} is given by:

$$\begin{aligned} \frac{\partial V_{H,t}^{a,l^F}}{\partial P_t^{a,l^F}} \quad = \quad & \sum_{s=0}^{\infty} \left(\beta \theta^H \right)^s \mathcal{E}_t \frac{\Lambda_{1,t+s} P_{t+s}^c Z_{t+s}^a}{\Lambda_{1,t} P_t^c Z_t^a} \left[X_{H,t+s} \left(\frac{P_t^{a,l^F}}{P_{t+s}^{a,F}} \right)^{-\epsilon^{a,F}} \left(X_{H,t+s} \right)^{-\epsilon^{a,F}} Y_{t+s}^{a,F} \right. \\ & \left. - \epsilon^{a,F} \left(P_t^{a,l^F} X_{H,t+s} - \lambda_{H,t+s}^{a,l^F} \right) \left(\frac{P_t^{a,l^F}}{P_{t+s}^{a,F}} \right)^{-\epsilon^{a,F}-1} \frac{1}{P_{t+s}^{a,F}} \left(X_{H,t+s} \right)^{-\epsilon^{a,F}} Y_{t+s}^{a,F} \right] = 0 \end{aligned}$$

This can be re-written as:

$$\begin{aligned} \frac{\partial V_{H,t}^{a,l^F}}{\partial P_t^{a,l^F}} \quad = \quad & \sum_{s=0}^{\infty} \left(\beta \theta^H \right)^s \mathcal{E}_t \frac{\Lambda_{1,t+s} P_{t+s}^c Z_{t+s}^a}{\Lambda_{1,t} P_t^c Z_t^a} \left[\left(1 - \epsilon^{a,F} \right) \left(\frac{P_t^{a,l^F}}{P_{t+s}^{a,F}} \right)^{-\epsilon^{a,F}} \left(X_{H,t+s} \right)^{1-\epsilon^{a,F}} Y_{t+s}^{a,F} \right. \\ & \left. + \epsilon^{a,F} \lambda_{H,t+s}^{a,l^F} \left(\frac{P_t^{a,l^F}}{P_{t+s}^{a,F}} \right)^{-\epsilon^{a,F}} \frac{1}{P_{t+s}^{a,l^F}} \left(X_{H,t+s} \right)^{-\epsilon^{a,F}} Y_{t+s}^{a,F} \right] = 0 \end{aligned}$$

which after factoring out $\left(P_t^{a,l^F}\right)^{-\epsilon^{a,F}}$ implies:

$$\sum_{s=0}^{\infty}\left(\beta\theta^H\right)^s\mathcal{E}_t\frac{\Lambda_{1,t+s}P_{t+s}^cZ_{t+s}^a}{\Lambda_{1,t}P_t^cZ_t^a}\left[\left(1-\epsilon^{a,F}\right)\left(\frac{1}{P_{t+s}^{a,F}}\right)^{-\epsilon^{a,F}}\left(X_{H,t+s}\right)^{1-\epsilon^{a,F}}Y_{t+s}^{a,F}\right.$$
$$\left.+\epsilon^{a,F}\lambda_{H,t+s}^{a,l^F}\left(\frac{1}{P_{t+s}^{a,F}}\right)^{-\epsilon^{a,F}}\frac{1}{P_{t+s}^{a,l^F}}\left(X_{H,t+s}\right)^{-\epsilon^{a,F}}Y_{t+s}^{a,F}\right]=0$$

which can be solved for P_t^{a,l^F}

$$P_{t+s}^{a,l^F}=\left(\frac{\epsilon^{a,F}}{\epsilon^{a,F}-1}\right)\frac{\sum_{s=0}^{\infty}\left(\beta\theta^H\right)^s\mathcal{E}_t\frac{\Lambda_{1,t+s}P_{t+s}^cZ_{t+s}^a}{\Lambda_{1,t}P_t^cZ_t^a}\lambda_{H,t+s}^{a,l^F}\left(\frac{1}{P_{t+s}^{a,F}}\right)^{-\epsilon^{a,F}}\left(X_{H,t+s}\right)^{-\epsilon^{a,F}}Y_{t+s}^{a,F}}{\sum_{s=0}^{\infty}\left(\beta\theta^H\right)^s\mathcal{E}_t\frac{\Lambda_{1,t+s}P_{t+s}^cZ_{t+s}^a}{\Lambda_{1,t}P_t^cZ_t^a}\left(\frac{1}{P_{t+s}^{a,F}}\right)^{-\epsilon^{a,F}}\left(X_{H,t+s}\right)^{1-\epsilon^{a,F}}Y_{t+s}^{a,F}}$$

Their optimization problem is given by the following Lagrangian :

$$\mathcal{L} = \mathcal{E}_0\sum_{t=0}^{\infty}\left(\beta\theta^H\right)^t\left\{P_t^{l,F}Y_t^{l,F}-\int_0^1\left[W_{j,t}^FN_{j,t}^{l,F}+f^{N^F}\left(\frac{N_{j,t}^{l,F}}{\overline{N}_j^{l,F}}\right)\right]dj^F-R_t^kK_{t-1}^{l,F}\right.$$
$$\left.+\tilde{\lambda}_t^H\left[z_t^{Y,F}\left(A_t^Y\right)^{1-\alpha^F-\alpha^{g,F}}\left(K_{t-1}^{l,F}\right)^{\alpha^F}\left(N_t^{l,F}\right)^{1-\alpha^F}\left(K_{t-1}^g\right)^{\alpha^{g,F}}-Y_t^{l,F}\right]\right\}$$

subject to

$$N_t^{l,F}=\left(\int_0^1\left(N_{j,t}^{l,F}\right)^{\frac{\epsilon^w-1}{\epsilon^w}}dj^F\right)^{\frac{\epsilon^w}{\epsilon^w-1}}$$

and with prices given by the following equation in the periods in which firms are not able to optimally adjust them

$$P_t^{l,F}=P_{t-1}^{l,F}\pi_{t-1}^{lH}\overline{\pi}^{(1-lH)}$$

The first order conditions with respect to $N_{j,t}^{l,F}$, and $K_{t-1}^{l,F}$ are given by:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial N_{j,t}^{l,F}} &= \left(\beta\theta^{H}\right)^{t} z_t^u \left(\frac{\nu}{c_{1,t}-\varsigma c_{1,t-1}}+\frac{1-\nu}{c_{2,t}-\varsigma c_{2,t-1}}\right) + \left(\beta\theta^{w}\right)^{t+1} \mathcal{E}_t z_{t+1}^u \varsigma \left(\frac{\nu}{c_{1,t+1}-\varsigma c_{1,t}}+\frac{1-\nu}{c_{2,t+1}-\varsigma c_{2,t}}\right) \\
&\quad - \left(\beta\theta^{w}\right)^{t} \tilde{\lambda}_t \left(1-\tau_t^c\right) P_t^c = 0 \\
\frac{\partial \mathcal{L}}{\partial K_{t-1}^{l,F}} &= \left(\beta\theta^{H}\right)^{t} \gamma_M \left(\nu\left(\frac{M_{1,t}}{P_t}\right)^{-\gamma_M} + (1-\nu)\left(\frac{M_{2,t}}{P_t}\right)^{-\gamma_M}\right) - \left(\beta\theta^{w}\right)^{t} \tilde{\lambda}_t P_t + \left(\beta\theta^{w}\right)^{t+1} \mathcal{E}_t \tilde{\lambda}_{t+1} P_t = 0
\end{aligned}
$$

They imply

$$
\begin{aligned}
& z_t^u \left(\frac{\nu}{c_{1,t}-\varsigma c_{1,t-1}}+\frac{1-\nu}{c_{2,t}-\varsigma c_{2,t-1}}\right) + \beta\theta^{w} \varsigma \mathcal{E}_t z_{t+1}^u \left(\frac{\nu}{c_{1,t+1}-\varsigma c_{1,t}}+\frac{1-\nu}{c_{2,t+1}-\varsigma c_{2,t}}\right) = \tilde{\lambda}_t \left(1-\tau_t^c\right) P_t^c \\
& \gamma_M \left(\nu\left(\frac{M_{1,t}}{P_t}\right)^{-\gamma_M} + (1-\nu)\left(\frac{M_{2,t}}{P_t}\right)^{-\gamma_M}\right) = \left(\tilde{\lambda}_t - \beta\theta^{w} \mathcal{E}_t \tilde{\lambda}_{t+1}\right) P_t \\
& \tilde{\mu}_t = \beta\theta^{w} \mathcal{E}_t \left(\tilde{\lambda}_{t+1} \left(\left(1-\tau_{t+1}^k\right) R_{t+1}^k + \tau_{t+1}^k \delta Q_t\right) + \tilde{\mu}_{t+1}(1-\delta)\right) \\
& \tilde{\lambda}_t P_t^I - \tilde{\mu}_t z_t^I \left(1 - f\left(\frac{I_t}{I_{t-1}}\right) - f'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)\right) = \beta\theta^{w} \mathcal{E}_t \left(\tilde{\mu}_{t+1} z_{t+1}^I f'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right) \\
& \tilde{\lambda}_t = \beta\theta^{w} \mathcal{E}_t \tilde{\lambda}_{t+1} R_t \\
& \tilde{\lambda}_t S_t = \beta\theta^{w} \mathcal{E}_t \tilde{\lambda}_{t+1} S_{t+1} R_t^*
\end{aligned}
$$

The first order condition with respect to $P_t^{l,F}$ is given by

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial W_{j,t}^F} &= (\beta\theta^w)^t \gamma_N \epsilon^w \left(\frac{W_{j,t}^F}{W_t^F}\right)^{-\epsilon^w-1} \frac{1}{W_t^F} N_t^{F,d} \left[\left(\frac{W_{j,t}^F}{W_t^F}\right)^{-\epsilon^w} N_t^{F,d}\right]^{\sigma_N} \\
&+\mathcal{E}_t (\beta\theta^w)^{t+1} \gamma_N \epsilon^w \left(\frac{W_{j,t+1}^F}{W_{t+1}^F}\right)^{-\epsilon^w-1} \frac{1}{W_{t+1}^F} N_{t+1}^{F,d} \left(g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}\right) \left[\left(\frac{W_{j,t+1}^F}{W_{t+1}^F}\right)^{-\epsilon^w} N_{t+1}^{F,d}\right]^{\sigma_N} \\
&+\mathcal{E}_t (\beta\theta^w)^{t+2} \gamma_N \epsilon^w \left(\frac{W_{j,t+2}^F}{W_{t+2}^F}\right)^{-\epsilon^w-1} \frac{1}{W_{t+2}^F} N_{t+2}^{F,d} \left(\left(g^z \bar{\pi}^{(1-\chi^w)}\right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w}\right) \left[\left(\frac{W_{j,t+2}^F}{W_{t+2}^F}\right)^{-\epsilon^w} N_{t+2}^{F,d}\right]^{\sigma_N} \\
&+\cdots \\
&+(\beta\theta^w)^t \tilde{\lambda}_t (1-\tau_t^w) \left(-W_{j,t}^F \epsilon^w \left(\frac{W_{j,t}^F}{W_t^F}\right)^{-\epsilon^w-1} \frac{1}{W_t^F} N_t^{F,d} + \left(\frac{W_{j,t}^F}{W_t^F}\right)^{-\epsilon^w} N_t^{F,d}\right) \\
&+(\beta\theta^w)^{t+1} \mathcal{E}_t \tilde{\lambda}_{t+1} \left(1-\tau_{t+1}^w\right) \left(-W_{j,t+1}^F \epsilon^w \left(\frac{W_{j,t+1}^F}{W_{t+1}^F}\right)^{-\epsilon^w-1} \frac{1}{W_{t+1}^F} N_{t+1}^{F,d} \left(g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}\right)\right. \\
&\left.+\left(g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w}\right) \left(\frac{W_{j,t+1}^F}{W_{t+1}^F}\right)^{-\epsilon^w} N_{t+1}^{F,d}\right) \\
&+(\beta\theta^w)^{t+2} \mathcal{E}_t \tilde{\lambda}_{t+2} \left(1-\tau_{t+2}^w\right) \left(-W_{j,t+2}^F \epsilon^w \left(\frac{W_{j,t+2}^F}{W_{t+2}^F}\right)^{-\epsilon^w-1} \frac{1}{W_{t+2}^F} N_{t+2}^{F,d} \left(\left(g^z \bar{\pi}^{(1-\chi^w)}\right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w}\right)\right. \\
&\left.+\left(\left(g^z \bar{\pi}^{(1-\chi^w)}\right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w}\right) \left(\frac{W_{j,t+2}^F}{W_{t+2}^F}\right)^{-\epsilon^w} N_{t+2}^{F,d}\right) \\
&+\cdots \\
&= 0
\end{aligned}
$$

This condition implies

$$
\begin{aligned}
&\gamma_N \epsilon^w \frac{1}{W_{j,t}^F} \left[\left(\frac{W_{j,t}^F}{W_t^F} \right)^{-\epsilon^w} N_t^{F,d} \right]^{1+\sigma_N} \\
&+ (\beta\theta^w) \gamma_N \epsilon^w \frac{1}{W_{j,t}^F} \mathcal{E}_t \left[\left(\frac{W_{j,t+1}^F}{W_{t+1}^F} \right)^{-\epsilon^w} N_{t+1}^{F,d} \right]^{1+\sigma_N} \\
&+ (\beta\theta^w)^2 \gamma_N \epsilon^w \frac{1}{W_{j,t}^F} \mathcal{E}_t \left[\left(\frac{W_{j,t+2}^F}{W_{t+2}^F} \right)^{-\epsilon^w} N_{t+2}^{F,d} \right]^{1+\sigma_N} \\
&+ \cdots \\
&= (\epsilon^w - 1) \tilde{\lambda}_t (1 - \tau_t^w) \left(\frac{W_{j,t}^F}{W_t^F} \right)^{-\epsilon^w} N_t^{F,d} \\
&+ (\epsilon^w - 1) (\beta\theta^w) \mathcal{E}_t \tilde{\lambda}_{t+1} \left(1 - \tau_{t+1}^w\right) \left(\frac{W_{j,t+1}^F}{W_{t+1}^F} \right)^{-\epsilon^w} N_{t+1}^{F,d} \left(g^z \bar{\pi}^{(1-\chi^w)} \pi_t^{\chi^w} \right) \\
&+ (\epsilon^w - 1) (\beta\theta^w)^2 \mathcal{E}_t \tilde{\lambda}_{t+2} \left(1 - \tau_{t+2}^w\right) \left(\frac{W_{j,t+2}^F}{W_{t+2}^F} \right)^{-\epsilon^w} N_{t+2}^{F,d} \left(\left(g^z \bar{\pi}^{(1-\chi^w)} \right)^2 \pi_{t+1}^{\chi^w} \pi_t^{\chi^w} \right) \\
&+ \cdots
\end{aligned}
$$

When unions can't adjust optimally their wages those are $W_{j,t+S}^F = W_{j,t}^F \prod_{s=0}^{S} g^z \bar{\pi}^{(1-\chi^w)} \pi_{t+s}^{\chi^w}$. Defining $X_{j,t,S}^F = \prod_{s=0}^{S} g^z \bar{\pi}^{(1-\chi^w)} \pi_{t+s}^{\chi^w}$, we can re-write those wages as $W_{j,t+S}^F = W_{j,t}^F X_{j,t,S}^F$, and the previous expression as

$$
\begin{aligned}
&\gamma_N \epsilon^w \frac{1}{W_{j,t}^F} \mathcal{E}_t \sum_{s=0}^{\infty} (\beta\theta^w)^s \left[\left(\frac{W_{j,t}^F X_{j,t,s}^F}{W_{t+s}^F} \right)^{-\epsilon^w} N_{t+s}^{F,d} \right]^{1+\sigma_N} \\
&= (\epsilon^w - 1) \mathcal{E}_t \sum_{s=0}^{\infty} (\beta\theta^w)^s \tilde{\lambda}_{t+s} \left(1 - \tau_{t+s}^w\right) \left(\frac{W_{j,t}^F X_{j,t,s}^F}{W_{t+s}^F} \right)^{-\epsilon^w} N_{t+s}^{F,d} X_{j,t,s}^F
\end{aligned}
$$

After further manipulation the expression can be re-written as:

$$\mathcal{E}_t \sum_{s=0}^{\infty} (\beta\theta^w)^s U_N\left(N_{j,t,s}^F\right)\left(X_{j,t,s}^F\right)^{-\epsilon^w}\left(W_{t+s}^F\right)^{\epsilon^w} N_{t+s}^{F,d}\left[\left(\frac{\nu}{MRS_{1,t+s}}+\frac{1-\nu}{MRS_{2,t+s}}\right)\frac{W_{j,t}^F X_{j,t,s}^F}{P_{t+s}^c}-\frac{\epsilon^w}{\epsilon^w-1}\right] = 0$$

where

$$\begin{aligned} U_N\left(N_{j,t+s}^F\right) &= -\gamma_N\left(N_{j,t,s}^F\right)^{\sigma_N} \\ U_c\left(c_{i,t+s}^F\right) &= \tilde{\lambda}_{i,t+s}\left(1-\tau_{t+s}^c\right)P_{t+s}^c \\ MRS_{i,t+s} &= -\frac{U_N\left(N_{j,t,s}^F\right)}{U_c\left(c_{i,j,t,s}\right)}\frac{\left(1-\tau_{t+s}^c\right)}{\left(1-\tau_{t+s}^w\right)} \end{aligned}$$

The corresponding expression in the original paper, which is in real terms is

$$\mathcal{E}_t \sum_{s=0}^{\infty} (\beta\theta^w)^s U_N\left(N_{j,t,s}^F\right)\left(\tilde{X}_{j,t,s}^F\right)^{-\epsilon^w}\left(w_{t+s}^F\right)^{\epsilon^w} N_{t+s}^{F,d}\left[\left(\frac{\nu}{MRS_{1,t+s}}+\frac{1-\nu}{MRS_{2,t+s}}\right)w_{j,t}^F \tilde{X}_{j,t,s}^F-\frac{\epsilon^w}{\epsilon^w-1}\right] = 0$$

where $\tilde{X}_{j,t,S}^F = \prod_{s=1}^{S}\left(\frac{g^z\bar{\pi}^{(1-\chi^w)}\pi_{t+s-1}^{\chi^w}}{\pi_{t+s}}\right)$ and $w_{j,t+s}^F = \frac{W_{j,t+s}^F}{P_{t+s}^c}$ and $w_{t+s}^F = \frac{W_{t+s}^F}{P_{t+s}^c}$.

Real terms, using P_t^c

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^{\infty}\mathbb{E}_0\left\{\beta^t u\left(c_t, N_t, m_t\right)+\lambda_t\left[\left(\left(1-\tau_t^k\right)r_t^k+\tau_t^k\delta\frac{q_{t-1}}{1+\pi_t}\right)K_{t-1}\right.\right. \\ &+ \left(1-\tau_t^w\right)\int w_{j,t}^F N_{1,j,t}^F dj + w_t^I N_{1,t}^I + \frac{\gamma^{1,O}}{1-\omega}e_t\overline{O}_t p_t^{O^*} + \frac{R_{t-1}}{1+\pi_t}b_{t-1} \\ &+ e_t\frac{R_{t-1}^*}{1+\pi_t^*}b_{t-1}^* + tr_{1,t} + \tilde{\xi}_t + \frac{m_{1,t-1}}{1+\pi_t} - (1+\tau_t^c)c_{1,t} - p_t^I I_t - m_{1,t} - b_t - e_t b_t^* \left.\left.\right]\right\} \end{aligned}$$

The numeraire in the economy is the price of consumption goods P_t^c. The Budget constraint in real terms of the households with access to credit markets and owning physical capital is given by:

$$
\begin{aligned}
(1+\tau_t^c)c_{1,t}+p_t^I I_t+m_{1,t}+b_t+e_t b_t^* &= \left[\left(1-\tau_t^k\right) r_t^k+\tau_t^k \delta \frac{q_{t-1}}{1+\pi_t}\right] K_{t-1} \\
&+ \left(1-\tau_t^w\right) \int w_{jt}^F N_{1,j,t}^F dj + w_t^I N_{1,t}^I + \frac{\gamma^{1,O}}{1-\omega} e_t \bar{O}_t p_t^{O^*} \\
&+ \frac{R_{t-1}}{1+\pi_t} b_{t-1} + e_t \frac{R_{t-1}^*}{1+\pi_t^*} b_{t-1}^* + tr_{1,t} + \tilde{\xi}_t + \frac{m_{1,t-1}}{1+\pi_t}
\end{aligned}
$$

where $p_t^I = \frac{P_t^I}{P_t^c}$ is the relative price of investment goods in terms of consumption goods; $m_{i,t}$ are real money balances for the household type $i = 1, 2$, b_t are real government's bonds holdings, b_t^* are real foreign bonds holdings, $e_t = \frac{S_t P_t^{c^*}}{P_t^c}$ is the real exchange rate, r_t^k is the real return from physical capital, $q_t = \frac{Q_t}{P_t^c}$ is the relative price of installed capital ; real wages for labor variety j in the formal and informal labor market are given by $w_{j,t}^F$ and w_t^I respectively, $p_t^{O^*} = \frac{P_t^{O^*}}{P_t^{c^*}}$ is the international relative price of commodity exports in terms of consumption, π_t is the (consumer) inflation rate between periods t and $t-1$, $tr_{i,t}$ are government transfers in real terms, and $\tilde{\xi}_t = \frac{\xi_t}{P_t^c}$ are real firms' profits.

A.3.6 Inflation Evolution and Price Distortions

From the optimization problem of the firms in sector $a \in \{X, S_i\}$ we know that the price index of in the sector (a, ii), $P_t^{a,ii}$, is given by

$$
P_t^{a,ii} = \left(\int_0^1 \left(P_t^{a,l^{ii}}\right)^{1-\epsilon^{a,ii}} dl^{a,ii}\right)^{\frac{1}{1-\epsilon^{a,ii}}}
$$

which can be re-written as

$$
\pi_t^{a,ii} = \left(\left(1-\theta^{ii}\right)\left(\tilde{p}_t^{a,ii,*}\pi_t^{a,ii}\right)^{1-\epsilon^{a,ii}} + \theta^{ii}\left(\pi_{t-1}^{lii}\overline{\pi}^{(1-lii)}\right)^{1-\epsilon^{a,ii}}\right)^{\frac{1}{1-\epsilon^{a,ii}}}
$$

We can define the price dispersion as

$$\nu_t^{a,ii,p} = \int_0^1 \left(\frac{P_t^{a,l^{ii}}}{P_t^{a,ii}} \right)^{-\epsilon^{a,ii}} dl^{a,ii}$$

$\nu_t^{a,ii,p}$ measures the welfare cost of price dispersion that makes $Y_t^{a,l^{ii}}$ differ across firms because of the different timing of their price revisions. Those that have not revised their prices in a long time produce more than if they had been able to revise each period (as their prices are below the average prices), and those that just have revised their prices are producing less than the they would otherwise (their prices are above the average prices). $\nu_t^{a,ii,p}$ is given by:

$$\nu_t^{a,ii,p} = \int_0^{1-\theta^{ii}} \left(\frac{P_t^{a,l^{ii},*}}{P_t^{a,ii}} \right)^{-\epsilon^{a,ii}} dl^{a,ii} + \int_{1-\theta^{ii}}^1 \left(\frac{P_{t-1}^{a,l^{ii}} \pi_{t-1}^{lii} \overline{\pi}^{1-lii}}{P_t^{a,ii}} \right)^{-\epsilon^{a,ii}} dl^{a,ii}$$

and this corresponds to

$$\nu_t^{a,ii,p} = \left(1-\theta^{ii}\right) \left(\frac{P_t^{a,ii,*}}{P_t^{a,ii}} \right)^{-\epsilon^{a,ii}} + \left(\pi_{t-1}^{lii} \overline{\pi}^{1-lii}\right)^{-\epsilon^{a,ii}} \int_{1-\theta^{ii}}^1 \left(\frac{P_{t-1}^{a,l^{ii}}}{P_t^{a,ii}} \frac{P_{t-1}^{a,ii}}{P_{t-1}^{a,ii}} \right)^{-\epsilon^{a,ii}} dl^{a,ii}$$

which can be expressed in recursive terms as

$$\nu_t^{a,ii,p} = \left(1-\theta^{ii}\right) \left(\frac{p_t^{a,ii,*}}{p_t^{a,ii}} \right)^{-\epsilon^{a,ii}} + \left(\frac{\pi_{t-1}^{lii} \overline{\pi}^{1-lii}}{\pi_{t-1}^{a,ii}} \right)^{-\epsilon^{a,ii}} \theta^{ii} \nu_{t-1}^{a,ii,p}$$

The total market supply of intermediate inputs produced by sector (a, ii) is given by $\int_0^1 Y_t^{a,l^{ii}} dl^{a,ii}$, and corresponds to

$$\int_0^1 Y_t^{a,l^{ii}} dl^{a,ii} = \int_0^1 \left[\left(\frac{P_t^{a,l^{ii}}}{P_t^{a,ii}} \right)^{-\epsilon^{a,ii}} Y_t^{a,ii,d} \right] dl^{a,ii} = \nu_t^{a,ii,p} Y_t^{a,ii,d}$$

A.3.7 Model Equilibrium

In equilibrium the budget constraints of the consumers, the government and the central bank are satisfied, the markets for services, for labor, and for capital clear, and the balance of payments is given.

We define aggregate consumption of exportable goods C_t^X, importable goods C_t^M, services $C_t^{S_i}$, aggregate labor supply in the formal labor market N_t^F, aggregate labor supply in the informal labor market N_t^I, and aggregate money demand M_t as

$$\begin{aligned}
C_t^X &= \int_j c_{j,1,t}^X dj + \int_j c_{j,2,t}^X dj = c_{1,t}^X + c_{2,t}^X \\
C_t^M &= \int_j c_{j,1,t}^M dj + \int_j c_{j,2,t}^M dj = c_{1,t}^M + c_{2,t}^M \\
C_t^{S_i} &= \int_j c_{j,1,t}^{S_i} dj + \int_j c_{j,2,t}^{S_i} dj = c_{1,t}^{S_i} + c_{2,t}^{S_i} \\
N_t^F &= \int_j N_{j,1,t}^F dj + \int_j N_{j,2,t}^F dj = N_{1,t}^F + N_{2,t}^F \\
N_t^I &= \int_j N_{j,1,t}^I dj + \int_j N_{j,2,t}^I dj = N_{1,t}^I + N_{2,t}^I \\
M_t^d &= \int_j M_{j,1,t} dj + \int_j M_{j,2,t} dj = M_{1,t} + M_{2,t}
\end{aligned}$$

The expressions for labor services j in the formal labor market $N_{j,t}^{F,d}$, the total labor demand in the formal labor market $N_t^{F,d}$, the total labor demand for labor services j in the informal labor market $N_{j,t}^{F,d}$, the total labor demand in the informal labor market $N_t^{F,d}$, and total capital demand K_{t-1}^{d} are as follows

$$N_{j,t}^{F,d} = \int_l N_{j,t}^{X,l^F} dl + \sum_i^S \int_l N_{j,t}^{S_i,l^F} dl = N_{j,t}^{X,F} + \sum_i^S N_{j,t}^{S_i,F}$$

$$N_{t}^{F,d} = \int_j N_{j,t}^{X,F} dj + \sum_i^S \int_j N_{j,t}^{S_i,F} dj = N_{t}^{X,F} + \sum_i^S N_{t}^{S_i,F}$$

$$N_{j,t}^{I,d} = \int_l N_{j,t}^{X,l^I} dl + \sum_i^S \int_l N_{j,t}^{S_i,l^I} dl = N_{j,t}^{X,I} + \sum_i^S N_{j,t}^{S_i,I}$$

$$N_{t}^{I,d} = \int_j N_{j,t}^{X,I} dj + \sum_i^S \int_j N_{j,t}^{S_i,I} dj = N_{t}^{X,I} + \sum_i^S N_{t}^{S_i,I}$$

$$\begin{aligned} K_{t-1}^{d} &= \int_l K_{t-1}^{X,l^F} dl + \sum_i^S \int_l K_{t-1}^{S_i,l^F} dl + \int_l K_{t-1}^{X,l^I} dl + \sum_i^S \int_l K_{t-1}^{S_i,l^I} dl \\ &= K_{t-1}^{X,F} + \sum_i^S K_{t-1}^{S_i,F} + K_{t-1}^{X,I} + \sum_i^S K_{t-1}^{S_i,I} = K_{t-1}^{X} + \sum_i^S K_{t-1}^{S_i} \\ &= K_{t-1}^{F} + K_{t-1}^{I} \end{aligned}$$

The equilibrium in the goods and services markets is given by the following expressions

$$\begin{aligned} Y_t^X &= C_t^X + I_t^X + G_t^X + I_t^{g,X} + C_t^{*,X} \\ Y_t^{S_i} &= C_t^{S_i} + G_t^{S_i} + I_t^{g,S_i} \end{aligned}$$

The equilibrium in the external sector is given by the Balance of Payments equation

$$s_t \left(nfa_t - nfa_{t-1} \right) = p_t^X C_t^{*,X} + s_t p_t^{O^*} \bar{O}_t - s_t \left(C_t^M + I_t^M + G_t^M + I_t^{g,M} \right) + s_t \left(\frac{R_{t-1}^*}{\pi_t^*} - 1 \right) nfa_{t-1}$$

The equilibrium in the formal and informal labor markets is given by

$$\begin{aligned} N_t^F &= N_t^{X,F} + \sum_i^S N_t^{S_i,F} = N_t^{F,d} \\ N_t^I &= N_t^{X,I} + \sum_i^S N_t^{S_i,I} = N_t^{I,d} \end{aligned}$$

The equilibrium in the capital market is given by

$$K_{t-1} = K_{t-1}^{X} + \sum_{i}^{S} K_{t-1}^{S_i} = K_{t-1}^{d}$$

The equilibrium in the domestic bond market is given by

$$B_t = B_t^h + B_t^{cb}$$

The equilibrium in the money market is given by

$$M_t^d = M_t$$

PUBLICATIONS

Informality and Shock Propagation in an Open Economy

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