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Inside (the) Money Machine: Modeling Liquidity, Maturity and Credit Transformations

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ABSTRACT: The key function of banks in the real world is endogenously creating (inside) money. But they do so facing solvency, liquidity and maturity risks and being subject to regulatory and demand constraints. These five aspects, representing the eventual breaks on banks' money-creation abilities, are tightly and nonlinearly interlinked. Yet, there is no tractable quantitative macro framework that models endogenous money creation while simultaneously addressing these interlinkages. In this paper we develop a tractable macro-banking model trying to fill this gap, emphasizing two key frictions: the capital adequacy constraint (generating a credit risk premium) and the central bank's collateral base constraint (generating a liquidity risk premium). The model simulations produce conclusions, about both normal times as well as stress episodes, many of which were frequently overlooked. For instance, it shows how – within capital requirements – setting lower risk weights on secured loans may lead to an expansion of unsecured loans. It also reveals subtle interactions between capital and liquidity regulations. The model also creates a certain bridge between a money-centered view of the price level and the fiscal theory of the price level.

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WORKING PAPERS

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1. Introduction

"All models are wrong, but some are useful"
George Box

What does a macro-banking model need to have to be useful? It needs to incorporate banks' main real-life function – endogenous (inside) money creation by liquidity, maturity and credit transformations, subject to regulatory and demand constraints.[2] The way that the macro modeling literature incorporates banks is mostly at odds with their real-life main functions (e.g. literal intermediation of real resources, *a la* loanable funds theory) or which capture only some parts of the above-mentioned aspects. In reality, banks create money, but do so facing solvency, liquidity and maturity risks, while being subject to regulatory constraints (e.g. capital adequacy requirements, liquidity requirements or central bank collateral eligibility policy, reserve requirements, etc.) and demand constraints (general macroeconomic conditions, including nonfinancial sector's reaction to interest rate adjustments). All these five aspects are tightly and nonlinearly interlinked.

Yet, there is no tractable equilibrium macro-banking model in the literature that addresses all those issues simultaneously and endogenously. Goodhart (2009) provides a summary of this shortcoming within monetary economics. This is where this paper tries to fill the gap. We develop a macro-banking theory, resulting in a tractable model that qualitatively and, in most cases, quantitatively provides realistic and intuitive causal answers to several dozen highly policy-relevant questions at the intersection of macroeconomics and financial stability. The model simulations produce conclusions, for normal times as well as crisis episodes, many of which were frequently overlooked. The model can be used for monetary and macroprudential policy analysis, as well as their interactions. It can be used for policy (counterfactual) simulations, e.g. understanding how a financial system may react to the introduction of a central bank digital currency (CBDC).

One of the many important messages from the model is that liquidity regulation (mandating banks to maintain low liquidity risk) and capital adequacy regulation (mandating banks to maintain low solvency risk) may be tightly interlinked. The reason is that the key overall fragility of banks comes from them creating (potentially too much) inside money by stretching their balance sheets, which can – to some extent similarly – be affected by both types of regulation.[3] In addition, the credit risk channel of monetary policy becomes part of the transmission mechanism. Namely, the model endogenously predicts, in line with empirical evidence, that

[2] As long as central banks defend financial stability (prevent systemic bank defaults), then inside money (created by private banks) and outside money (created by central banks), from the public's viewpoint, are considered to be the same (due to at-par convertibility). Hence, in some contexts, we can drop the word "inside". Also note that one bank's lending decisions create money at the system level, even if its own balance sheet responds differently.

[3] In an extreme example, if the bank must hold equity equal to the entire loan (non-HQLA) portfolio (≈100% capital requirement), it means all the deposits will now be backed by liquid riskless securities (HQLA) portfolio, automatically making liquidity risk insured (≈100% liquidity/reserve requirement).

when monetary policy becomes too loose, banks increase leverage too much (in search for yields) that increases their fragility (solvency risk). Thanks to strong nonlinearities, the model can also easily generate a credit crunch or sharp deleveraging scenario even without large shocks, if initial conditions are fragile (e.g. due to banks' excessive optimism about the credit risk). This can be useful for central banks conducting forecasting and policy analysis rounds, highlighting potential tail risks, and how to respond to them appropriately in real time.[4] The framework can also understand an asymmetric speed of impacts from monetary tightening versus easing, as well as why central banks may need to pay close attention to the size of their collateral base. The tradeoffs associated with a narrow banking approach also become evident. Moreover, we uncover macro-banking determinants of the neutral policy rate, which has frequently been missing from monetary policy discussions.

The model also implies that when fiscal policy issues more public debt to finance deficit spending, it raises the price level, other things equal. This inflationary impact doesn't necessarily come from a standard New Keynesian logic present in most DSGE models. This is very much in line with the fiscal theory of the price level (FTPL), which implies a direct link between public debt and the price level, even under flexible prices (e.g. Cochrane, 2023). Unlike the FTPL, however, the main channel in our model is through endogenous money creation: when the government issues a bond *and* spends the proceeds (i.e. deficit spending), it eventually contributes to creating broad money,[5] which may increase nominal demand beyond the impact implied by the risk-free (policy) rate. This excess nominal demand lingers, until either the price level sufficiently increases to absorb the extra level of the public debt, or private credit is crowded out pushing broad money back down to its initial level. Hence, our model is also able to motivate why monetary policy may (at least, theoretically) still be able to defend price stability: by sufficiently tightening monetary policy and, hence, private lending rates, it can moderate the extra money creation induced by the public debt expansion. This means that, unlike FTPL where active fiscal policy fully determines the price level and monetary policy just ensures debt sustainability, in our model monetary policy can still maintain price stability even when fiscal policy doesn't cooperate. But this can just happen at the expense of a potentially costly private sector deleveraging. Discussion of additional model implications are relegated into the appendix.

The paper is organized as follows: section 2 briefly discusses the related literature, to emphasize the relative novelty of the model developed here; section 3 builds the financial sector of our macro-banking model; section 4 outlines the nonfinancial sector of the model; section 5 shows some key policy simulations to demonstrate the usability and tractability of the model,

[4] Naturally, any new model would first need time and scrutiny to prove its usefulness for a real-time policymaking. Also, the current version abstracts from some important details, e.g. open economy considerations.

[5] In reality, this is not always the case. Namely, if it's the non-bank private sector who buys newly-issued government bonds, then new money creation is not taking place. In this case bond issuance absorbs existing money (of the non-bank private sector) and government spending then re-injects it back again. In this case, however, money changes hands (e.g. from wealthy investors to less wealthy households) and may potentially lead to an acceleration of velocity, with similar effects as in the cases when money creation does happen.

while section 6 concludes and discusses potential extensions and research avenues for the future. Appendix 1 briefly summarizes all directly visible (monetary, macroprudential, fiscal) policy messages coming from the model, while appendix 2 details the derivations of the key interest rate equations.

2. Literature summary

"Among the worst are… standard theory of the evolution of money"
Charles Goodhart

The macro-financial literature to date has modeled banks in various ways. Bernanke *et al* (1999) is a seminal work that shows how asymmetric information in financial intermediation can lead to state-dependent external finance premium for firms (financial accelerator of nonfinancial shocks). See also an earlier similar paper by Carlstrom and Fuerst (1997). But these papers model nonfinancial companies' net worth and treat banks essentially as a veil.[6] Kiyotaki and Moore (1997) also does a similar job but with the assumption that firms face a collateral constraint when taking out a (in equilibrium, risk-free) loan from a bank. Again, the focus is on nonfinancial firms and less so on banks. Also, this and similar models that build on Kiyotaki-Moore framework try to model collateral constraints on risk-free debt, while in reality risk-free debt sometimes (e.g. if issued by a sovereign) is itself something that *relaxes* (not tightens) collateral constraints (on risky debt with actual defaults), because it is itself used as a collateral accepted even by central banks.

In addition, at least since the Global Financial Crisis (GFC), we understand very well that banks themselves (and their balance sheets) can also be a source of fluctuations on their own. One of the most influential papers that evolved on this premise, Gertler and Kiyotaki (2010), then modeled interlinkages between macroeconomics and banking where banks could generate macro fluctuations. Their model incorporates financial frictions due to borrowing constraints, where firms depend on banks for external financing, with an agency problem between the two, leading to monitoring costs that affect credit availability by banks. But there is also an agency problem between depositors and banks which can lead to a situation where an intermediary's balance sheet limits its ability to obtain deposits, that *then* hurts credit extension. See also Gertler and Karadi (2011) for a DSGE model where banks' balance sheets constrain their lending decisions but feature no role for bank runs and liquidity crunches.

Gertler and Kiyotaki (2015) do introduce a possibility of a bank run in a similar model, but there is no quantitative role for the central bank's lender of last resort monetary operations that, in reality, are tightly linked to bank runs and their macro implications. Also, banks in their model,

[6] Carlstrom *et al* (2016) provide a starker critical evaluation of the Bernanke *et al* (1999) paper, arguing that the financial accelerator in that model disappears when the privately optimal contract is incorporated.

as in most other models, intermediate real resources,[7] and are not modeled as (constrained) creators of broad (inside) money, which in reality they are. This downplays the role of banks in quickly generating credit booms if they decide to do so, e.g. due to excessively optimistic views about risks.[8] That by implication also downplays the role of banks, or a lack of proper regulation, in creating grounds for a subsequent financial crisis and quick deleveraging. Mistaking real savings (from the National Accounts) with financial savings can be dangerous, especially during those times when we need an understanding of macro-financial linkages the most. Still another influential paper is Cúrdia and Woodford (2010) which extends a standard DSGE model by introducing financial intermediaries that set a spread between the interest rates they charge on borrowers and the risk-free rate. This spread is influenced by their own balance sheet conditions. But here again, banks are assumed to “attract” deposits first and only make loans afterwards (i.e. intermediating real resources), which is at odds with the idea of endogenous money creation (see McLeay *et al*, 2014). Again, this downplays the role of banks in credit booms and busts.

DSGE models by Benes *et al* (2014) as well as Jakab and Kumhof (2015) are the closest ones to what we develop in this paper. However, their models only feature solvency risks (credit loss absorbing capacity) of banks that generate nonlinear macro-financial dynamics. Their models allow for an accumulation of vulnerabilities that can lead to crises as leverage builds up during boom periods. They also emphasize banks’ role in endogenously creating money through lending, with these newly created deposits being used as a medium of exchange. But they abstract from the limited liability aspect, which rarely but still can become an issue. More importantly, as mentioned, they don’t say anything about the liquidity and maturity risks that banks may face, what they depend on and how they influence banks’ money creation abilities/incentives. Hence, they remain essentially silent on the liquidity and maturity transformation aspects. For instance, using a similar model, Benes and Kumhof (2012) argue for a narrow banking approach, which would shut down the endogeneity of money creation by private banks. But they abstract from a liquidity transformation aspect (and benefits coming from it), which could be really the most important downside of such a narrow banking approach. On the other hand, Brunnermeier and Sannikov (2016) focus on liquidity and maturity

[7] That’s why they could make an assumption that assets the banks hold (meant to capture loans) are such assets that the households can hold as well (even if with some management costs). In reality, the difference between banks’ lending and households’ lending is that the former creates broad money (because bank lending is funded by issuing a deposit liability that acts as a medium of exchange), while the latter does not. That is a fundamental difference.

[8] Note the difference between physical capital and financial capital: the first one is an accumulation of past (labor/goods), while the second one is a promise about the future (cash flows). Past (physical capital) cannot be changed instantaneously (it is a predetermined variable), while promises about the future can be made in vast amounts quickly (as long as they are worth anything; and bank liabilities are worth as much as being accepted as a medium of exchange). That’s why banks are more elastic in credit extension, since they expand their balance sheets with financial assets and liabilities. Again, this is important for thinking about credit booms and their speed better. See Borio and Disyatat (2011).

transformations and how inside money is endogenously created, but abstract from banks' solvency and capital adequacy issues.

To summarize the macro-financial literature for our purposes, without spending too much space on discussing many more interesting but broadly similarly-spirited papers, it seems the most important issue with these models is that they do not directly and simultaneously model the most important micro-foundations of what banks really do – endogenously creating money with liquidity, maturity and credit transformations, subject to regulatory and demand constraints.[9] Instead, most of them try to approximate important final results with simple but abstract technical modeling choices. While useful in some cases, this approach faces significant limitations, especially when trying to answer policy-relevant questions with sufficient detail about the real underlying mechanism. An example would be modeling liquidity risks. In the real world, liquidity risks that banking systems as a whole face mostly depend on a central bank collateral eligibility policy for their monetary operations,[10] but that's not covered in most models.

That's why the model, or at least the banking part of it, we build below starts from scratch, not constrained by existing DSGE approaches that are usually abstract and relatively unrealistic. On the other hand, corporate finance literature has also modeled banks, sometimes in more "micro-founded" ways, but they usually remain only conceptual in nature (e.g. Kashyap *et al*, 2002, Diamond and Dybvig, 1983, Drechsler *et al*, 2021), that doesn't really allow them to be directly used in a quantitative business cycle model or in everyday policy analysis and simulations, unlike our model here. Gross and Siebenbrunner (2022) have emphasized many of the same issues we discuss here, however using an Agent-Based Model that is less well explored in the central banking community. Meanwhile, policy institutions have frequently emphasized the importance of all those aspects mentioned above (e.g., see Ihrig *et al*, 2021, Doherty *et al*, 2018, Borio and Disyatat, 2015 or Bundesbank, 2017), putting a premium on developing more policy-relevant general equilibrium models of macro-banking interactions.

3. Modeling setup: financial sector

The main idea that we model is banks creating/issuing one type of financial liability (which will be liquid, short-term, low risk, used as a medium of exchange) and using it to fund "buying" someone else's liability against this bank that will be an asset for the bank (which will be illiquid, long-term, riskier, but profit-generating). This is liquidity, maturity and credit transformation that

[9] Again, some important papers deal with some of these issues but not with all of them together, which is key.

[10] In theory, central banks have an unlimited power to create central bank money. However, in practice, they have to either buy assets to create central bank money, or lend against collateral. In both cases, they are expected not to take any credit risk. But since risk-free assets (to buy or take as collateral) aren't in unlimited quantity, this means central banks do have some constraints in terms of money creation. It's only in combination with fiscal policy (that *can* take credit risk) when central banks become truly unlimited in terms of money creation power.

banks make profit out of. Hence, for us to be able to model banks, we need to track their balance sheets (assets and liabilities) and their main items in sufficient detail, which is discussed first. Of course, discussion of balance sheets should include that of a central bank as well, since liquidity risk, as discussed above, is dependent on it. Then we introduce demand constraints and a couple of financial frictions one by one, which introduce breaks to banks' money-creation abilities. The key frictions are central banks' collateral base constraint and capital adequacy constraint.

3.1. Balance sheets and timeline

In the model, the financial system is populated by many commercial banks and the central bank, each having its own balance sheet, in the spirit similar to Mkhatrishvili *et al* (2024). Each bank is similar ex-ante, but differ ex-post, depending on risks materializing. All the balances sheets are finally aggregated and consistently capture stock-flow relationships. These balance sheets are shown in Figure 1, with Q_t, S_t, L_t^S and L_t^R being a bank's assets (liquidity/reserve balances, risk-free government securities,[11] private safe/secured loans and private risky/unsecured loans, respectively), while D_t, R_t and E_t being the bank's retail deposits, wholesale funding or a central bank discount window to get liquidity (refinancing loans) and bank equity/capital,[12] respectively, at a time instant that is at the beginning of period t.[13] R_t^X and CM_t stand for the central bank's FX reserves (net foreign assets) and currency (cash and potentially CBDC).[14]

The bank maximizes its after-tax return on equity, over the loan maturity, by choosing its balance sheet item quantities and interest rates, subject to demand and regulatory constraints (discussed below) as well as three key sources of uncertainty related to: [1] potential collateral-induced wholesale liquidity stress/runs (uncertainty: natural/structural demand for central bank money relative to the collateral base for monetary/wholesale operations), [2] potential panic-induced retail bank runs (uncertainty: retail depositors' trust), and [3] potential credit risk-induced default (uncertainty: nonperforming risky loans, i.e. borrower defaults).

[11] Of course, any security has some default risk even when issued by a strong sovereign, but we take the lowest-risk securities as a risk-free benchmark. Modeling potential sovereign defaults can be a future extension of the model.

[12] Here we'll be using the terms bank equity and capital interchangeably, unless it is explicit that by capital we refer to physical capital stock.

[13] When defining the banks' variables we will omit the i subscript that is usually used to denote the individual bank-specific variables, because each bank in this model will have similar optimization problems *ex-ante*, for the reasons discussed below. Due to this, we will have aggregate optimality conditions that are identical to individual ones, that make complicating the equations with the i subscripts everywhere unnecessary.

[14] The version of the model here depicts a financial system that is in a structural liquidity deficit, with central bank liquidity provision, like in many emerging small open economies. In that environment, buying risk-free securities and lending to banks against risk-free securities as collateral are close to equivalent for a central bank. That's why our central bank balance sheet doesn't have securities in it, only lending to banks against securities as collateral. But, in principle, we could also easily introduce an ample reserves regime with a central bank holding large amounts of risk-free securities, e.g. to model QE operations.

Figure 1 – Balance sheets of commercial banks and the central bank in the model

A commercial bank

	Assets	Liabilities	
Central bank reserve balances	Q_t	D_t	Deposits (retail funding)
Risk-free securities portfolio	S_t	R_t	Refinancing loans (wholesale funding)
Safe / secured loan portfolio	L_t^S	E_t	Bank equity / capital
Risky / unsecured loan portfolio	L_t^R		

The central bank

	Assets	Liabilities	
Refinancing loans	R_t	Q_t	Central bank reserve balances
FX reserves stock	R_t^X	CM_t	Currency (cash / CBDC)

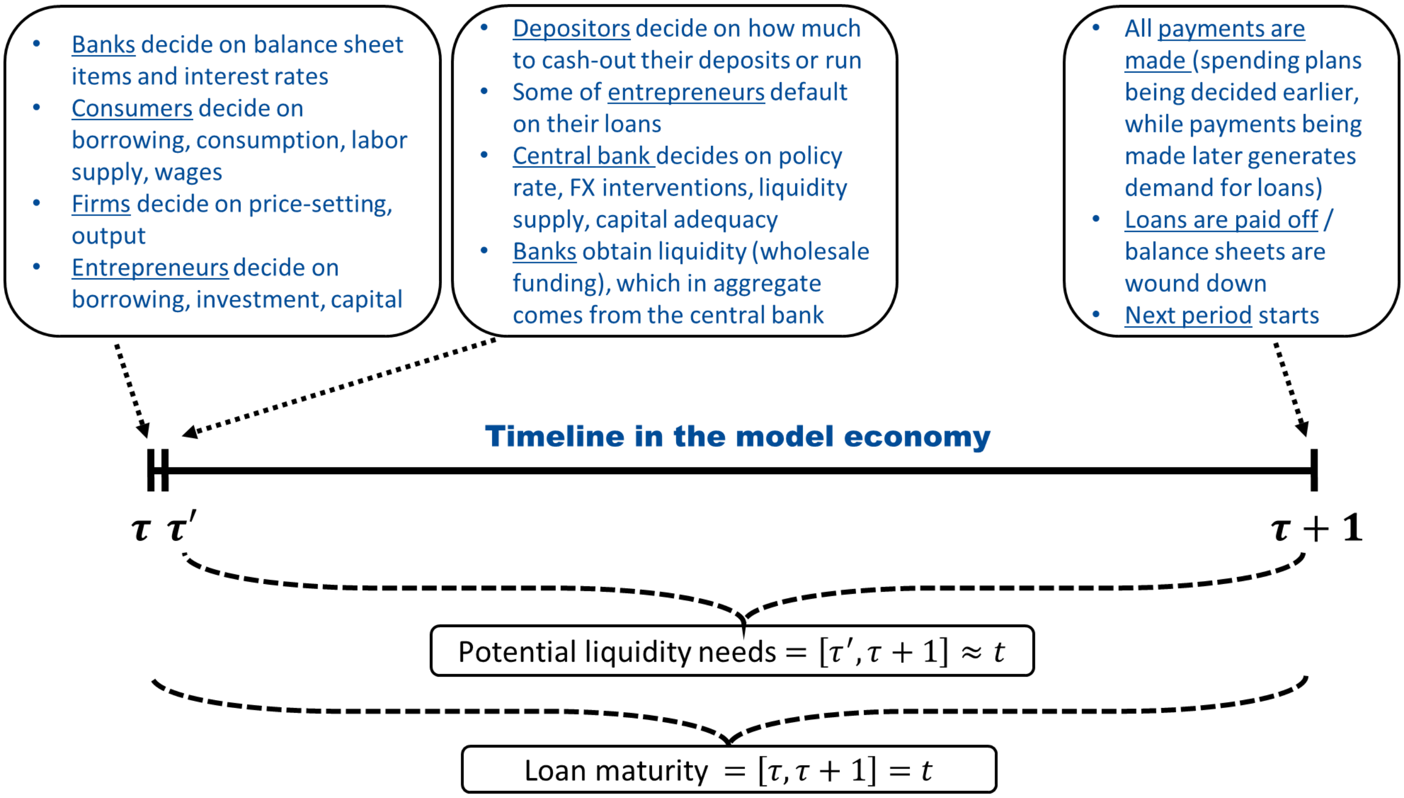
Source: author's construction

Note that, whenever a bank is solvent, it doesn't *automatically* default when it faces a bank run. The reason is our assumption that the central bank checks the solvency of this bank the best it can right after the eruption of its bank run and provides lending of last resort after confirming its solvency, but with a penalty rate/conditions. The only consideration for this bank is then not a default, but an extra cost (penalty rates, as discussed below) it will incur when a bank run happens. Hence, Bagehot principle applies in our model. Correia *et al* (2024) empirically support this characterization of bank failures. However, even if liquidity-induced solvency crisis doesn't happen *automatically*, it also doesn't mean that a bank run never results in a bank default in our model. It certainly can, and in most cases does in our baseline calibration of the penalty costs. As bank runs create an extra cost for banks, there will always be some banks for which this extra cost (reduction in profitability) is all it takes to push their equity level below the (regulatory) minimum.[15] Hence, a liquidity crisis can create a solvency crisis for weaker banks but likely not for strong banks (with strength measured in terms of equity buffers).

To fully grasp how the model economy and its banking system functions, it is best to first describe the timing conventions in the model and then introduce the objective function, constraints and sources of uncertainty one by one. Figure 2 shows the timeline, i.e. the ordering and list of choices by economic agents in the model, as well as maturities of the bank balance sheet items.

[15] Here we assume that equity ratio falling below the regulatory minimum is when supervisors take over and restructure a bank into a new one (i.e. the bank, in its current form of management and ownership, ceasing to exist). The model can be adjusted to incorporate bank defaults only after equity falls below 0% instead, but this wouldn't have changed our qualitative results while complicating some of the derivations.

Figure 2 – Timeline in the model



Source: author's construction

This way of ordering in decisions in the timeline technically leads to those three sources of uncertainties we mentioned above. This will help us in generating some important risk premia that will define banks behavior and potentially nasty nonlinearities. With these preliminaries (the composition of balance sheets and the timeline), we can proceed with stating the objective function of banks, but first, for expositional purposes, without demand constraints or sources of uncertainty (which will be introduced afterwards one by one).

3.2. Objective function of banks

Subject to its balance sheet identity constraint, at a minimum, banks decide on their balance sheet quantities (both types of loans, security holdings, deposits, refinancing loans, equity) and interest rates (loan rates, security yields, deposit rates) to maximize their risk-adjusted after-tax return on equity (ROE).[16] This is an important point. In standard macro-financial models, commercial banks are assumed to maximize just profit, but in reality, they maximize profit *per equity* (i.e. ROE). The latter usually leads to those banks underinvesting in their equity

[16] Of course, this assumption doesn't apply to all types of banks, some of which may have other considerations (other than ROE) when deciding on lending.

position,[17] as we can show within the baseline version of the model as well. Without any uncertainty and credit/deposit demand and regulatory constraints, each of the banks' optimization problem looks as follows:

$$\max_{Q_t, i_t^S, S_t, i_t^{SL}, L_t^S, i_t^{RL}, L_t^R, i_t^D, D_t, R_t, E_t} \mathbb{E}_t \frac{i_t^S\, S_t + i_t^{SL}\, L_t^S + i_t^{RL}\, L_t^R - i_t^D\, D_t - i_t^R\, (R_t - Q_t) - \psi}{E_t} (1-\tau) \tag{1}$$

$$subject\ to \qquad Q_t + S_t + L_t^S + L_t^R = D_t + R_t + E_t \tag{2}$$

where $\mathbb{E}_t$ is an expectations operator with information set available at time instant at the beginning of period t, ψ represents overhead (net non-interest) costs, while τ is the profit tax rate applied to banks. Also, i_t^S, i_t^{SL}, i_t^{RL}, i_t^D and i_t^R are interest rates on securities, safe loans, risky loans, deposits and central bank discount window (refinancing loans), respectively.[18]

The constraint (2) is just a balance sheet identity equation, which captures the idea of endogenously creating money with credit extension: banks can easily create new money by expanding their balance sheet through lending (to the government or to the private sector) and simultaneously issuing deposit liabilities to those borrowers. No need for prior *real* savings. Hence, if there is no uncertainty (which we will introduce only in the later subsections) and if we assumed that the banks didn't face regulatory and credit demand constraints, then the bank would be able to really create money ex-nihilo (out of thin air) without any limits. Indeed, mathematically, the solution to the above (simple) optimization problem is expanding the balance sheet infinitely largely (extending infinitely many loans), while setting the equity ratio at 0% (assuming only 0-100% range is allowed). Hence, banks would be able to make infinite profits with no equity. Naturally, this specification, while a good starting point to look at ROE and balance sheet identity, is not sufficient to make sense.

Indeed, in reality banks do not *limitlessly* create inside money, even if endogenous money creation does happen. Regulatory and demand constraints do exist in the real world, and they create a set of limits on banks' ability to create money seemingly out of nothing. For instance, if banks continue creating money, they should continue to find new borrowers who are willing to borrow, which they won't find unless the banks reduce loan interest rates. However, if they do that, it will reduce their profit. Hence, profit-maximization incentive coupled with borrowers' demand being negatively dependent on interest rates create the first set of limits on endogenous money creation.

[17] The lower the amount of equity the higher is ROE, even with the nominal amount of profits being unchanged.

[18] Note that the levels of interest rate on deposits (i_t^D) and central bank discount window/refinancing loans (i_t^R) that banks take into account when pricing their products, will depend on either actual current policy rate, if the central bank makes a policy rate decision at the very beginning of the period, or, as is in this model assumed, the expected policy rate, if the central bank makes a policy rate decision after the start of the period. Banks will face interest rate risk only in the second case (i.e. in our model as well).

3.3. Introducing demand constraints to banks

Downward sloping demand schedules that banks face imply a first limit on banks' ability to expand their balance sheets and hence create inside money. We model these constraints in a simple way. Namely, we assume that households borrow from each bank with an imperfect substitution between any two banks' loans (using standard CES aggregator), and compare different savings instruments (that are also imperfect substitutes) within the financial system,[19] resulting in the following:

$$S_t \leq f^S(i_t^S) \qquad (3)$$

$$L_t^S \leq f^{SL}(i_t^{SL}) \qquad (4)$$

$$L_t^R \leq f^{RL}(i_t^{RL}) \qquad (5)$$

$$i_t^D \geq f^D(i_t^R) \qquad (6)$$

where the demand constraints (3), (4) and (5) have the following standard CES property: $\frac{\partial f^x(i_t^x)}{\partial i_t^x} = -\varepsilon^x \frac{f^x(i_t^x)}{i_t^x}$, where $x = S, L^S, L^R$. The demand constraint (6) simply puts a lower bound on the deposit rates, including through competition from currency or money market mutual funds (MMMF), e.g. $i_t^D \geq \frac{\varepsilon^D - 1}{\varepsilon^D}\, i_t^R$ that is essentially a markdown on the central bank's policy rate. The most important aspect of these constraints is the negative relationship between credit demand and credit interest rates. Similar constraints in Mkhatrishvili *et al* (2024) are shown to be binding in equilibrium. Constraints (3), (4) and (5) come from competitive forces in the credit market (endogenous money creation), while (6) comes from competitive forces in the money market, where *existing* money can take different forms (deposits/MMMF shares/cash/CBDC). This difference is important, as emphasized by Mkhatrishvili and Boonstra (2022).

For illustration purposes, let's assume at this point that a fraction ($\hat{c}$) of deposits that are withdrawn from each bank, after they create it via lending, is constant (until we introduce the liquidity risk in the next subsections). Based on the balance sheet identities of the banks and the central bank, we then have $R_t - Q_t = \hat{c}\,(TA_t - E_t) - (1-\hat{c})R_t^X$ and $D_t = (1-\hat{c})(TA_t - E_t + R_t^X)$, where $TA_t = S_t + L_t^S + L_t^R$ represents total assets/lending portfolio. With these, the first order conditions of the above optimization problem lead to the following interest rate pricing equation:

$$i_t^x = \frac{\varepsilon^x}{\varepsilon^x - 1}\left((1-\hat{c})\, i_t^D + \hat{c}\, i_t^R\right) \qquad (7)$$

[19] Usual practice in DSGE model-building is to assume there is a separate economic agent doing the aggregation, even though it does not really do any other meaningful business. Operating under perfect competition, it produces an aggregated loans/deposits for nonfinancial agents, and use bank services with its demand schedules in mind. The CES aggregation, however, is applied only to the same type of loans (e.g. different banks' safe loans are aggregated that way) and it doesn't apply to different types of loans (e.g. risky vs safe).

for $x = S, L^S, L^R$. These optimal interest rates then lead to finite value solutions for the balance sheet items/quantities, based on the balance sheet identity and demand constraints, (2), (3)-(5). This looks intuitive, loan interest rates/yields being a (potentially time-varying) markup over the weighted average cost of funds (deposits/retail and refinancing/wholesale). As balance sheets are now finite, there's no longer limitless money creation. But at the same time, it is still overly simplistic, as it doesn't capture other important limits on the banks' incentives/ability to extend loans and create money.[20] These additional limits are liquidity risk (liability side) and credit risk (asset side), which we introduce next one by one.

3.4. Introducing exogenous bank runs

In terms of uncertainty, we first introduce an exogenous (at least, from the commercial banks perspective) source of uncertainty – panic-induced retail bank run risk.[21] This particular exogenous risk is not a particularly important part of macro-banking linkages in this model, but because of its simplicity and relevance for CBDC discussions (as we show later in the related upcoming paper), we still introduce it first. Here we follow the literature describing bank deposits as information-insensitive short-term debt (e.g. Gorton *et al,* 2012). In this spirit, depositors in our model start information acquisition about the bank's health if the cost of doing so is less than the expected loss due to potential bank default if they don't do anything. With $\hat{\sigma}_t$ denoting a depositor's perceived probability of a bank default (which can be a function of banks reputation or rumors), bank run (i.e. depositors starting the information acquisition process) happens if:

$$(1-\hat{\sigma}_t)\, D_t\, (1+i_t^D) \leq \; (1-\hat{\sigma}_t)\, D_t\, (1+i_t^D) + \; \hat{\sigma}_t\, D_t\, (1+i_t^{CB}-\phi) - \gamma\, D_t \qquad (8)$$

where i_t^{CB} interest rate on central bank money (cash/CBDC) holdings,[22] ϕ the cost of holding cash/CBDC (explicit or implicit),[23] and γ is the cost of information acquisition (digging deeper into the bank's balance sheet). The left-hand side is the gross return a depositor gets if he/she doesn't do anything (i.e. getting the gross deposit interest rate in case the bank doesn't default and getting nothing if it does). The right-hand side is the gross return the depositor gets if he/she does start information acquisition process (i.e. getting the gross deposit interest rate if he/she finds out the bank is solvent, while withdrawing the deposit and getting the gross cash/CBDC interest rate if he/she finds out it isn't solvent, with all these minus the information acquisition costs that are proportional to the deposit portfolio).

[20] Even though, the constraints (3), (4) and (5) have already resulted in a finite solution for lending volumes, liquidity and credit risk would compress these volumes even further.

[21] For the central bank, managing this risk may very well be an endogenous problem, dependent on e.g. CBDC policy choices/variables.

[22] Interest rate on cash is naturally zero, while it *can* be nonzero for CBDC.

[23] This cost *can* be different for cash versus CBDC. This can have an important impact on the likelihood of bank runs in our model.

In other words, a depositor should acquire information if the cost of doing so is less than what one is expected to gain with perfect information. If you realize the bank is healthy you will keep your deposits, if not you will immediately convert them into cash, CBDC or, more likely, another bank's deposit before others do, so that you avoid losing money. Now, if I know that others will start acquiring information as well, I may be left with worthless deposits if I'm one of the last ones to go to the bank. Therefore, I convert my deposits into cash/CBDC/another bank right away even without actual information acquisition. Hence, the bank run ensues, as bank deposits begin to no longer be "immune to adverse selection in trading", *a la* Gorton *et al* (2012). This is the same as saying that a bank run happens once deposits (that initially were information-insensitive short-term debt) become information-sensitive, i.e. they lose their money-ness or no-questions-asked property (see Gorton *et al*, 2022). Simplifying Equation (8) with only $\hat{\sigma}_t$ on the right hand side and assuming $\hat{\sigma}_t$ is a random variable with a distribution $F_{\hat{\sigma}}$, then the probability that a fear-induced retail bank run does NOT happen is:[24]

$$F_\sigma \equiv F_{\hat{\sigma}}\left(\frac{\gamma}{1+i_t^{CB}-\phi}\right) \tag{9}$$

For now, let's assume that the central bank passively provides liquidity to the banks at the policy rate in any amount they demand (i.e. there is no collateral constraint in the central bank's liquidity operations). This means that the only impact a (retail depositors) bank run has on the bank is by forcing it to switch to more costly wholesale funding. If we adjust the banks' optimization problem (the *expected* return on equity) with this probability, the optimal interest rate setting, i.e. equation (7), becomes:[25]

$$i_t^x = \frac{\varepsilon^x}{\varepsilon^x-1}\left(F_\sigma(1-\hat{c})\, i_t^D + \big(1-F_\sigma(1-\hat{c})\big)\, i_t^R\right) \tag{10}$$

Hence, if retail depositor-runs force banks to switch to more expensive wholesale funding whenever these runs happen, then the optimal interest rate on loans would be proportionally higher as the probability-weighted average cost of funds increases. But this is an only marginal effect (assuming F_σ is relatively close to one, as it should be in normal times) and nowhere near the interesting macro-financial questions relevant for policymakers, except for the questions of optimal CBDC policy, which is deferred for a later discussion.[26]

[24] No matter where this fear comes from, rational or irrational.

[25] Note that the cumulative (CDF) and probability (PDF) distribution function-related variables throughout the text do not show a time t subscript only for expositional ease. They do depend on time-varying variables and, hence, are themselves time-varying as well.

[26] Indeed, if introducing CBDC increases i_t^{CB} (i.e. it is interest-bearing) and reduces ϕ (since withdrawing CBDCs can be very easy/costless), then F_σ can drop quite a bit below one, when its impact and, more importantly, fluctuations become much more visible.

3.5. Introducing endogenous bank runs

In terms of the second source of uncertainty, endogenous wholesale bank runs and collateral insufficiency issues create a liquidity risk premium (even when the banks are assumed to be risk-neutral) and an important set of nonlinear macro-banking interlinkages. The relevance of this risk depends on the demand for central bank money (cash, CBDC or banks' reserve balances) and the collateral base for the central bank's monetary operations. The former is mostly a stochastic process for the bank, but dependent on the central bank decisions as well (e.g. about CBDC design). The latter (dollar-value of the central bank's collateral base) partly depends on the banks' decisions (about their asset portfolios' composition), but is also a product of central bank decisions. The central bank's collateral base is just a haircut- and policy rate-adjusted sum of the banks' *collateral-eligible* assets. Hence, for each bank the collateral constraint (relevant when trying to access wholesale funding[27], R_t) is the following:

$$CB_t \equiv \frac{\eta^S\, S_t + \eta^{SL}\, L_t^S + \eta^{RL}\, L_t^R}{1+i_t^R} \geq R_t \qquad (11)$$

where η^S, η^{SL} and η^{RL} define central bank collateral-eligibility of the assets (i.e. one minus the respective haircuts). Usually, government securities are part of the collateral base, i.e. $\eta^s = 1$, while the loan assets are not collateral-eligible ($\eta^{SL} = \eta^{RL} = 0$), but this is not always the case.

The bank takes this collateral constraint into account when it needs to borrow liquidity (i.e. the constraint is endogenous to the bank's optimization problem), since lending decisions have an impact on the probability whether the collateral constraint starts binding or not. From the central bank balance sheet identity ($R_t + R_t^X = Q_t + CM_t$) and assuming that the demand for currency is a random $\hat{c}$ fraction ($\hat{c} \sim F_{\hat{c}}$) of overall broad money ($CM_t + D_t$) while deposits are the rest ($1 - \hat{c}$ fraction),[28] the bank will have sufficient collateral if:

$$R_t = \hat{c}(TA_t - E_t + R_t^X) + Q_t - R_t^X \leq CB_t \qquad \rightarrow \qquad \hat{c} \leq \frac{CB_t \quad - rr(TA_t - E_t) + (1-rr)R_t^X}{(TA_t - E_t) - rr(TA_t - E_t) + (1-rr)R_t^X} \qquad (12)$$

where TA_t stands for the total assets of the bank excluding the liquid balances (which is not a relevant variable for defining the collateral base), and $TA_t - E_t = (1 - rr)D_t + R_t$, based on the bank balance sheet identity and the no-excess reserves assumption. rr is the reserve requirement ratio imposed by the central bank and we assume that the central bank doesn't remunerate the excess liquidity/reserve balances (monetary policy operates under the corridor system). Hence, Q_t will optimally be maintained at the required level $rr \cdot D_t$ and not more. Also, since Q_t is remunerated with the same interest rate as R_t, reserve requirements have no direct

[27] Of course, in aggregate, refinancing needs can only be satisfied by the central bank, as the monopoly supplier of the reserve money.

[28] Note that, using both central bank and commercial bank balance sheet identities, broad money equals $CM_t + D_t = S_t + L_t^S + L_t^R - E_t + R_t^X = TA_t - E_t + R_t^X$.

impact on net interest margin.[29] See Mkhatrishvili *et al* (2024) for a similar discussion. Relaxing this particular assumption is relatively straightforward, e.g. to model QE.

If $CB_t \geq TA_t - E_t$, which is almost never the case, then the bank will never run out of collateral even if it faces a bank run. Otherwise, the bank will either definitely have collateral sufficiency issues (if there's a panic-induced retail bank run, i.e. $\hat{c} = 1$) or will face the following probability that it will NOT have collateral sufficiency issues (if there's no panic-induced retail bank run):

$$F_c \equiv F_{\hat{c}}\left(\frac{CB_t \quad -rr(TA_t - E_t) + (1-rr)R_t^X}{(TA_t - E_t) - rr(TA_t - E_t) + (1-rr)R_t^X}\right) \tag{13}$$

In other words:

$$F_{R|\sigma} = \begin{cases} 1, & if\ CB_t \geq TA_t - E_t \\ F_c, & otherwise \end{cases} \qquad F_{R|1-\sigma} = \begin{cases} 1, & if\ CB_t \geq TA_t - E_t \\ 0, & otherwise \end{cases} \tag{14}$$

where $F_{R|\sigma}$ is the probability of no collateral sufficiency issues given there's no panic-induced bank run, while $F_{R|1-\sigma}$ is the same thing given there is a panic-induced bank run.

What happens if the bank hits the collateral constraint? It experiences a (wholesale) bank run, leading to a spike in wholesale funding costs for this bank. This extra cost is either the central bank stepping in with the lender of last resort at the penalty rate and supervisory actions, or the market penalizing this particular bank with exceedingly high (unsecured) interbank rates. At this point, this extra cost can just lead to high loan interest rates for this bank, but once we introduce bank defaults in the next subsection, it can also lead to bank insolvency (i.e. liquidity-induced solvency crisis). Adding the collateral constraint to the problem, as shown below, results in interest rates including a nonlinear liquidity risk premium that incorporates many of the important macro-banking interlinkages. Hence, at this point the optimization problem is already much more realistic, but optimal amount of equity is still zero within this specification. Hence, the fact that in reality banks hold a positive amount of equity (even above the regulatory minimum) motivates us to include the next source of uncertainty in our model. Hence, we show how to solve this optimization problem in detail once we add the next component as well.

3.6. Introducing bank defaults

Here we model the nonfinancial sector's default probability in an exogenous way. This assumption can be relaxed following some standard approaches (e.g. costly state verification, etc.). This will make the nonfinancial sector much more realistic and be even more useful for real time policy applications. However, nonfinancial sector is not our focus in this paper and is only modeled in a relatively simple way. For our purposes what matters more, since it is what's lacking more from the literature, is to show how banks' money creation incentive/ability reacts to

[29] It may still have an indirect impact on profit due to its impact on the probability of experiencing collateral sufficiency issues, as shown below.

their borrowers' potential defaults. If many borrowers default at once, then the banks' equity drops too much and creates solvency crisis. In our model, a bank will not default if the ex-post[30] realized credit risk (i.e. the fraction $\hat{\delta} \sim F_{\hat{\delta}}$ of risky loans that become nonperforming) does not exceed the capital buffer over and above the prudential norm (capital adequacy requirement):

$$\hat{\delta}\, L_t^R \leq\, E_t - e^m\, RWA_t \qquad (15)$$

where e^m is the minimum capital adequacy ratio, in percent of risk-weighted assets $RWA_t = \omega^S\, S_t +\, \omega^{SL}\, L_t^S +\, \omega^{RL}\, L_t^R$, with ω's being the respective risk weights assigned by the central bank/prudential authority). Hence, NO bank-default with the following probability:

$$F_\delta \,\equiv\, F_{\hat{\delta}}\left(\frac{E_t - e^m\, RWA_t}{L_t^R}\right) \qquad (16)$$

This, in turn, will lead to banks voluntarily holding equity buffers and incorporating a nonlinear credit risk premium into the interest rate setting.

3.7. Putting everything together

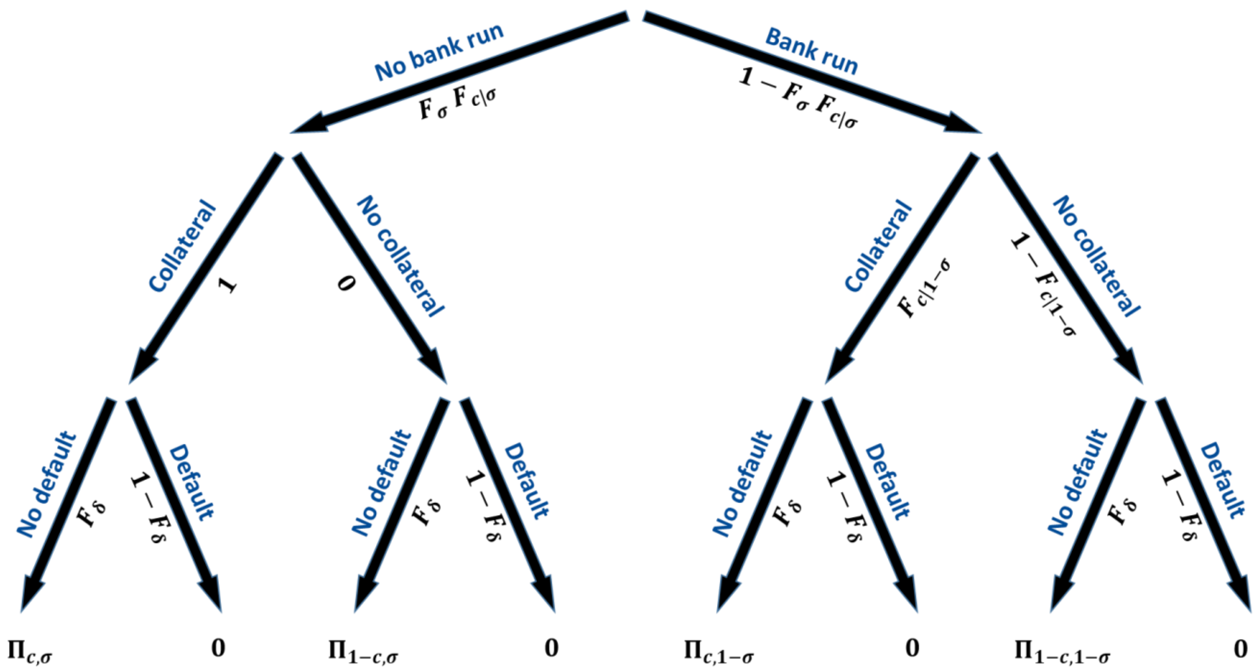
Now we put all the constraints and sources of uncertainty together, that will result in key macro-banking interlinkages. Figure 3 summarizes the above three sources of uncertainty and the bank's potential profit outcomes, with $\Pi_{c|\sigma}$ being the profit if there's no panic-induced (retail) bank run and given collateral is sufficient (i.e. no wholesale bank run either), $\Pi_{1-c|\sigma}$ – profit if there's no panic-induced bank run but collateral is insufficient (i.e. including the penalty, χ fraction of R_t, for experiencing a collateral/liquidity crisis or a wholesale bank run), $\Pi_{c|1-\sigma}$ – profit with a panic-induced bank run but with collateral being sufficient (i.e. no wholesale bank run), $\Pi_{1-c|1-\sigma}$ – profit with a panic-induced bank run as well as collateral being insufficient (i.e. including the penalty, χ fraction of R_t, for experiencing a collateral/liquidity crisis or a wholesale bank run). Due to limited liability, in case of bank default its profit is zero. χ is a particularly important component of the bank's optimization problem and, as explained above, can be interpreted from the market[31] as well as the regulatory[32] perspectives. All these will be used to formulate the bank's optimization problem (probability-weighted ROE) subject to demand and regulatory constraints.

[30] It is not necessary for the nonperforming loan's maturity to come to an end for the bank to realize credit losses for this particular loan. In fact, IFRS 9 requires banks to reflect these losses on their balance sheets in advance. Hence, expected credit losses may reduce bank equity quickly, well before other loans generate interest income/profit and increase equity back again.

[31] The cost of regaining the market access after losing it.

[32] The cost of the supervisor taking over and/or charging penalty rates on the lending of last resort.

Figure 3 – Banks' profit probability tree



Source: author's construction

Assuming that $CB_t < TA_t - E_t$,[33] the bank realizes profit in only two cases (out of eight on the figure): when there's no bank run (neither retail nor wholesale) and no default ($\Pi_{c|\sigma}$), and when there is a bank run (either retail or wholesale) but no default ($\Pi_{1-c|1-\sigma}$). If we take the bank run-probability weighted after-tax profit per equity,[34] we get an expected ROE given there's no default. Hence, the bank will maximize the following objective function (default probability-weighted expected ROE):

$$F_\delta \, ROE_t = F_\delta \frac{F_\sigma \, F_c \, \Pi_{c|\sigma} + (1 - F_\sigma \, F_c)\Pi_{1-c|1-\sigma}}{E_t} (1 - \tau) \tag{17}$$

As for the profit (net interest income minus overhead costs) in each of the two cases:

$$\Pi_{c|\sigma} = i_t^S \, S_t + i_t^{SL} \, L_t^S + \left(i_t^{RL} - \hat{\delta}\right) L_t^R - i_t^D \, D_t - i_t^R \, (R_t - Q_t) - \psi \tag{18}$$

$$\Pi_{1-c|1-\sigma} = i_t^S \, S_t + i_t^{SL} \, L_t^S + \left(i_t^{RL} - \hat{\delta}\right) L_t^R - (i_t^R + \chi) \, R_t - \psi \tag{19}$$

[33] The other (much less realistic) case shuts the liquidity risk premium down and is less interesting.

[34] We take the probability weighted ROE, since banks are assumed to be risk neutral. With risk averse banks, the channels we emphasize below would have been even stronger.

Again, based on the balance sheet identities, we have $R_t - Q_t = \hat{c}\,(TA_t - E_t) - (1-\hat{c})R_t^X$ and $D_t = (1-\hat{c})(TA_t - E_t + R_t^X)$ when there's no bank run, but $R_t = TA_t - E_t$ and $D_t = 0$ when there is a bank run. With that, after denoting the profit-relevant interest revenues with $IR_t \equiv i_t^S\, S_t + i_t^{SL}\, L_t^S + \left(i_t^{RL} - \hat{\delta}\right) L_t^R$:

$$\Pi_{c|\sigma} = IR_t - \left(i_t^D\,(1-\hat{c}) + i_t^R\,\hat{c}\right)(TA_t - E_t) + (i_t^R - i_t^D)(1-\hat{c})R_t^X - \psi \qquad (20)$$

$$\Pi_{1-c|1-\sigma} = IR_t - \left(i_t^R + \chi\right)(TA_t - E_t) - \psi \qquad (21)$$

Hence, the optimization problem of the bank becomes the following:[35]

$$\max \mathbb{E}_t\, F_\delta\, ROE_t = \mathbb{E}_t F_\delta \frac{IR_t - F_\sigma F_c\left[\left(i_t^D(1-\hat{c}) + i_t^R \hat{c}\right)(TA_t - E_t) - \left(i_t^R - i_t^D\right)(1-\hat{c})R_t^X\right] - (1 - F_\sigma F_c)\left(i_t^R + \chi\right)(TA_t - E_t) - \psi}{E_t}(1-\tau) \qquad (22)$$

$$\textit{subject to} \qquad S_t \le f^S(i_t^S) \qquad (23)$$

$$L_t^S \le f^{SL}(i_t^{SL}) \qquad (24)$$

$$L_t^R \le f^{RL}(i_t^{RL}) \qquad (25)$$

$$i_t^D \ge f^D(i_t^R) \qquad (26)$$

and definitions of CDFs. Appendix 2 shows how this results in the following equation describing securities yields:

$$i_t^S = \frac{\varepsilon^S}{\varepsilon^S - 1}\Big(F_\sigma\, F_c\,(1-c)\, i_t^D + \big(1 - F_\sigma\, F_c\,(1-c)\big)\, i_t^R + (1 - F_\sigma\, F_c)\,\chi + \frac{1}{1-rr}\, f_c\, F_\sigma\, \Big(\frac{CB_t + R_t^X}{TA_t - E_t + R_t^X} - \eta^S\Big)\left[(i_t^R - i_t^D)\,(1-c) + \chi \frac{TA_t - E_t}{TA_t - E_t + R_t^X}\right] + \frac{f_\delta}{F_\delta}\, \omega^S\, e^m \frac{E_t}{L_t^R}\, \frac{ROE_t}{1-\tau}\Big) \qquad (27)$$

We have a similar equation for the equilibrium interest rate on safe bank loans:

$$i_t^{SL} = \frac{\varepsilon^{SL}}{\varepsilon^{SL} - 1}\Big(F_\sigma\, F_c\,(1-c)\, i_t^D + \big(1 - F_\sigma\, F_c\,(1-c)\big)\, i_t^R + (1 - F_\sigma\, F_c)\,\chi + \frac{1}{1-rr}\, f_c\, F_\sigma\, \Big(\frac{CB_t + R_t^X}{TA_t - E_t + R_t^X} - \eta^{SL}\Big)\left[(i_t^R - i_t^D)\,(1-c) + \chi \frac{TA_t - E_t}{TA_t - E_t + R_t^X}\right] + \frac{f_\delta}{F_\delta}\, \omega^{SL}\, e^m \frac{E_t}{L_t^R}\, \frac{ROE_t}{1-\tau}\Big) \qquad (28)$$

[35] Here the expected value of demand for central bank money (c), is not an unconditional mean of the distribution, but a conditional one, conditional on the bank having sufficient collateral. Hence the mean will be $\int_0^{\frac{CB_t - rr(TA_t - E_t) + (1-rr)R_t^X}{(1-rr)(TA_t - E_t) + (1-rr)R_t^X}} x\, f_{\hat{c}}(x)\, dx$ (with normalized $f_{\hat{c}}$) and not $\int_0^1 x\, f_{\hat{c}}(x)\, dx$. However, given that the collateral constraint will be binding very infrequently (the end point in the first integral being on the very right of the distribution), any movement in $\frac{CB_t - rr(TA_t - E_t) + (1-rr)R_t^X}{(1-rr)(TA_t - E_t) + (1-rr)R_t^X}$ will have only a very small impact on this conditional mean. Hence, we abstract from this small detail in the model derivations.

But the equilibrium interest rate on risky bank loans is slightly different, due to borrower default probability (with δ being the expected default rate) and an *extra* credit risk premium:

$$i_t^{RL} = \frac{\varepsilon^{RL}}{\varepsilon^{RL}-1}\Big(\delta + F_\sigma\, F_c\,(1-c)\, i_t^D + \big(1 - F_\sigma\, F_c\,(1-c)\big)\, i_t^R + (1 - F_\sigma\, F_c)\,\chi + \frac{1}{1-rr}\ \ f_c\, F_\sigma\,\Big(\frac{CB_t + R_t^X}{TA_t - E_t + R_t^X} - \eta^{RL}\Big)\Big[(i_t^R - i_t^D)\,(1-c) + \chi\frac{TA_t - E_t}{TA_t - E_t + R_t^X}\Big] + \frac{f_\delta}{F_\delta}\Big(\omega^{RL}\, e^m \ + \frac{E_t - e^m\, RWA_t}{L_t^R}\Big)\frac{E_t}{L_t^R}\,\frac{ROE_t}{1-\tau}\Big) \qquad (29)$$

These equations are very rich but at the same time quite tractable and very intuitive.[36] Lending interest rates should compensate for (1) banks' profit margin due to monopolistic competition in the credit market ($\frac{\varepsilon^{RL}}{\varepsilon^{RL}-1}$), (2) *expected* borrower credit risk (δ), (3) retail funding costs, or interest rate on deposits if there is neither bank run ($F_\sigma\, F_c$) nor a normal expected deposit conversions into the central bank money ($1-c$), (4) interest rate on wholesale funding when there's a retail deposit outflow, (5) penalty rates for the cases when the liquidity crisis happens, (6) liquidity risk (collateral service) premium, or internalizing a marginal impact of lending on the probability of a bank run that can be either favorable (if the haircut on this type of asset is less than the "average") or adverse (if the haircut on this type of asset is more than the "average"),[37] and (7) credit risk premium, or internalizing a marginal impact of the loan extension on the probability of a bank default (due to its impact on the capital adequacy ratio as well as on the overall exposure to credit risk). More importantly, these effects are interlinked and sometimes quite nonlinear. Again, even though banks are assumed to be risk-neutral, (liquidity and credit) risk premia still feature prominently in their optimal interest rate setting.

3.8. Law of motion for banks' equity

If each bank that survived the previous period's default probability were to freely choose an optimal amount of equity, they would choose a level that incorporates a positive buffer over and above the regulatory minimum, due to the endogeneity of the credit risk premium shown above. But this is not the only way to model bank equity here. Instead, one could have the bank equity follow a law of motion where it mostly depends on endogenously decided retained earnings. In the full model we follow the second approach. This is not necessarily a simplifying assumption. It is both more realistic as well as policy relevant to assume that banks cannot easily get fresh new equity without accumulating it internally via profits. It is more realistic because it is in line with the literature discussing the frictions present when trying to attract new equity, at least in the short run (regulatory constraints, timing issues, investor sentiment, cost of issuance, etc.). It is more policy relevant because the same literature shows that just when banks are having

[36] Even though these optimality conditions are for each individual bank, the aggregate optimality conditions, describing the financial system-wide interest rates and balance sheets, will look the same, given that all the banks that survived the default probability in the previous period have now the same *ex-ante* optimization problems.

[37] This marginal change in the probability of a bank run, e.g. if its reduced, has two benefits for the bank: it saves money by having retail deposits with less interest cost instead of refinancing loans that have higher interest cost ($(i_t^R - i_t^D)\,(1-c)$) and it also saves money by not having to pay the penalty rate during a bank run ($\chi\frac{TA_t - E_t}{TA_t - E_t + R_t^X}$).

trouble, they do not issue new equity but instead accumulate profit (or sell assets) to boost their equity position (e.g. see Baker and Wurgler, 2002), and that's exactly the time when policymakers want to understand macro-financial linkages the most. The reason is that the impact of equity on bank lending decisions is highly nonlinear: if capital is abundant, being able to attract more of it has only tiny impact on bank lending decisions, but, if there is a capital shortfall, even a tiny further decline in it has a huge impact on those decisions.

We also assume that whenever one bank defaults, another one enters the market, so that the number of banks is kept the same in equilibrium (e.g. due to competitive forces in the banking market, which makes sure investors exploit the market position whenever an opportunity arises). The newly entered bank starts with an amount of equity similar to other banks. With this approach to modeling bank equity, and also assuming an exogenously given dividend policy (it being some fraction of equity), we arrive at the following law of motion for the bank equity:

$$E_{t+1} = F_\delta \left(E_t + (ROE_t - Dividend_t)E_t + \epsilon_t^K\right) + (1 - F_\delta)E_t \tag{30}$$

where ϵ_t^K is the exogenous equity injection/destruction term that, due to the reasons discussed above, is assumed to not be seen by banks as a choice variable.

This equation may make it look as if the bank capital is just exogenously given. In fact, equation (30) itself gives us no steady state level of equity, yet it does have a steady state in the model. It is indeed the bank's balance sheet decisions that drive its equity to a steady state (which can change implicitly depending on a policy framework and parameters). Is this (indirectly) bank-decided level of (steady state) equity relative to its assets equal to the regulatory minimum? No, because the banks internalize the impact of equity ratio on the probability of default. If equity is at the regulatory minimum, then even a tiny portion of loans becoming nonperforming (which is practically sure to happen) will make the bank default and forego its profit.

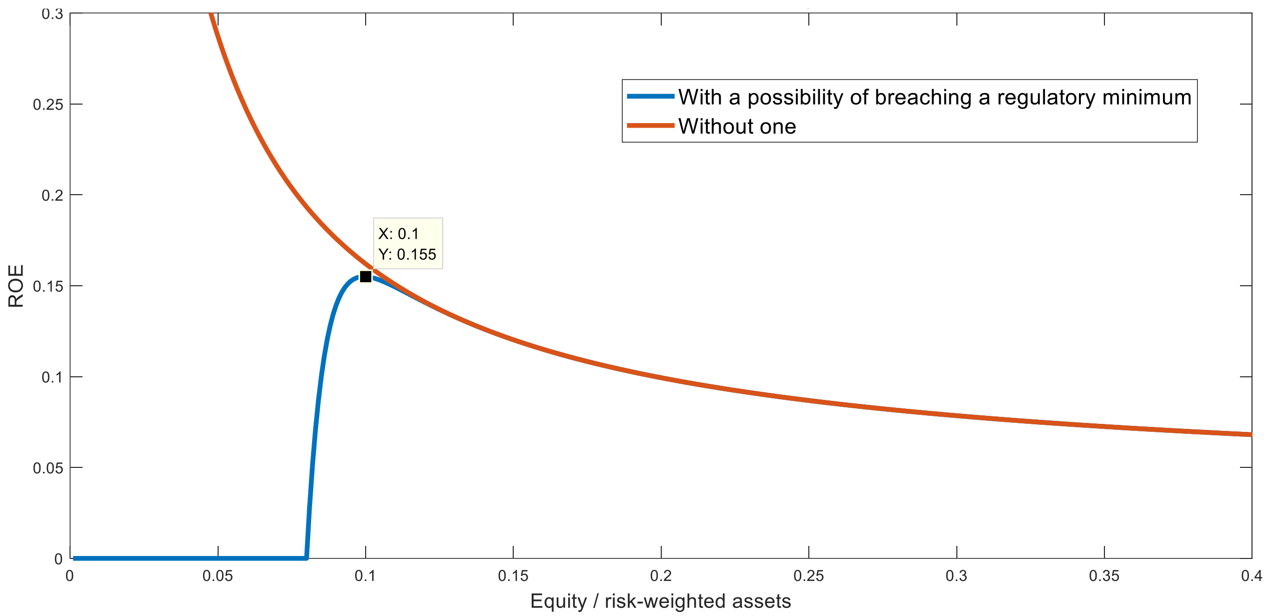
In other words, the bank internalizes that increasing the equity ratio, while lowering the ROE in a non-default state, also lowers the probability (risk) of a default state, which is beneficial because the bank only gets profit in a non-default state. This logic can also easily be shown if we look at the bank's ROE as a function of equity under two assumptions: when the bank internalizes its decisions' impact on the default probability (as in the full model) and when it doesn't. Indeed, as Figure 4 shows,[38] a bank in our model would voluntarily choose to hold an equity ratio (in this example at about 10%) well above its regulatory minimum (8% of RWA in this calibration of the model), while a bank without internalizing a possibility of breaching a regulatory minimum would choose capital as small as possible. The latter case assumes (of course, wrongly) that there is no probability of bank default and, hence, concludes that the less the capital is the more the ROE would be.[39] But of course, in reality, as in our model, this is not the case. There is an endogenous capital buffer that depends on the economic conditions and policy environment as

[38] Values for all the variables in calculating ROE (except equity, naturally) are equal to the respective steady states.

[39] If this were true, an investor could generate an infinite ROE by opening up a bank with zero equity.

well as the bank's perceptions of borrower-default probability. This is a key aspect of this model that can also explain why a credit crunch or sharp deleveraging can happen.[40] It also implies that Modigliani-Miller does not hold in our model.

Figure 4 – Bank's default risk-adjusted ROE and its choice of optimal level of equity



Source: author's construction

Also, we do not *need* to assume exogenously given exits from the banking system to avoid banks accumulating equity to 100% of assets if they are allowed to live indefinitely. Instead, banks here operate as long as there are no big defaults on their loan portfolios, but being on the market indefinitely doesn't lead to them accumulating 100% equity financing either, as some agents do in DSGE models. This is because banks see that there is such a thing as optimal finite equity buffer above the regulatory minimum.

An important assumption to note is that in this model the banks do not internalize the impact of their interest rate decisions on future potential profits via the law of motion of capital. This assumption can be defended by the corporate governance literature, which argues that corporate managers predominantly have short-term shareholder value/profit maximization focus (e.g. Bebchuk and Fried, 2006 or DeYoung *et al*, 2013). Also, relaxing this assumption would likely not change the qualitative features of the model. Additionally, we know that the banks that survive the last period's defaults may end the period with different equity positions, because each bank has its own realization of the default rate from the default probability distribution. But

[40] This is when the banking system, after earlier optimism about the credit risk in the macroeconomy, realizes that the capital buffer they hold is now no longer big enough and they start deleveraging. This (nonlinearly large) deleveraging happens in our model endogenously.

we assume that after this point the bank shareholders/investors reallocate their equity positions between different (but solvent) banks. Namely, they reduce equity in banks that experienced below-average borrower defaults and use these funds to increase equity in banks that experienced above-average borrower defaults (but remained solvent). The result of this reallocation is that the solvent banks start the next period with the same equity positions. This resets the symmetry in *ex-ante* decisions (but not ex-post) and makes the optimization problems much more tractable.

4. Modeling setup: nonfinancial sector

Nonfinancial sector is standard relative to other DSGE models,[41] with standard notation for variables as well, except for one single part (shown in red in the optimization problems below) that generates a simple link with the banking sector: demand for loans (money balances for payment purposes) that depends on consumption/investment expenditures and loan interest rates. This is minimally necessary to create an endogenous (and nonlinear) feedback loops between the macroeconomy and banks, thanks to an elaborate modeling of banks.

4.1. Households

Households maximize lifetime utility, a function of consumption (with external habit formation) and number of hours of work:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-h) \frac{(C_t - h\,\bar{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{\eta+1}}{\eta+1} \right]$$

subject to intertemporal budget constraint and a downward-sloping labor demand curve:

$$P_t\, C_t \left(1 + \frac{1}{2}\xi_{SL} \left(\frac{M_t}{P_t C_t} - \phi_{SL}\right)^2\right) + M_t' + i_t^{SL} L_t^S = (1 + i_{t-1}^M)\, M_{t-1}' + W_t N_t \left(1 - \frac{1}{2}\xi_W \Omega_{W,t}^2\right) + \bar{\Gamma}_t$$

$$N_t = \left(\frac{W_t}{\overline{W}_t}\right)^{-\frac{\mu}{\mu-1}} \bar{N}_t$$

[41] This is on purpose – to show how easily the banking part (which is far more realistic than in existing DSGE models) can be incorporated into otherwise standard macro models.

where $M_t = L_t^S - E_{t-1} + R_{t-1}^X + S_t$ is the beginning-of-the-period (domestic currency) money balances held by the household,[42] M_t' are the end-of-the-period money balances, while $\Omega_{W,t} = \log(W_t/W_{t-1}) - \log(W_{t+1}/W_t)$ is the wage inflation adjustment cost. If we were to assume (as in the simulations below) that the banks are owned by investors other than households (e.g. foreigners) then $M_t = L_t^S + R_{t-1}^X + S_t$. On the other hand, $i_{t-1}^M = \hat{c}_t i_t^{CB} + (1 - \hat{c}_t)\, i_t^D$ is the interest rate on money balances. More specifically it is a weighted average of an interest rate paid on central bank-issued money (cash/CBDC) and an interest rate on bank deposits, with the weight being the exogenous random variable mentioned above (reflecting payment technology needs). Note that household borrowing from the banks is assumed to have no credit risk (e.g. these loans being collateralized by sufficient physical capital and firms' ownerships/dividend flows). That's what the superscript *S* stands for (i.e. "safe" or "secured"). We will have credit risk when it comes to borrowing by entrepreneurs in the next subsection, with superscript *R* ("risky").

The red (new) part reads as follows: households face quadratic adjustment costs when the spending needs are not adequately backed by money balances at the beginning of the period when the payments are being committed to (representing money as a medium of exchange, obtaining of which has a marginal cost equal to a loan interest rate). This not only generates demand for loans, but also makes consumption decisions depend on loan extension and, hence, deposit money creation. In reality (financial) wealth (in this case money balances) is frequently a key factor influencing consumption decisions, but absent from standard DSGE models/dynamic IS curves. See Tchanturia *et al* (2024) for direct empirical evidence that the consumption function, linking consumption to net wealth, is an almost perfect characterization of the US personal consumption expenditure data.

It is crucial to emphasize that households make decisions on both loans and deposits/money. That makes these variables jump variables instead of predetermined ones as in more standard DSGE models with bank balance sheets. In other words, the specification here acknowledges that, on the one hand, households need money balances on its own but, on the other hand, deposit balances depend on loan decisions by the household since loan extension is what creates deposit balances (this, as a flip side, is also reflected into the banks' balance sheet identity). Hence, if one wants to have a sudden credit boom, this doesn't require depositors to refrain from consumption to accumulate "loanable funds". Instead, households will simply borrow more and create *new* money balances/deposits this way (assuming the banks are willing to lend given their risk preferences and regulations, as discussed in the banking section). This is

[42] The reason why we have bank equity as a determinant for the household money balances is that more equity financing means that someone should have converted their deposit money into bank equity and this someone in the model can be households as the only owners of the banks. We can also easily relax this assumption later, e.g. by assuming foreign ownership of banks. Also, the central bank continuously holds a portfolio of FX reserves which creates a continuous supply of broad money on top of that due to bank lending. Finally, when government borrows to finance budget deficit and then spends this money, we can assume this money goes to the households in the form of transfers, wages or anything similar. Alternatively, we may assume (as we will be doing during the simulations) that the money created by government's deficit spending is allocated to the households and the entrepreneurs according to their weights in the real economy.

a simple but endogenous money creation-type of a setup for a DSGE model. See Jakab and Kumhof (2015) for a similar discussion.

Maximizing the Lagrangian[43] yields the following first order conditions:[44]

$$M_t': \qquad \frac{1}{1+i_t^M} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$$

$$C_t: \quad (1-h)(C_t - h\,\bar{C}_{t-1})^{-\sigma} \approx \Lambda_t\, P_t \left(1 - \xi_{SL}\phi_{SL}\left(\frac{M_t}{P_t C_t} - \phi_{SL}\right)\right)$$

$$L_t^S: \qquad L_t^S = \left(\phi_{SL} - \frac{i_t^{SL}}{\xi_{SL}}\right) P_t C_t + E_{t-1} - R_{t-1}^X - S_t$$

$$W_t: \qquad \mu \frac{N_t^\eta}{\Lambda_t\, W_t} - 1 \approx (\mu - 1)\, \xi_W \left(\Omega_{W,t} - \beta \Omega_{W,t+1}\right)$$

Just for illustration purposes, if we log-linearize the above equations, we'll get the following two intuitive equilibrium conditions for consumption smoothing and labor supply:

$$\hat{c}_t = \frac{h}{1+h}\,\hat{c}_{t-1} + \frac{1}{1+h}\,E_t\hat{c}_{t+1} - \frac{1-h}{(1+h)\sigma}\left(i_t^M - E_t\pi_{t+1} - \rho - \xi_{SL}\phi_{SL}\,(\widehat{m}_t - \widehat{m}_{t+1})\right)$$

$$\pi_t^W = \frac{1}{1+\beta}\pi_{t-1}^W + \frac{\beta}{1+\beta}E_t\pi_{t+1}^W + \frac{1}{(1+\beta)(\mu-1)\xi_W}\left(\eta\,\hat{n}_t + \frac{\sigma}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) - \widehat{rw}_t\right)$$

where $\widehat{m}_t = \left(\frac{M_t}{P_t C_t} - \phi_{SL}\right)$ is the deviation of real (inside and outside) money balances of households from its long-run equilibrium (which is ϕ_{SL} fraction of C_t), with the rest of the variables defined as is standard in the literature. This money balance's partial impact is the only addition to an otherwise standard dynamic IS curve (with habit formation) and the wage Phillips curve is essentially the same as in standard DSGE models with wage rigidities. In other words, now consumption doesn't just depend on the permanent income and long-term interest rates, but also on the evolution of real money balances. Many economists would agree that, while monetary aggregates may not matter as much in the short-term, they do have some role in the longer-term, unless there are structural changes, which is part of our model as well, in the form of ϕ_{SL} which can easily be made time-varying – equivalent to a time-varying money velocity, potentially with a trend. Also note that real money balance's impact resembles that of

43 With Λ_t being the Lagrange multiplier.

44 We use first order approximation only when it comes to quadratic adjustment costs, since they make the equations unreadable but do not change any qualitative (and in many cases even quantitative) results.

preference shocks in standard DSGE models, but here its impact is derived from households/banks behavior and payment needs, not just being exogenously given.

4.2. Producers

There are no changes for this type of economic agents, as well as for the next one, relative to standard DSGE models. Competitive producers maximize profit by choosing the quantity of output and inputs subject to a standard technology (production function):

$$\max\ P_t^Y\, Y_t\ - W_t\, N_t - Q_t\, K_t$$

$$s.t.\quad Y_t = A_t\, N_t^{\alpha}\, K_t^{1-\alpha}$$

FOCs yield demand schedules for labor and physical capital (in addition to the production function itself), while cost minimization problem gives the marginal cost equation which is the same as the price of output due to competition:

$$N_t: \qquad \alpha\, P_t^Y\, Y_t = W_t\, N_t$$

$$K_t: \qquad (1-\alpha)\, P_t^Y\, Y_t = Q_t\, K_t$$

$$P_t^Y\ = \frac{1}{\alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\,A_t}\,W_t^{\alpha} Q_t^{1-\alpha}$$

4.3. Distributors

Retail distributors operate under monopolistic competition. They buy output from the producers, repackage it and resell at a markup. Retailers are subject to downward-sloping demand curve. Technically, their only role is to generate price stickiness. Hence:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[P_t\, Z_t \left(1 - \frac{1}{2}\xi_P \Omega_{P,t}^2\right) - P_t^Y Z_t\right]$$

$$s.t.\quad Z_t = \left(\frac{P_t}{\bar{\bar{P}}_t}\right)^{-\frac{\epsilon}{\epsilon-1}}\ \bar{Z}_t$$

where $\Omega_{P,t}\left(= \log\frac{P_t}{P_{t-1}} - \log\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}\right)$ is a price inflation adjustment cost. The FOC leads to:

$$P_t: \qquad \epsilon\frac{P_t^Y}{P_t} - 1 \approx (\epsilon-1)\,\xi_P\big(\Omega_{P,t} - \beta\Omega_{P,t+1}\big)$$

This is pretty much a standard New-Keynesian Phillips Curve. Overall demand $Z_t = C_t + I_t + BD_t$, which in equilibrium should be equal to Y_t, where BD_t is the government's budget deficit (i.e. net expenditures).

4.4. Entrepreneurs

Entrepreneurs make investments to accumulate capital stock that is then rented to producers for output production. They maximize their profit, with respect to investment and physical capital (without making financial savings decisions), subject to capital's law of motion. Entrepreneurs' profit, just like households' budget constraint, depends on adequately backing investment expenditures with money balances at the beginning of the period. Otherwise, they'll face quadratically increasing adjustment costs. Hence their loan demand will depend on nominal investment expenditures and loan interest rates. Optimization problem reads:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[Q_t\, K_t - P_t I_t \left(1 + \frac{1}{2} \xi_I \Omega_{I,t}^2 + \frac{1}{2} \xi_{RL} \left(\frac{L_t^R}{P_t I_t} - \phi_{RL} \right)^2 \right) - \left(i_t^{RL} - \hat{\delta}_t \right) L_t^R \right]$$

$$s.t. \quad K_{t+1} = (1 - \delta_K) K_t + \left(1 - \hat{\delta}_t\right) I_t$$

where $\Omega_{\mathrm{I,t}} = \log \frac{I_t}{I_{t-1}}$ is an investment adjustment cost and $\hat{\delta}_t$ is a random exogenous variable representing the share of entrepreneurs that fail in their work of transforming investment expenditures into capital (i.e. a failed project that didn't result in what was hoped for). When that failure happens, entrepreneurs default on their loans as well, since they don't get any income from the new project (while the physical capital is eventually owned by households and entrepreneurs just operate on their behalf). The only idea is to have a type of loans that is unsecured and risky, while that of households is secured, as mentioned above. This is a simple way of generating credit risk in the model. Even if this credit risk is fully exogenous here, credit risk premium (set by banks) is still fully endogenous and highly nonlinear.

With Θ_t being the Lagrange multiplier on the physical capital's law of motion (and, hence, being the price of capital), we have the following FOCs (in addition to the capital's law of motion):

$$K_{t+1}: \qquad \Theta_t = \frac{1}{1 + i_t^M} \left(Q_{t+1} + (1 - \delta_K) \Theta_{t+1} \right)$$

$$I_t: \qquad \left(1 - \hat{\delta}\right) \frac{\Theta_t}{P_t} - 1 + \xi_{RL} \phi_{RL} \left(\frac{L_t^R}{P_t I_t} - \phi_{RL} \right) \approx \xi_I \left(\Omega_{I,t} - \beta \Omega_{I,t+1} \right)$$

$$L_t^R: \qquad L_t^R = \left(\phi_{RL} - \frac{i_t^{RL} - \hat{\delta}_t}{\xi_{RL}} \right) P_t I_t$$

4.5. Macro policies

There are several policy instruments in the model. The central bank's *monetary policy* follows a Taylor-type reaction function of the following form (reacting to a one-period ahead inflation forecast with its policy rate):

$$i_t^P = \rho_i \, i_{t-1}^P + (1-\rho_i)\big(r^P + \pi + \eta_\pi(\mathbb{E}_t\pi_{t+1} - \pi) + \eta_y \, \widehat{mc}_t\big)$$

where $\widehat{mc}_t$ is marginal cost gap (meant to measure the real economy's cyclical position). The central bank's *liquidity operations* define what type of assets the central bank takes as collateral when lending reserve balances to banks and with how much haircut. Its *macroprudential policy*, on the other hand, defines the minimum capital adequacy ratio banks must satisfy at all times and how high risk-weights are for each asset on the banks' balance sheets. *Fiscal policy*, on the other hand, determines the amount of budget deficit such that it would stabilize public debt as some implicit targeted level.

5. Key policy-relevant simulations

In this section we first discuss how the model is calibrated, followed by impulse response analysis (i.e. responses to temporary shocks) as well as comparative statics and transition dynamics (i.e. responses to permanent shocks/recalibrations). These are meant to demonstrate the applicability of the model as well as to show some of the novel messages it incorporates relative to more standard models.

5.1. Calibration

Calibration of the nonfinancial sector is pretty standard and mostly in line with literature. Part of the financial sector is easily calibrated based on the most frequent regulatory and policy setups, while another part of the financial sector (e.g. parameters of distribution functions, etc.) are based on judgment and the broad empirical literature, but robustness checks are also done to see how much of the results depend on a specific calibration. The model is calibrated with quarterly dynamics and steady states in mind. The following table summarizes the calibration:

Table 1 – Calibration of the full set of model parameters

	Parameter	Value[45]	Description
Fin. sector	$\bar{\sigma}$	5%/4	Mean of the retail bank run probability (exponential) distribution $F_{\hat{\sigma}}$
	$\bar{c}$	25%/4	Mean of the central bank liquidity demand (exponential) distribution $F_{\hat{c}}$
	$\bar{\delta}$	5%/4	Mean of nonperforming loans/borrower defaults (exponential) distribution
Liquidity risk	γ	20%/4	Cost of information acquisition about a bank's balance sheet health
	ϕ	10%/4	Full cost of handling cash (*can be different for CBDC*)
	η^S	100%	Portion of government securities included in the central bank collateral
	η^{SL}	0%	Portion of secured retail loans included in the central bank collateral

[45] Since our goal is to demonstrate the quantitative capabilities of the model, making the calibration accurately represent any particular country is not an aim here, which can be a future application if need be.

	η^{RL}	0%	Portion of risky retail loans included in the central bank collateral
	rr	0%	Reserve requirement ratio
	χ	10%/4	Penalty rate due to lending of last resort or market cost of liquidity stress
Competition in fin. sector	ϵ^{D}	5	Elasticity of substitution in the deposit market
	ϵ^{S}	10	Elasticity of substitution in the government securities market
	ϵ^{SL}	5	Elasticity of substitution in the secured retail loans market
	ϵ^{RL}	5	Elasticity of substitution in the risky retail loans market
Parameters for optimal capital	e^{m}	8%	Minimum capital adequacy ratio requirement (in % of RWA)
	ω^{S}	0%	Risk weight applied to government securities
	ω^{SL}	100%	Risk weight applied to secured retail loans
	ω^{RL}	100%	Risk weight applied to risky retail loans
	d	15%/4	Steady state dividends per equity (investors' required ROE in SS)
	ψ	2%	Overhead costs for banks per equity
SS of macro policy variables	π	2%/4	Quarterly inflation target
	R^{X}	0	Steady state level of central banks net FX reserves
	τ	20%	Corporate profit tax rate applied to banks
	BD	0%	Steady state budget deficit
	S	5	Steady state government securities overall portfolio
Persistence parameters	ρ_{i}	0.9	Persistence of policy rate
	ρ_{R^X}	0.9	Persistence of FX reserves process
	ρ_{BD}	0.7	Persistence of budget deficit process
	ρ_{c}	0.7	Persistence of currency demand process
	ρ_{A}	0.9	Persistence of technology process
	$\rho_{\phi^{SL}}$	0.7	Persistence of consumption expenditures' share to be covered by money
	$\rho_{\phi^{RL}}$	0.7	Persistence of investment expenditures' share to be covered by money
Nonfinancial sector parameters	h	0.7	Habit persistence
	σ	2	Relative risk aversion of households
	β	0.995	Discount factor
	η	0	Elasticity of labor supply
	μ	2	Elasticity of substitution in the labor market
	ξ_{W}	20	Adjustment cost applied to wage changes
	ξ_{P}	400	Adjustment cost applied to prices changes
	ξ_{I}	10	Adjustment cost applied to investment changes
	ξ_{SL}	0.5	Adjustment cost applied to mismatch between money and consumption
	ξ_{RL}	0.5	Adjustment cost applied to mismatch between money and investment

	ϕ^{SL}	1	Share of consumption expenditures to be covered by money balances
	ϕ^{RL}	1	Share of investment expenditures to be covered by money balances
	A	1	Steady state of technology process
	α	0.5	Share of labor in the production function
	ϵ_P	1.5	Elasticity of substitution in the product market
	δ_K	3%	Depreciation of physical capital
	η_π	1.5	Policy rate's reaction to inflation
	η_y	0.5	Policy rate's reaction to real marginal cost gap

Source: author's construction

5.2. Impulse response analysis

Usually, the first step in macro modeling literature to understand how well the model captures the propagation mechanism in the economy is to look at the impulse responses to monetary policy shocks. In our model though, the monetary transmission mechanism is much more nuanced, making it possible to tell a richer story. But that also means there are a variety of instruments in our model that the central bank can use, as in real world, that can have different transmission to the economy. Figure 5 shows the impulse responses to a (standard) temporary quarterly policy rate shock of the size of 1 percentage point (annualized 4 pp).

The transmission to output and prices is in line with other models, since those models have always tried to generate the final responses in line with the empirical literature on monetary policy effects. However, there are some transmission channels that usually are absent in virtually all standard macro models. For example, the credit risk channel of monetary policy operating via banks searching for yield and taking on more (bank default) risk. Borio and Zhu (2018) have emphasized this kind of risk-taking channel of monetary policy empirically for the EU. In our model this results in banks reducing their capital buffers, reducing capital adequacy ratio closer to the regulatory minimum. This leads to the probability of bank default increasing, from less than 0.5% to about 1.5% temporarily. The reason for this is that banks see loan interest rates declining more than the deposit rates, compressing interest margins and ROE.[46]

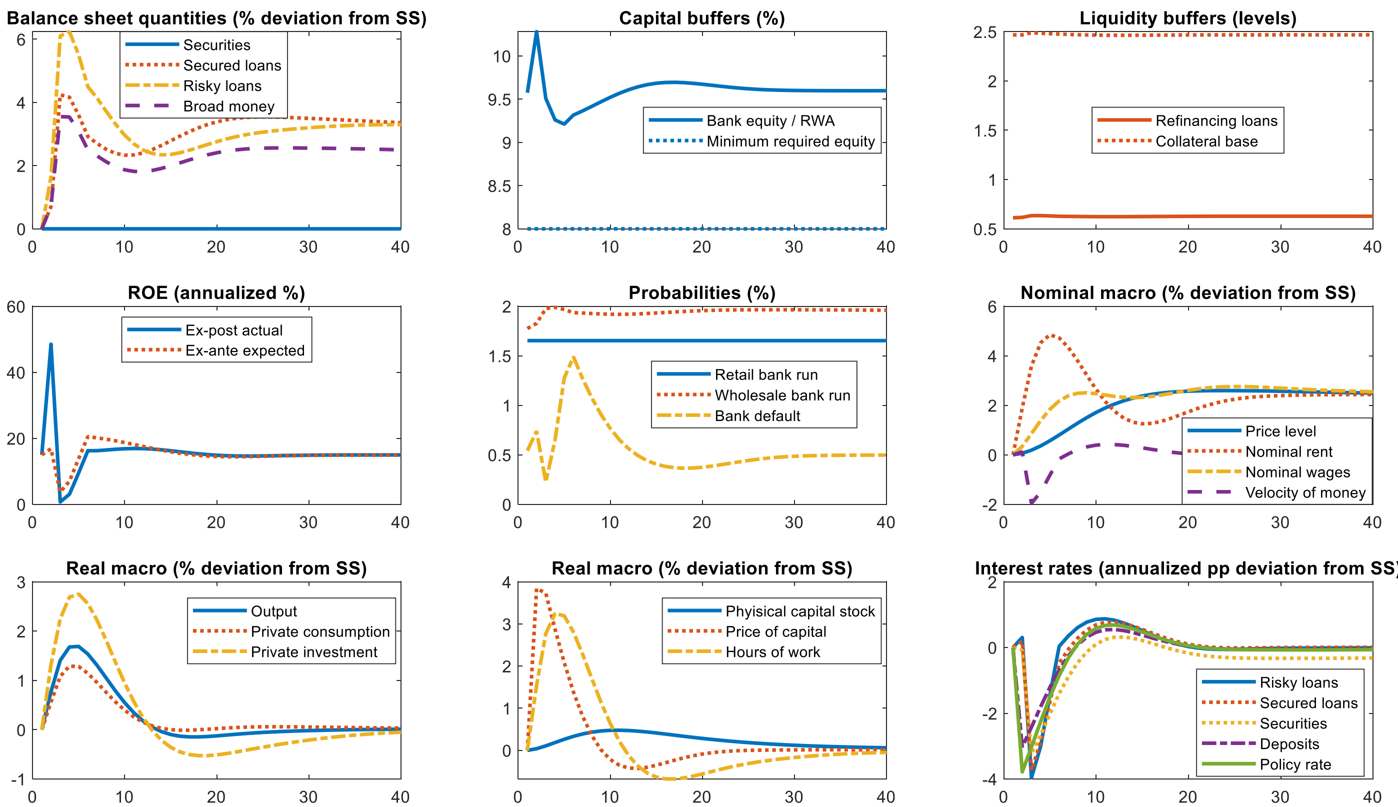
Partly because of this credit risk taking channel, even though all loan types accelerate, risky loans pick up the most (by about 5 pp). These risky loans in this model are used by entrepreneurs,[47] so that investment increases the most by almost 3%. However, after the policy rate starts going back to its steady state, then the lending conditions tighten even more than the policy rate increases itself. This is because banks are now facing the eventual increase in

[46] Despite the fact that ROE initially jumps up because of a positive revaluation effect coming from unexpected interest rate cuts (i.e. interest rate risk materializing on the positive side).

[47] This, of course, is not a necessary part of the model. Other models (even with the same approach to modeling banks as in here) can assume risky loans to be used by some other agents, which may modify the macro story.

interest rates with compressed margins and profitability. This higher credit risk makes banks with weaker initial conditions more reactive to later policy rate hikes and risky loans decline back again, leading to investment later being weak quite persistently. This fully neutralizes any positive effect of initial policy easing on capital accumulation. Hence, the initial stimulus turns out to lead to permanent price level increase (that also inflates nominal money and credit permanently) without any benefit for the real economy. The same happens with real asset prices, jumping up significantly initially in response to policy easing, only to crash later even below the initial steady state, staying there persistently. This demonstrates that the excessive monetary policy easing can be costly in the medium term, even for the real economy itself, due to its negative implications for financial stability and (credit) risk taking that start having a negative impact on the real economy down the road.

Figure 5 – Impulse responses to a quarterly policy rate shock of 1pp (4pp annualized)



Source: author's construction

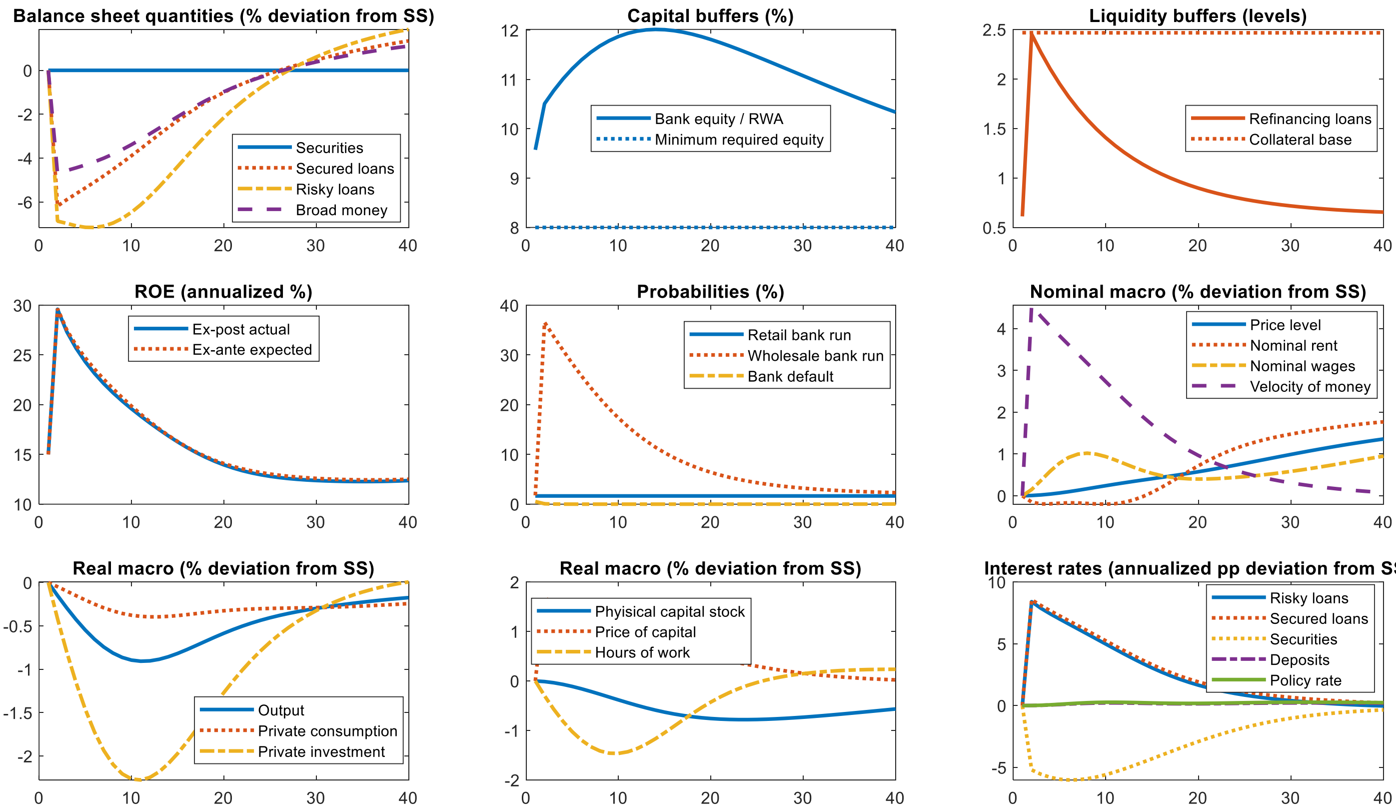
The other channel, which is silent in this particular simulation, is the liquidity risk channel. Since the policy rate shock itself isn't assumed to generate any movements in the government securities portfolio (it is assumed that the government stabilizes the debt at some pre-defined level), the central bank's collateral base doesn't change much either (apart from small valuation changes in the market value of the portfolio due to interest rate movements). Also, neither demand for cash nor reserve requirements change and, hence, central banks refinancing loans are almost the same. This means that the difference between liquidity borrowings from the central bank and the maximum liquidity they could normally get from it (i.e. collateral base) is also unchanged. In short, liquidity buffers do not move much. That's why this channel is silent in this simulation. However, it will be very much active under some other shocks, generating real effects that, once again, virtually all standard macro models miss.

We discuss the latter issue next within the impulse responses to the demand for cash, pushing the overall liquidity demand (refinancing loans) close to the central bank's collateral constraint (see Figure 6). In other words, the shock is that nonfinancial sector starts to massively switch to outside money (that only the central bank creates) instead of inside money (that commercial banks create). For banks to be able to satisfy such huge deposit outflows, they need to borrow from the central bank, which itself is constrained by the fact that it cannot take on credit risk (i.e. lending to banks must be collateralized). But the issue is that the collateral base is finite and usually much less than the size of overall deposit portfolio. This means the banks, if they need it in the near future, will not be able to borrow liquidity from the central bank anymore, since they have already hit the collateral constraint. In other words, while the central bank continues to fully provide the liquidity that is demanded *as of today*, the current liquidity demand inching all the way up to the collateral constraint means that banks won't be able to satisfy any *extra* deposit outflows from tomorrow on. This fear of further deposit outflows, which can already precipitate a full scale bank run, leads to banks increasing their liquidity risk premium and reducing collateral service premium. This is true only until the cash demand declines back again.

Hence, the demand for cash increasing so much as to make collateral constraint bind (i.e. to lead to complete, but temporary, exhaustion of "liquidity buffers") is equivalent to constraining banks in their liquidity transformation business. The result is higher interest rates on those bank assets that are generating liquidity risk. Both types of loans here, as in real world, in fact, are such assets, because their extension means more deposit-money creation, which leads to more probability of *extra* deposit outflows (at least for the part of the newly created deposits). As a result, interest rates on both types of loans jump up on impact, by about 8 pp – a sudden deleveraging.[48] This leads to the lending portfolio and broad money creation declining by almost 5% on impact (which is a lot for a temporary shock to cash demand), before gradually recovering once liquidity buffers are recovered, even with secured loans recovering much faster. This deleveraging, in turn, leads to output contracting by about 1%, and quite persistently. Investments, on the other hand, decline by much more – in excess of 2%.

[48] An important point to emphasize here is that this model abstracts from banks' non-price credit rationing. Hence, interest rates here jumping up by 8 pp may be interpreted as banks tightening overall lending conditions (both interest and non-interest terms) that have an overall impact equivalent to 8 pp increase in interest rates. This is a simplification (resulting in overestimation of ROE in stress episodes when interest rates increase) that can be relaxed in the future, e.g. by introducing non-price credit rationing as done by Benes *et al* (2014). However, in any case, level of credit itself would still show a similar dynamic.

Figure 6 – Impulse responses to a temporary shock to cash demand (pushing liquidity demand to collateral constraint)



Source: author's construction

A reason for this asymmetry is a certain degree of equivalence between accepting credit risk and accepting liquidity risk. This connection is not frequently appreciated in the macro-financial literature, because, as mentioned above, these two (credit and liquidity) risk premia are usually not modelled together in quantitative business cycle models. But in this model when banks realize they no longer have sufficient liquidity buffers, they start deleveraging/reducing the balance sheet with the intent to build up capital position (doubling the capital adequacy buffer in this simulation), reducing the bank default risk essentially to zero. Why? Because the more capital they hold, for given asset portfolio size, the less is dependence on run-prone information-insensitive liabilities (i.e. deposits). This, in turn, *partially* mitigates the elevated liquidity risk. And what's a better way of rebuilding capital buffers? Cutting back more on risky loans, which, in this model, finance entrepreneurs' investments. That's a key reason why investment expenditures decline much more than the overall output. Lower investments mean lower physical capital accumulation, resulting in the initial output decline being quite persistent.

Another interesting aspect of this liquidity risk-related simulation is that this doesn't translate into all interest rates being higher, even as liquidity risk premium jumps up in general. In this case, interest rate (yield) on government securities declines, not increases, significantly, by about 5 pp on impact. The reason for this is that the government securities provide a liquidity risk insurance service, unlike the other assets on the banks' balance sheet. Since banks are now super hungry for liquidity risk insurance, they're willing to accommodate extremely low yields on their securities holdings, as long as these holdings help on the liquidity risk side. This picture resembles a key aspect of the GFC, when risky interest rates shot up, while risk-free securities yields dropped significantly after the liquidity crunch.

Still another important simulation that demonstrates the workings of the model is to show how banks would react to a sudden (proper) realization of their previously-underpriced credit risk. The simulation assumes that the starting point is the situation where everything is as in the baseline version of the model, except that banks think that the standard deviation of the distribution that governs borrowers default is tiny. Then, after period 0, they realize the proper size of this standard deviation of the default probability distribution. This particular simulation also assumes that banks take a couple of quarters to gradually adjust their private sector lending interest rates. This is the only thing that happens here. No shock, other recalibration or a model change is used. Still, the simulation shows that, after the proper realization of the previously-underpriced credit risk, a sudden deleveraging happens, resulting in a drop of secured lending by about 8% and risky credit declining by 12%.[49] Broad money declines by only 6-7%, because bank lending to the government (i.e. securities portfolio) is unchanged, which is also a source of money creation on its own. Hence, the overall decline on the liability side (excluding the equity) becomes only 6-7%, instead of 8-12%.

This sharp deleveraging is a flip side of the fact that private sector lending interest rates jump up by 10-15 pp (at an annualized rate), with risky loan interest rates getting a particular hit (since its this type of loans that creates solvency concerns). This deleveraging is what banks do to quickly build up a capital buffer,[50] which they realized was very small initially, and to reduce the probability of a solvency crisis from an initial excessive level. Indeed, the capital adecuacy ratio increases from close to a regulatory minimum of 8% initially to over 13% in a couple of years (through temporarily higher retained earnings). That decline in money and credit creation, then, feeds into output declining as well, which drops by about 0.7%, whereas investments („financed" by risky loans) decline by 1.2%. Gross (2022) emphasize a similar point on credit risk-driven cycles using a type of disequilibrium macro-financial model where banks create money.

Output declining by less than 1% when money creation declines by almost as much as 7% is only made possible by private nonfinancial economic agents, at least in this version of the model, reducing their demand for money due to high interest rates and, hence, increasing the money velocity (economizing on money balances).[51] That's why this deleveraging has little impact on inflation and, hence, policy rate reacts only marginally. If we were to assume that

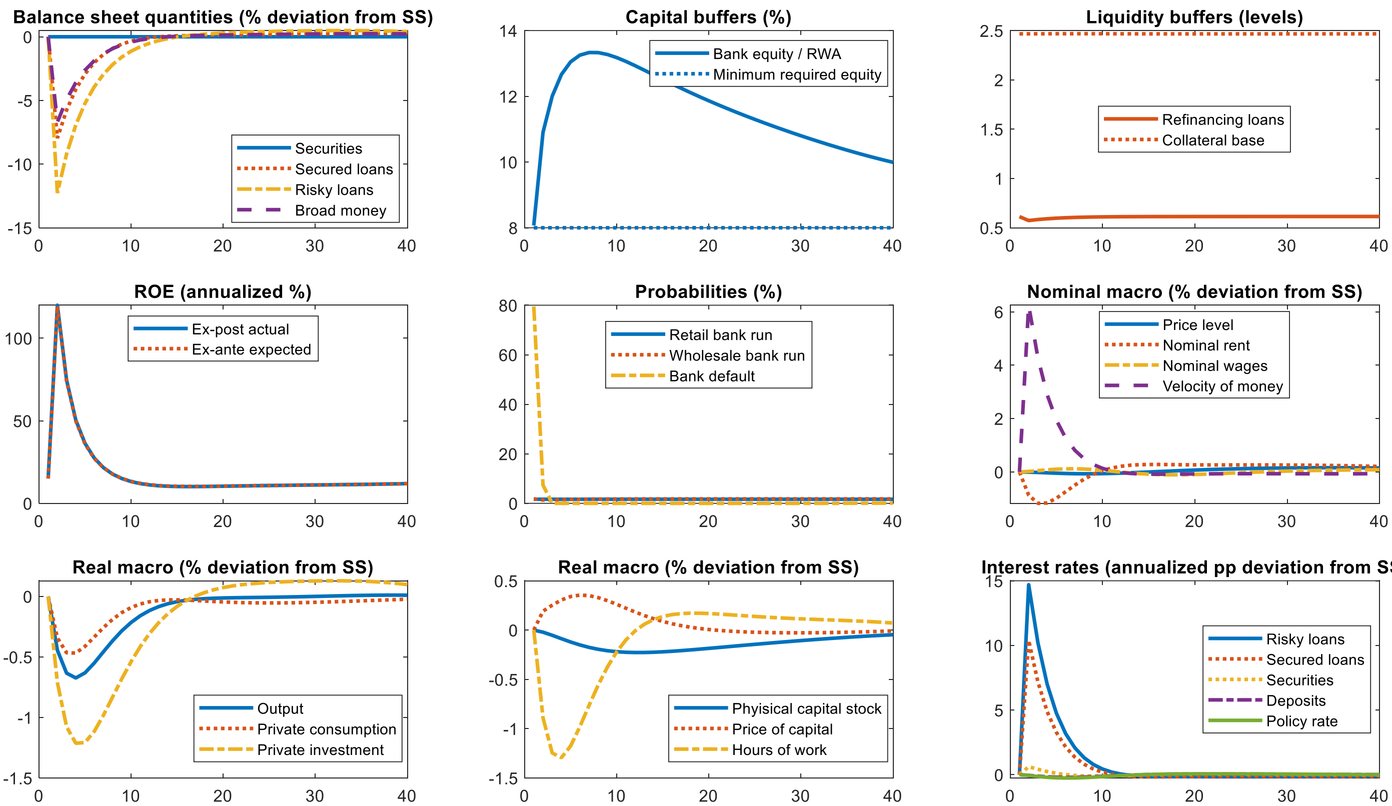
[49] This is in line with the empirical evidence by López-Salido *et al* (2017) and earlier contributions of Minsky (1986).

[50] That's why risky lending declines particularly more.

[51] Evidently, a similar issue was present during the liquidity stress simulation as well.

money demand and money velocity are kept stable throughout the simulation,[52] then the decline in output would be much larger. Indeed, when money velocity is assumed to not increase as much, then the simulated output contracts by a whopping 5.8%, with a full recovery taking about 5 years. This of course is a huge response, emphasizing the cost of a (potential) solvency crisis, especially since the simulation doesn't use any shock other than a one-off change in the banks' perceptions of the size of the credit risk. What's more, during a financial crisis money demand not only may not decline, but it may actually increase, worsening the decline in output.

Figure 7 – Impulse responses to banks suddenly reappraising uncertainty around their borrowers' defaults



Source: author's construction

5.3. Transition dynamics

Another set of simulations, to showcase the applicability of the model, is considering what transition dynamics will look like when there's a permanent change in the policy framework, parameters or the steady states of some variables. An example would be what would happen if

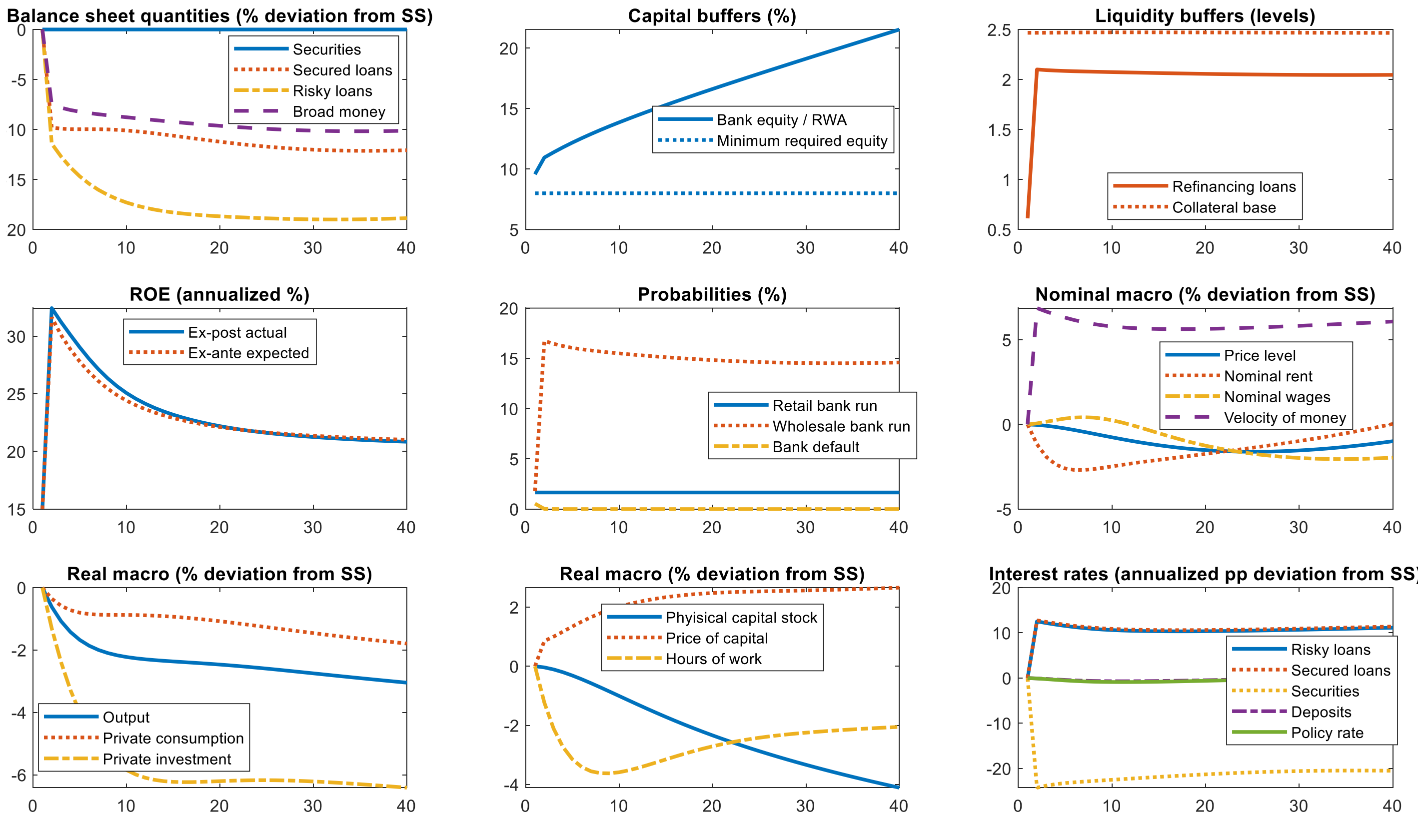
52 As mentioned above, the nonfinancial sector is a weakness of this model, since our focus has been on the financial sector. A more realistic nonfinancial sector should better model the demand for money (and, hence, money velocity).

the central bank started increasing the reserve requirement ratio (RRR) from the current calibration of 0% to higher and higher number. In standard models, RRR has literally no impact on the financial system and the real economy. The reason is that these standard models assume that the central bank faces no constraint in liquidity provision: if RRR becomes higher, the central bank will provide correspondingly more liquidity to make sure that liquidity supply and demand remain in sync and short-term market interest rates remain close to the policy rate, without any impact on other interest rates as well. However, in reality, central banks' standard liquidity injection operations require eligible assets (for them to either buy or take as collateral). When liquidity demand becomes so high that these eligible assets start becoming very scarce, then RRR will start having real effects, even under interest rate targeting frameworks.

Indeed, in our model, when RRR is increased from 0% to 10%, broad money creation declines by 2.5%, but when an RRR increase doubles (i.e. increases by another 10 pp), the decline in broad money creation more than quadruples (i.e. exceeds 10%). See Figure 8. This more than 10% drop in broad money, which proves to be essentially permanent, translates into permanent and significant declines in output, consumption and investments. Similar to previous simulations, here as well, money velocity increases as a result of economic agents economizing on money balances in response to high lending interest rates (which is their marginal cost of obtaining money balances). If this increase in velocity were not to happen, then the decline in (nominal) output would have been close to 10%.

This simulation highlights the costs of a central bank switching to a narrow banking approach, where RRR increases to 100%. For a given size of the risk-free asset portfolio (in this case government securities) in the financial system, that is less than the value of broad money, such a move to a narrow banking results in a very strong deleveraging, because banks will be forced to stop their liquidity transformation business. Again, this much deleveraging leads to a permanent decline in output, by more than 3%. Investments, on the other hand, decline permanently by more than 6%. The latter (investments declining much more than consumption) is caused by risky loans seeing much stronger deleveraging. And the reason for this, in turn, is the same message we discussed above: a trade-off between banks taking a liquidity risk versus them taking a credit risk. In other words, as banks are now facing much more stringent conditions on the liquidity transformation side (because of high RRR), they gradually switch to equity financing, which, in turn, leads to lower solvency risk (with the probability of a bank default going all the way down to 0%). The latter is good news, of course, on its own, but it is a reaction of banks to the fact that they no longer can rely on inside money creation for supporting the economy with money balances. Overall, the economy suffers.

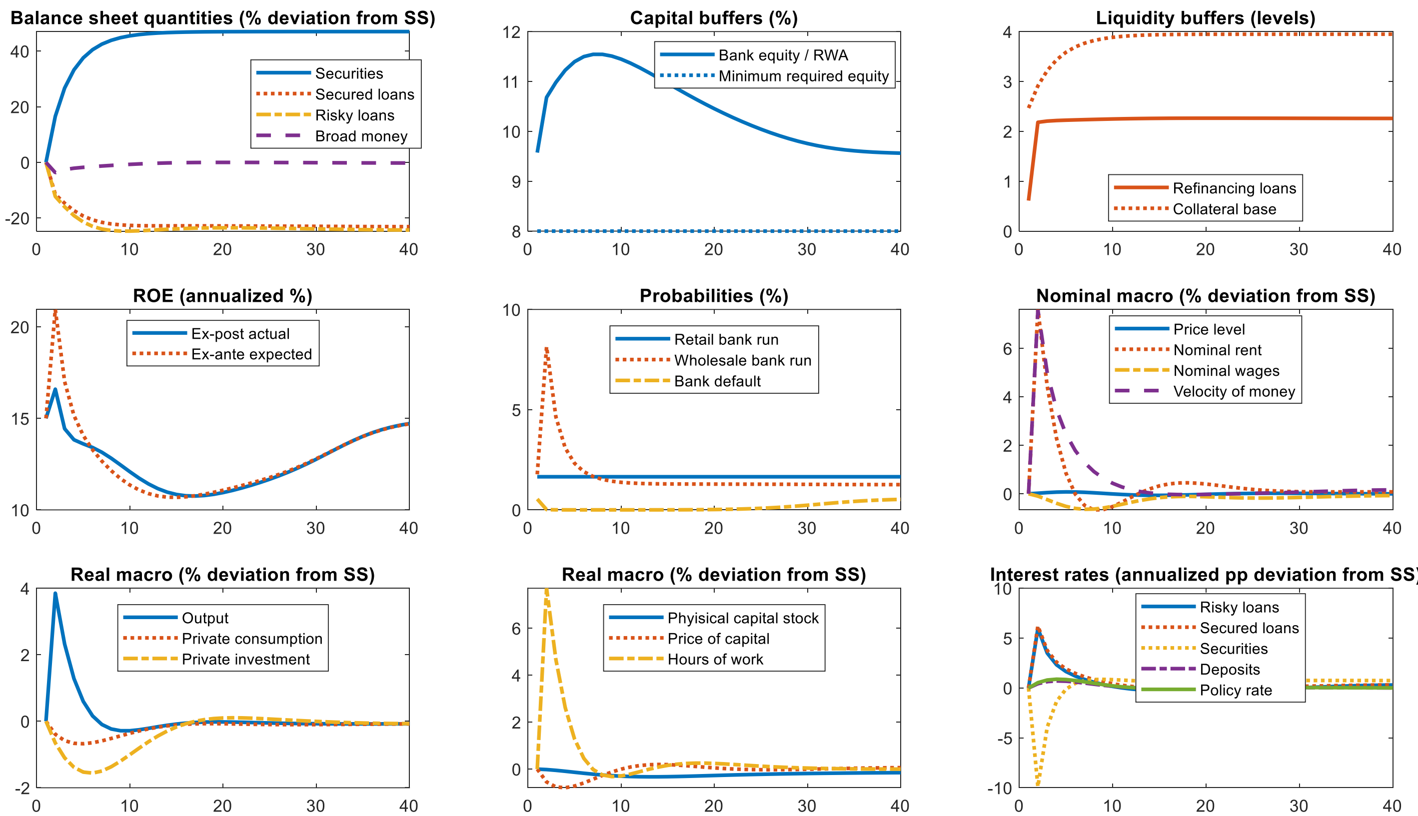
Figure 8 – Transition to a permanently higher RRR (pushing liquidity demand close to collateral constraint)



Source: author's construction

Can the authorities help to reduce the cost of switching to a narrow banking approach? Fiscal policy can, by significantly increasing the size of government debt portfolio. This can also be visible by looking at the securities yield in the simulation – it declines significantly, meaning that the financial system is super hungry for such risk-free assets. In other words, a decline in broad money, induced by private sector deleveraging, can be countered by government sector leveraging. The latter would result in directly creating new broad money (and, hence, counterbalancing the previously mentioned downward pressure on it), but will also result in banks holding more government securities on their balance sheets that relax their collateral constraint on central bank operations. Indeed, as Figure 9 shows, when the same simulation (of significantly higher RRR) is couple with a permanent increase in the government debt level (by deficit spending), then the permanent declines in output, consumption and investments vanish. If anything, in the short term, output gets a boost (due to deficit spending, even if private consumption and investment decline temporarily).

Figure 9 – Transition to a permanently higher RRR, with corresponding expansion of the government securities

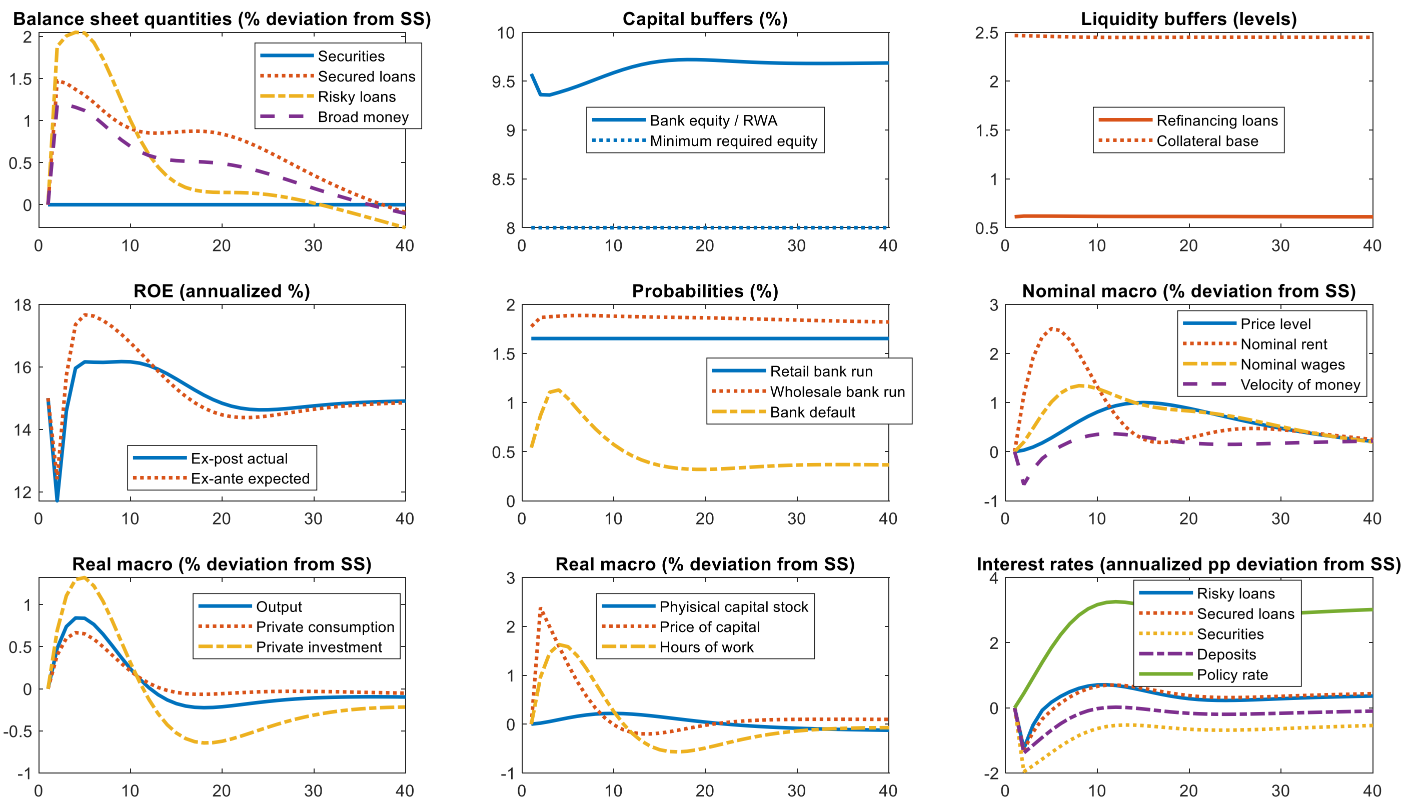


Source: author's construction

Hence, a huge expansion of the government debt portfolio can neutralize the cost of switching to a narrow banking approach. In other words, liquidity risk premium declines back again, meaning that all the interest rates converge back to initial levels, while broad money remains stable, along with the price level (to which the central bank is reacting). However, then the question becomes more of a political economy nature: who should receive the newly created money from such a huge deficit-spending? Should entrepreneurs receive it since they were who would have got hit the most? That is, money and credit allocation decisions would, at least during the transition, go into the government's hands, which (even when well-intended) typically is much less efficient than the private sector banks. This cost, which we don't model here, then still remains relevant in the real world. Another potential cost of such a policy is a perceived riskiness of government debt (sovereign default, which we don't model here either) after that amount of fiscal expansion.

Another interesting simulation that the model can deliver is to see what happens if banks face less competition in the deposit market. Fleckenstein and Longstaff (2024) argue that unsophisticated depositors lead to higher deposit rate markdowns and hence more profit for banks. Our model can directly speak about this topic. If less competition really leads to more profit for banks, does it have any implications for the rest of the economy? The answer is on Figure 10. It shows that, indeed, bank profits/ROE increase for several years following a permanent reduction in the degree of competition in the deposit market. This is supported by a significant decline in deposit interest rate and, hence, interest costs for banks. However, the increase in banks profitability is partially counterbalanced by two factors: (i) banks reduce lending interest rates as well so that they pass on a portion of this extra profit to borrowers, to incentivize them to borrow more and expand the bank balance sheet; and (ii) the central bank (seeing this decline in lending interest rates and the resulting moderate boom in output, consumption, investments and inflation) in response tightens its policy rate, which leads to interest rates gradually going back up. As policy rate increases over time this also generates some losses for banks due to their exposure to interest rate (maturity) risk.[53]

Figure 10 – Transition to a permanently less competition in the deposit market



Source: author's construction

[53] That's why on the figure ex-post ROE is somewhat lower than the ex-ante ROE.

Eventually, as the central bank reacts to this moderate (around 1%) expansion in credit/money creation and, hence, in the real economy,[54] all variables are pushed back very close to their initial steady states. The only variable that significantly changes in terms of the terminal level is the policy rate. The reason is that the level of competition has resulted in a permanent shift in banking sector interest rates. And since its these interest rates that pin down the real economy variables in the steady state, the policy rate will have to be adjusted permanently so that these banking sector interest rates go back to their starting point (which was consistent with the central bank objectives or initial steady states). This means that neutral level of the policy rate, among standard macroeconomic determinants, also depends on banking sector structure and the degree of competition in it. This point has usually been missing from monetary policy discussions. In this example, neutral policy rate becomes 3pp higher – a very big impact.[55]

For this section last, but certainly not least, we do a simulation that shows the reaction of money and credit to a permanent change in the government debt level. This simulation is part of the earlier one, where we discussed the impact of excessive levels of RRR. However, it still deserves a separate figure, since it encompasses an important message: a certain missing bridge between the money-centered view of the price level (emphasizing monetary policy) and the fiscal theory of the price level (emphasizing government's expected future primary surpluses). Our model, in principle, encompasses both in a unifying framework. Namely, price level in our model depends on both monetary policy-induced creation of money (via private credit) as well as fiscal policy-induced creation of money (via government debt). In FTPL terms, government debt leads to price level adjustments when it's not properly backed by expected future primary surpluses, e.g., as suggested by the following asset pricing equation:

$$\mathbb{E}_{\boldsymbol{t}} \sum_{\boldsymbol{i=t}}^{\infty} \boldsymbol{\beta^{i}} \,(\,\boldsymbol{primary\ surplus_{i}}\,) = \frac{Gov.Bonds_t + Outside\ money_t}{P_t} = \frac{\boldsymbol{S_t} \ + \boldsymbol{R_t^X}}{\boldsymbol{P_t}}$$

The last equality simply translates the terms in our model's language (without banks). However, what we add here, in principle, is that deposit money created by private bank credit is a contingent liability for consolidated governments: they are who promise to ensure one-to-one convertibility from banks' liability (deposits) into consolidated government's liability (cash/CBDC). Hence, in a unifying framework, where government debt is related to future primary surpluses, this debt should be the sum of conventional government debt (plus outside money) and privately created bank deposits. What also needs to be added is the private bank assets that back deposits. These assets represent the future private cash flows of the bank. In terms of the asset pricing equation:

[54] Note that as the dynamics start by banking sector interest rates declining on impact, the impulse responses resemble that of a monetary policy rate shock.

[55] And the reason why other variables are not impacted much more significantly is exactly because most of the (permanent) shock is absorbed by the policy rate itself.

$$\mathbb{E}_t \sum_{i=t}^{\infty} \boldsymbol{\beta^i} \left(\boldsymbol{primary\ surplus_i} + private\ cash\ flows_i \right)$$
$$= \frac{Gov.Bonds_t + Outside\ money_t + Privately\ created\ inside\ money_t}{P_t}$$
$$= \frac{S_t + R_t^X + L_t^S + L_t^R - E_t}{P_t} = \frac{\boldsymbol{BM_t}}{\boldsymbol{P_t}}$$

The last two equalities are translating the terms in our model's language, suggesting that it is the broad money that, in this case, tracks the consolidate government's "overall debt". One of our main results then emerges after assuming that private cash flow expectations do not change materially and simply follow a balanced growth path, which admittedly is an extreme example quantitatively.[56] But in that case, when the government increases its nominal (conventional type of) debt, without promising to increase primary surpluses in the future, this may not necessarily lead to a permanent increase in the price level (which, in this simulation, the central bank directly reacts to). Instead, more money creation via more government debt may be counterbalanced by less money creation via private sector credit. Indeed, that's what happens on Figure 11 as well. These two forces balancing each other leads to the broad money variable remaining stable over the simulation horizon, even though the increase in the government debt portfolio is very large (20% increase in nominal government debt).

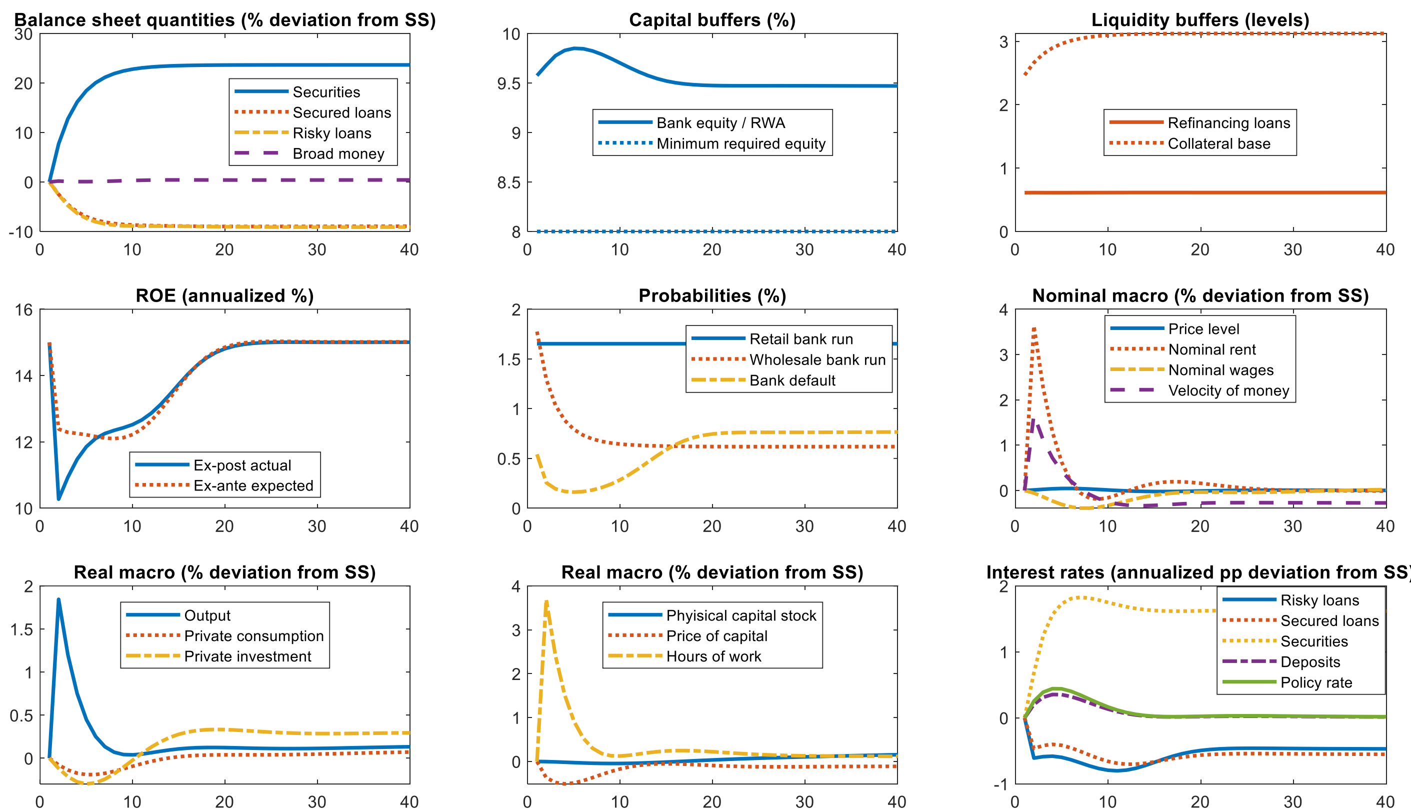
Public and private debt moving in opposite directions is not something new – that's what the crowding out literature is all about. However, in our model this doesn't come from standard reasoning of governments using up scarce loanable funds. In other words, crowding out in our model doesn't happen because of governments forcing banks to supply less credit to the private sector. Instead, this happens because the government's deficit-spending leads to more money creation going into the private nonfinancial sector hands, meaning that the latter economic agents no longer need to borrow from banks as much (which is costly) in order to obtain money balances for their transactions. In other words, it's a credit demand story, not a credit supply one, which has implications about where interest rates will go in response. Indeed, while broad money doesn't change much and, hence, the real economy quickly goes back to the initial steady state (along with virtually no impact on the price level), there are still variables that see a permanent adjustment. These are mostly banking sector interest rates, since government debt accumulation leads to lower liquidity risk premium as well as collateral service premium. This means that government securities yields increase permanently (by almost 2 pp), while private sector lending interest rates decline permanently (by about 0.5 pp).[57] From another perspective, this also looks very much like a securities supply effect, when someone "sells" more of its

[56] Qualitatively though, what matters is that the present value of all future private bank cash flows to not follow policy variables (e.g. government debt) one-to-one, which is a realistic assumption. This is equivalent to taxpayers occasionally paying the bill for ensuring financial stability.

[57] Note that secured and risky loans' interest rates decline similarly. This is because these two interest rates usually only differ in terms of the credit risk, whereas the changes we are talking about here come from liquidity risk, not from credit risk. That's why both interest rates and both types of loans adjust similarly.

securities, its price should decline. In our model this effect is brought by the presence of liquidity risk in banks liquidity transformation business.

Figure 11 – Transition to a permanently higher (nominal) government debt level



Source: author's construction

Finally, we may ask, why isn't this crowding out more painful for the real economy? This is because in our model we have homogenous households and entrepreneurs. This means that when the government spends newly created money after its debt expansion, this money goes directly into the hands of those households and entrepreneurs that were borrowing from the bank (at quite high interest rates). Hence, when these economic agents get (part of) the money they need for their transactions, they simply reduce borrowing from the banks, leading to an "immaculate deleveraging" (crowding out). But what if deficit spending were to go to economic agents that are wealthy and not borrowers themselves? Then, the deleveraging would be much uglier – for it to happen it would take a deliberate central bank action to increase interest rates strongly enough to engineer a sharp private sector deleveraging, so that the sum of government bonds and privately created deposit money, as a whole, do not change. This makes sure that the price level also doesn't get hit. In any case, it still says that monetary policy, in theory, can control the price level even when fiscal policy isn't cooperating. It's just that the cost of doing so may be extremely high, making it more of a political economy issue that we don't cover here.

6. Conclusions

We have developed a tractable macro-banking model that treats banks as the major creators of (inside) money, that face solvency, liquidity and maturity risks, and are subject to regulatory and demand constraints (i.e. incremental profit prospects). These five aspects, as discussed above, are tightly and nonlinearly interlinked. The model developed here provided causal answers to many highly policy-relevant questions at the intersection of macroeconomics and financial stability. The simulations, showing broad applicability of the model, demonstrated how the model can help in terms of understanding both normal as well as stress episodes. Many of the policy messages coming out of the model have usually been overlooked. Two financial frictions were key: central banks' collateral base constraint (generating a liquidity risk premium) and capital adequacy constraint (generating a credit risk premium).

In terms of the caveats, an important drawback of this model, relative to literature, is its rudimentary treatment of the nonfinancial sector. While this was not the focus here, a more realistic modeling of the nonfinancial sector can reveal even more intricacies that may be relevant for policymaking. The closed economy nature of the model is also limiting the analysis of the open economy-related questions. Also, the banking sector here abstracts from FX lending and, hence, creation of FX deposits. This issue is particularly important for emerging and developing economies. It is also important since the creation of FX deposits by domestic banks, which are not covered by a central bank of that currency, means potentially more liquidity and (currency induced) credit risks, which are worth studying separately. Introducing potential sovereign defaults, generating term premium, as well as stickiness in interest rate settings may also be interesting to study within this model. All these, along with bringing the model to data for estimation/filtering purposes, are avenues for future research.

Appendix 1 – summary of all model implications

The ability of this model to speak to different macro-banking issues is very broad. Hence, we could not show all the interesting simulations separately because of space constraints of the paper. Hence, here we just provide only very brief summaries of all the model-based policy-relevant messages one could easily think of, starting with the ones we did show simulations for. Some of these messages are nothing new, while others have been frequently overlooked:

[1] Credit risk channel of monetary policy: overly loose monetary policy can lead to banks taking on more credit and, hence, bank default risk (e.g. see Borio and Zhu, 2018 for empirical evidence). This works via excessively low rates compressing banks' profitability and increasing search for yield. As a result, bank leverage increases and capital adequacy

ratio declines. Therefore, it's risky lending and, hence, risky borrowers' (e.g. entrepreneurs) activity that experience the most pronounced boom-bust cycles.

[2] Strength of monetary tightening VS easing: in line with empirical evidence (e.g. Alessandri *et al*, 2025), a very strong monetary tightening is visibly more impactful on output relative to a similarly strong monetary easing. The reason is the asymmetric response of credit spreads to monetary tightening versus easing. When policy is tightened unexpectedly, this leads to banks seeing capital losses, to which they react more (in order to avoid breaching the capital requirement). On the other hand, capital gains due to an unexpected monetary easing do not lead to such abrupt reaction from banks.

[3] Demand for currency and liquidity risk premium: when demand for currency increases substantially, this can lead to banks tightening their lending conditions excessively. The reason is the collateral constraint on central bank's liquidity provision. Namely, since only a portion of banks' assets (usually government securities) are part of the central bank's collateral base, if currency demand is big enough to lead to deposit outflows in value more than those particular assets, banks will find it difficult to borrow the needed liquidity from the standard central bank discount window (and, hence, on the market as well). To avoid the liquidity stress, banks increase lending interest rates to deleverage.[58] This, in turn, reduces deposit (money) creation and, hence, dependence on runnable liabilities.

[4] Link between credit risk and liquidity risk: for banks, there is a certain degree of equivalence between accepting a credit risk and accepting a liquidity risk. For example, when banks want to reduce liquidity risks, they start deleveraging with the intent to build up capital position, reducing the bank default risk as well. The latter helps since the more capital they hold the less is dependence on run-prone information-insensitive liabilities like deposits. This, in turn, partially mitigates the liquidity risk. In other words, banks' decisions about the size of the balance sheet usually affect credit and liquidity risks in the same direction, meaning that if an exogenous driver pushes only one of those risks up, banks will respond by reducing both. Hence, they will end up operating with still slightly more of one risk and slightly less of the other risk. Put differently, banks may tolerate higher credit risks if liquidity risks are kept low by something else (e.g. implicit guarantees from the central bank), or vice versa – a certain kind of *within-sector regulatory arbitrage* if credit *and* liquidity risks aren't both properly and simultaneously regulated.

[5] Liquidity risk and collateral service: liquidity risk premium is usually a flip side of collateral service premium. Namely, while banks charge higher interest rates on those types of lending that create runnable deposits but are not part of the central bank's collateral base, the same banks will charge lower interest rates on those types of lending that also create runnable deposits but *are* part of the central bank's collateral base. The first type of lending, usually to the private sector, *generates* liquidity risk and, hence, incorporates

[58] In reality they will also increase term deposit rates, but this type of deposits is not part of the baseline version of our model here.

liquidity risk premium (upward pressure on rates). The second type of lending, usually to governments, *insures* liquidity risk and, hence, incorporates collateral service premium (downward pressure on rates). Indeed, during the GFC risky rates jumped up, while risk-free ones dropped.

[6] Underpricing of risks and subsequent credit crunch: when banks underprice the credit risk of their borrowers, this leads to a gradual erosion of bank capital. With little capital buffers to begin with, a later reappraisal of the credit risk becomes particularly impactful. Namely, to quickly rebuild the capital buffers and avoid the solvency crisis, the fastest way would be an immediate and sharp deleveraging (credit crunch). And unlike the liquidity stress, when secured as well as risky loan interest rates increase similarly (since they both similarly lead to more deposit creation and, hence, liquidity risk), during the solvency stress, risky loan interest rates increase particularly more. The aim is to cut back on the type of lending that was eating up capital buffers the most.

[7] Reserve requirements and money supply: it is already a widely held view that, under interest rate targeting framework, the level of reserve requirements doesn't matter for money supply. Which is true in most cases. Yet our model shows that it does matter in some (potentially rare) cases. The issue is that central banks' standard liquidity injection operations require eligible assets (for them to either buy or take as collateral). When liquidity demand, which is partly determined by the level of reserve requirements, becomes so high that these eligible assets start becoming very scarce, then these requirements will start having real effects, even under interest rate targeting frameworks. These real effects come from banks deleveraging in response to them realizing that liquidity risks are now too high.

[8] Narrow banking and credit crunch: whenever the size of a risk-free asset portfolio (usually, government securities) in the financial system is much less than the value of broad money, moving to a narrow banking approach may result in a sharp deleveraging. Narrow banking essentially means an end to banks' liquidity transformation (creation) business. As a result, banks will gradually[59] switch to equity financing, meaning no more inside money creation, which the economy depended on for transaction purposes. Since the central bank alone cannot usually create unlimited amounts of liquidity, unless it's backed by risk-free assets (which are scarce), the overall broad money declines as well, permanently. Hence, the economy gets a hit permanently as well (output and/or prices).

[9] Fiscal expansion and costs of narrow banking: fiscal policy can mitigate the above-mentioned cost (credit crunch) of switching to a narrow banking approach, by significantly increasing the size of government debt portfolio. This would lead to pushing broad money back up again (counterbalancing the previously mentioned downward pressure on it). Banks holding more government securities on their balance sheets would also relax their

[59] Gradually, because of new equity injection frictions.

collateral constraint on central bank operations. As a result, overall money creation doesn't get hit and the real economic effects become minimal as well. On the positive side, as a byproduct, banks' solvency risk goes to essentially zero. But what about political economy costs? The need to decide who receives the newly created money from such a huge deficit-spending (this is equivalent to centrally deciding on credit allocation)? Also, what if perceived probability of sovereign default increases in response to such a huge fiscal expansion? These questions are beyond this particular model.

[10] Deposit market competition and bank credit: increasing the share of unsophisticated depositors leads to less deposit market competition and, hence, lower deposit rates. This causes banks profitability to go up, but not very much due to two reasons: (i) in response, lending interest rates also decline (so borrowers also profit, not just banks, to increase loan demand); and (ii) the central bank hikes the policy rate (in response to accelerating credit and, eventually, inflation), resulting in negative revaluation effects due to interest rate (maturity) risk that banks have on their balance sheet.

[11] Macro-banking determinants of a neutral policy rate: the model clearly shows that any macro-banking adjustment that leads to permanent changes in deposit or lending interest rates (other than policy rate movements themselves), will lead to adjustments in a neutral level of the monetary policy rate. The reason is that its these deposit/lending interest rates that pin down the real economy variables, like consumption and investment. If their spread below/over the policy rate changes permanently, then the policy rate will have to be adjusted permanently as well, so that those banking sector interest rates go back to their steady states, equilibrating the real economy.

[12] Bank credit and the fiscal theory of the price level: our model provides a unifying framework where it incorporates a certain bridge between the money-centered view of the price level (emphasizing monetary policy) and the fiscal theory of the price level (emphasizing government's expected future primary surpluses). Namely, price level in our model depends on both monetary policy-induced creation of money (via private bank credit) as well as fiscal policy-induced creation of money (via government debt). This means that, when the government increases its nominal debt, without promising to increase primary surpluses in the future, this may not necessarily lead to a permanent increase in the price level. Instead, more money creation via government debt may be counterbalanced by less money creation via private sector credit. In FTPL terms, the key is that deposit-money (created by private bank credit) is a kind of a contingent liability for governments: they (through a central bank) promise to ensure one-to-one convertibility from banks' liability (deposits, or inside money) into consolidated government's liability (cash/CBDC), as if it *is* outside money.

[13] Public debt and crowding out: crowding out literature usually emphasize that, by public debt expansion, governments are using up scarce loanable funds, which forces banks to supply less credit to the private sector. However, this is not what happens in our model. Instead, crowding out may happen because the government's deficit spending leads to

more money creation going in the private nonfinancial sector hands, meaning that the latter economic agents no longer need to borrow from banks as much (which is quite costly) in order to obtain money balances for their transactions. Hence, crowding out is not a credit supply effect, but a credit demand effect materializing. This is important to understand the impact on lending interest rates.

[14] Public debt and crowding in: on the other hand, public debt can even support crowding in, if the above-mentioned (crowding out) effect will be absent (which, of course, usually isn't). The reason for this (potential) crowding in from government debt accumulation is that it leads to lower liquidity risk for banks. This means that government securities yields increase (since collateral service becomes marginally less valuable), while private sector lending interest rates decline (since the liquidity risk they generate is now marginally less scary). Lower lending rates may incentivize private credit.

[15] Immaculate deleveraging, or not: public debt-induced deleveraging (crowding out), in our model, happens with little cost to the real economy. But this is only due to our (strong) nonfinancial sector modeling assumptions (homogeneity of households and entrepreneurs). When the government spends the newly created money in our model, it goes directly to those households and entrepreneurs that were borrowing from the banks (at some interest cost). Hence, when these economic agents get (part of) the money they need for their transactions, they simply reduce borrowing from the banks, leading to an "immaculate deleveraging". But if deficit spending goes to economic agents that are wealthy and not bank borrowers themselves, then the deleveraging would be much uglier – it would need to happen via strong deliberate central bank actions, increasing interest rates to engineer a sharp private sector deleveraging of other economic agents.

[16] QE and its impact on broad money: in the model, even if the central bank buys government securities and increases the liquidity supply via QE, this doesn't necessarily impact broad money itself. The reason is that in our model, as banks are assumed to be risk-neutral, we do not have a term premium (even though we do have an interest rate risk). But without an adjustment to the term premium, there is no impact on bank credit, meaning that broad money doesn't change either.[60] In other words, this emphasizes the endogeneity of money creation via the banking system, instead of looking just at the supply of reserve balances by the central bank. On the other hand, if QE involves buying assets outside of central bank's collateral base (especially if it's bought from the nonfinancial sector), then this particular type of QE does affect liquidity risk premium, banking sector interest rates and inside money creation (broad money).

[17] Credit booms and their predictive power: after the GFC, it became already widely acknowledged that credit booms predict subsequent financial crises (e.g. Jordà *et al,*

[60] Another assumption is that the central bank buys these bonds from commercial banks. However, if it buys those bonds from non-bank private agents, then money supply may increase directly, without any change in private credit.

2010). The model can endogenously understand this empirical regularity, as mentioned above. However, the model also incorporates an asymmetry in a credit boom-bust cycle. Namely, when banks start underpricing the credit risk, the credit boom that follows is relatively moderate, however once they start to (properly) re-appraise the credit risk then the credit crunch is disproportionately larger. The reason is the nonlinearity on the credit transformation side: when banks become overly optimistic they marginally reduce lending rates, not to hurt profitability too much, which may accelerate demand for credit only moderately; however, when they realize that capital buffers have declined too much (as ex-post credit risk continuously surprised on the upside) then they sharply increase interest rates to quickly rebuild their capital position via a sharp deleveraging.[61]

[18] <u>Credit market competition and bank risk-taking</u>: when competition among banks (in credit extension) accelerates permanently, a trade-off emerges. The result of more intense competition is a permanent expansion of credit volumes (especially risky credit) and real economic activity. Borrowers enjoy lower lending interest rates. However, the downside of this is that banks respond to this heightened competition by increasing their risk-taking behavior. As a result, they lower capital buffers and get closer to the regulatory minimum. This increases the chances of bank defaults down the road, potentially significantly. Hence the trade-off, which the model simulations directly show.

[19] <u>Bank opacity and money creation</u>: inside money trades at par with outside money. However, the former is created by banks and for it to be widely accepted its backing should never be questioned. Otherwise, there would be bank runs. That is why bank assets are optimally opaque (Gorton, 2013). Indeed, if we reduce the cost of banks' information acquisition in our model, it predicts deleveraging, as banks start building up capital position to reduce dependence on runnable deposits. In other words, they will incorporate higher liquidity risk premium into lending interest rates. In addition, it will increase the banks' weighted average cost of funds, since their expected dependence on more costly wholesale funding increases. The result is less money creation (deleveraging).

[20] <u>Borrower default risk and the macroeconomy</u>: borrower default risk is a powerful driver for the model economy. When it worsens permanently, along with banks' perceptions about it, the risky bank lending sees a sharp permanent deleveraging, without a commensurate increase in other types of lending. The reason is the banks' heightened desire to (voluntarily) accumulate more capital buffers, above the regulatory minimum. Hence, overall money creation declines, leading to a permanent drop in output and prices. Due to persistent disinflationary pressures, the policy rate also declines very persistently. Observationally, this is equivalent to saying that a worsening in the borrower default risk

[61] If banks were able to attract new equity injections without frictions, then they wouldn't have to generate deleveraging. They would have rebuilt their capital adequacy ratio by increasing the numerator, instead of reducing the denominator. However, we know that when banks are undercapitalized that's exactly the time that investors become anxious and unwilling to increase their capital exposure even more.

leads to a decline in neutral policy rate, making the probability of hitting the effective lower bound on interest rates, which this model does not cover, higher.

[21] Asymmetric effects of capital requirements: when a prudential authority changes its capital adequacy requirements, the effect on bank balance sheets within the model is very asymmetric. If the requirement is relaxed, the effect on credit and money creation is very gradual and initially small. On the other hand, if the requirement is tightened, the effect on credit and money creation is abrupt and quite large. That's why tightening of these requirements are usually introduced very gradually and communicated early, so that banks have a chance to accumulate capital over time, without trying to generate an increase in a capital adequacy ratio via the denominator (i.e. deleveraging).

[22] Collateral base and its impact on credit allocation: it is frequently overlooked how the central bank's collateral framework can have a significant impact on the private credit allocation. And it is even less well understood that this impact on credit allocation is quite nuanced, as captured by our model. Namely, when the central bank includes a new asset into its collateral base, it incentivizes banks to reallocate where they divert credit, even when nothing else changes. However, this reallocation depends on how much liquidity buffers change. If the central bank introducing a new asset into its collateral base increased liquidity buffers (i.e. the overall size of the collateral base) so much that liquidity risk premium declined essentially to zero, then the central bank has just incentivized banks to increase their exposure to those assets that generate liquidity risk, including to those assets that are still not part of the collateral base (which may initially have sounded counterintuitive). On the other hand, if the collateral base still remained small and liquidity risk premium still sizeable, then the central bank has just incentivized banks to increase their exposure to that particular asset that was recently included in the collateral base.

[23] Risk weights and its impact on credit allocation: within the capital adequacy regulation, and unlike collateral base changes, the impact that a risk weight change has on credit allocation has received more attention. However, similar to collateral base changes, its nuanced nature has been less well acknowledged. According to our model, if the change in the risk weight is such that it increases/frees up overall capital buffers (e.g. secured loans now receiving much lower risk weight, hence, instantaneously increasing equity/risk-weighted asset ratio), the impact on credit allocation would be to incentivize banks to reallocate credit more generally to those borrowers that generate credit risk. Sounds counterintuitive, but a closer look shows the underlying intuition. Indeed, if banks are now less worried about violating the capital adequacy regulation (after equity ratio suddenly jumping up), then they will exploit this by taking on more credit risk, instead of reducing it by switching to safer loans (i.e. defeating the purpose of the initial risk weight change). On the other hand, if the risk weight changes are such that the overall capital buffers do not change (e.g. secured loans now receiving lower risk weight, but risky loans now receiving higher risk weight), then the central bank has just incentivized banks to reallocate credit from risky borrowers to safer borrowers, as intended.

[24] Access to wholesale funding and credit growth: as would be expected, if the cost of accessing the wholesale funding market increases, then overall credit extension and money creation decline, even if there are no retail deposit outflows. The reason is that liquidity risk premium becomes higher. The latter happens not because of higher amount of liquidity risk that banks face, but because of higher price of liquidity risk. Banks would reduce credit in the hope of increasing the capital adequacy ratio and, hence, dependence on equity on the liability side. Naturally, because of less money creation, output and prices decline as well.

[25] FX interventions (FXI) and the real economy: FXI can have a significant impact on the real economy, depending on the size of a central bank's collateral base, even without considering exchange rate-related effects. For example, within our model and its current calibration, a one-off FXI worth of 2% of annual GDP can either increase output by about 0.2% if the central bank buys FX or reduce it by about 1% if the central bank sells FX. The channel goes through our liquidity risk premium. That's why the impact is asymmetric. Namely, when the central bank buys FX it supplies newly created liquidity to banks and reduces their dependence on central bank discount borrowing (which uses up their scarce collateral). But if liquidity buffers were adequate to begin with, then any extra buffers are not valued as much. Hence, liquidity risk premium declines only modestly and, therefore, credit and money creation accelerate only marginally as well. On the other hand, when the central bank sells FX it widens the structural liquidity deficit in the system, making the scarce collateral significantly more valuable, as banks are now forced to use the central bank discount window more aggressively. Hence, liquidity risk premium, because of its nonlinearity, jumps up and induces credit and money creation to decelerate significantly, hurting output. Weaker demand reduces prices, which would actually make this impact in line with the objectives of selling FX in a real world example (central banks usually sell FX when they want to defend the exchange rate due to inflation fears). However, this impact comes at a price of slowing down credit and output.

[26] Profit tax and bank credit: reducing the profit tax rate applied to banks leads to more risk-taking, on the side of banks, in terms of accelerating the risky credit extension in particular. However, this does not lead to a higher probability of bank default. If anything, this probability marginally declines. The reason for both (acceleration of risky loans but a reduction in bank default risk) happening at the same time is that lower tax rate implies higher after-tax ROE, other things equal. Higher retained earnings, in turn, lead to higher capital. More capital is partly used to take on more borrower risk, while another part is used to maintain higher capital adequacy ratio, implying more robustness of banks in facing the credit risk. Obviously, the downside of this is marginally higher price level and/or the need for monetary policy tightening and fiscal consolidation in other parts of the economy (as lower bank profit tax rate implies lower overall tax revenues, other things equal). Also, if banks suddenly increase dividend payouts, then the solvency risk would no longer decline.

[27] Investors' required return and bank default risk: if investors suddenly and permanently increase their required returns on bank equity (which in the model is achieved through higher SS dividend payouts per equity share), the result will be banks increasing their lending interest rates to generate higher ROE. However, higher ROE doesn't result in higher capital adequacy ratio, since this extra profit is not kept as retained earnings but rather paid out as dividends. As a result of higher dividend payouts, capital adequacy ratio actually declines, resulting in higher bank default probability on the margin. In other words, when investors increase their *perception* of banking sector risk (hence, requiring higher returns), the banking sector risk, in response, then actually does increase.

[28] Bank overhead costs and financial stability: a decline in banks' overhead costs (or net non-interest costs) leads to higher ROE/retained earnings and, hence, an accumulation of capital buffers. These buffers are then partially used to extend more credit, particularly risky credit, without negatively impacting bank default risk (as capital is now higher than initially). The flip side of this up-leveraging is lower interest rates, hence borrowers benefiting from this as well. Unlike lowering profit tax, here there's no fiscal cost. If anything, higher ROE means banks will be paying higher taxes as well. Hence, work on reducing bank overhead (net non-interest) costs may be a particularly efficient way of helping credit while maintaining financial stability.

[29] Changes in potential output and bank leverage: potential output adjustments may have an impact on banks credit risk-taking, however the size of this impact depends on the nature of the driver behind such adjustments. For example, if the potential output changes due to one-off adjustments in the level of inputs into production (e.g. level shift in physical capital, labor force, or productivity), then output and prices move in opposite directions and bank credit doesn't change much, since they depend on nominal output. However, if we instead have a change in the expectations about the future potential output without much changes instantaneously, then output and prices move in the same direction in this transition period and, hence, this may have an outsized impact on bank credit. If this is in the downward direction, then banks see a decline in profitability (as lower demand for credit leads to lower lending interest rates) and increase their risk-taking, in the form of reducing their (voluntary) capital buffers.

[30] Money velocity and liquidity/credit risk premia: in an interest rate targeting framework, volatility in money velocity should be absorbed by corresponding changes in the supply of money. As inside money (created by banks) represents the biggest part of the overall broad money, this means that changes in money velocity should be absorbed by corresponding changes in bank credit. If the change in money velocity is large, then bank credit will have to adjust in similarly large amount. However, an abrupt and substantial change in bank credit means liquidity and capital buffers also change in relation to the overall size of the bank balance sheet. E.g. if money velocity declines, bank credit should increase by the same amount, instantaneously. But as accumulating liquidity and capital buffers, on the other hand, takes time for the bank, it will then have to temporarily operate will higher risks on both liquidity as well as credit transformation sides. The result is higher

risky lending rates (due to higher default risk premium) and lower risk-free yields (due to higher liquidity risk premium, which is the opposite of collateral service premium).

[31] FXI and financial stability: FX interventions by the central bank may affect not only the real economy, but the solvency risk of the banking system, at least during a transition period. Namely, if the central bank buys FX to hold it permanently (i.e. accumulates FX reserves), this means overall broad money demand is satisfied more with FX reserves stock and the need for private credit extension becomes less. Lower credit, for a given level of bank capital, means higher capital adequacy and less probability of bank default. It is true, the bank will then respond to this situation by relaxing credit conditions (reducing lending interest rates) and pushing bank default probability back to its equilibrium. However, at least during the transition period (i.e. when FXI is unanticipated, which in many cases they are), financial stability risks are somewhat mitigated. The reverse is true for FX sales.

[32] Interest rate risk and policy transmission: while the presence of interest rate risk may once in a while generate financial stability considerations, they can also strengthen the transmission of monetary policy. For instance, if policy rate unexpectedly tightens, this leads to banks experiencing losses due to negative realization of their interest rate risk. This leads to an erosion of capital and, hence, even more tightening of bank credit than it would otherwise lead. The reverse is also true when policy is eased, but maybe working at a slower pace. This is so since, in the model, bank capital being insufficient is much more risky, to which banks reacts immediately, while excess capital may be used for credit acceleration more gradually (unless investors ask for an immediate dividend payouts). In any case, whether policy rate changes are anticipated or unanticipated matters for the strength of the transmission.[62]

[33] Inside money and its endogeneity: according to this model, there can be four aggregate variables that directly lead to inside money creation. These are private credit extension, government deficit spending (debt accumulation), central banks' FX reserves accumulation and contractions in bank equity. In other words, while some models have rightly emphasized the key feature of money creation (that it is done via private bank credit), there are a couple of key factors that can generate leakages/divergence between money and private credit. This, as this model also underscores, shouldn't make us think that money creation is a process exogenous to private bank credit decisions.

[34] Inflation target and financial stability: higher inflation is usually associated with higher seigniorage only for central banks. But the model underlines that commercial banks may also benefit from seigniorage income. This is the case since, as inflation increases, deposit interest rates remain partially compressed due to money demand, while lending interest rates increase more forcefully. In other words, other things equal, nominal credit

[62] Another consideration is how frequent it is for banks to issue floating interest rate loans. This is present particularly in countries with high and/or volatile inflation. Floating rate loans also make policy transmission stronger, but from nonfinancial side, not from the banks' side.

spreads are positively related to inflation. This means that, if the central bank increases its inflation target or allows higher inflation, banks' ROE increases pushing capital up and reducing the bank default probability. This is akin to "financial dominance", when central banks tolerate higher inflation because of financial stability considerations.

[35] Money multiplier VS money divisor: does the model tell the mechanics of money creation process? It does and suggests that the so-called money multiplier understanding of money creation explains the process totally upside down. In the words of Goodhart (2017), while the money multiplier can be calculated based on an identity, "the causal direction works in exactly the reverse direction to that normally supposed". More specifically, first banks lend and create broad money, and only then do central banks supply the necessary base money to support their interest rate target. Eventually, we may still have a picture where base money is a small fraction of broad money, but the process of getting there is completely in reverse. Put differently, "it is not so much a money multiplier, as a money divisor" (Goodhart, 2017). This means that, in principle, the process starts with banks and that's why we need to understand their behavior first.

[36] Comovement of consumption and investment: a standard banking theory of loanable funds would suggest that for investment to pick up, first consumption should slow down. The reason is that those models confuse financial saving with real saving. Namely, in these models, households would need to save – in the form of less consumption – so that banks have sufficient funds to lend out later. And since investment depends on lending, we get a negative comovement between investment and consumption in a credit boom-bust cycle. Yet, our model shows that this is not necessarily the case. The reason is that in our model it is financial saving that funds investment.[63] But financial saving is created by banks expanding the size of their balance sheets, which – while have limitations of their own – are not really constrained by the amount of real saving in the economy.

This list, while very rich in policy implications, isn't full, as we have only considered the impacts of single shocks or re-calibrations. Generating plausible scenarios (with multiple shocks) may generate some extra interesting policy implications, due to strong nonlinearities. All these emphasize the usefulness of the features that the model incorporates in terms of understanding the intricacies of the macro-banking realm.

Appendix 2 – deriving key interest rate equations

Recall that the optimization problem of the bank is:

[63] In standard macro textbooks, real saving being equal to investment is not a causal model/link, it is just an identity.

$$\max \mathbb{E}_t\, F_\delta\, ROE_t = \mathbb{E}_t F_\delta\, \frac{IR_t - F_\sigma\, F_c\left[\left(i_t^D(1-\hat{c})+i_t^R\hat{c}\right)(TA_t-E_t)-\left(i_t^R-i_t^D\right)(1-\hat{c})R_t^X\right] - (1-F_\sigma\, F_c)\left(i_t^R+\chi\right)(TA_t-E_t) - \psi}{E_t}\,(1-\tau)$$

$$subject\ to \qquad S_t \le f^S(i_t^S)$$

$$L_t^S \le f^{SL}(i_t^{SL})$$

$$L_t^R \le f^{RL}(i_t^{RL})$$

$$i_t^D \ge f^D(i_t^R)$$

Before solving this optimization problem, let's first collect the key partial derivatives to make the math easier:

$\frac{\partial\, IR_t}{\partial\, i_t^x} = (1-\varepsilon^x)\, x$ where $x = S_t, L_t^S$ and $\frac{\partial\, IR_t}{\partial\, i_t^{RL}} = \left(1-\varepsilon^{RL} + \varepsilon^{RL}\frac{\hat{\delta}}{i_t^{RL}}\right) L_t^R$

$\frac{\partial\, TA_t}{\partial\, i_t^x} = -\varepsilon^x \frac{x}{i_t^x}$; $\frac{\partial\, CB_t}{\partial\, i_t^x} = -\eta^x\, \varepsilon^x \frac{x}{i_t^x}$; $\frac{\partial\, RWA_t}{\partial\, i_t^x} = -\omega^x\, \varepsilon^x \frac{x}{i_t^x}$ where $x = S_t, L_t^S, L_t^R$

$$\frac{\partial\, F_c}{\partial\, i_t^x} = f_{\hat{c}}\left(\frac{CB_t - rr(TA_t-E_t)+(1-rr)R_t^X}{(1-rr)(TA_t-E_t)+(1-rr)R_t^X}\right)\frac{1}{1-rr}\left(\frac{\partial\, CB_t}{\partial\, i_t^x}\frac{1}{(TA_t-E_t+R_t^X)} - \frac{\partial\, TA_t}{\partial\, i_t^x}\frac{CB_t+R_t^X}{\left(TA_t-E_t+R_t^X\right)^2}\right) \text{ where } x = S_t, L_t^S, L_t^R$$

$$\frac{\partial\, F_\delta}{\partial\, i_t^x} = f_{\hat{\delta}}\left(\frac{E_t - e^m\, RWA_t}{L_t^R}\right)\left(-\frac{e^m}{L_t^R}\right)\frac{\partial\, RWA_t}{\partial\, i_t^x} \qquad \text{where } x = S_t, L_t^S$$

$$\frac{\partial\, F_\delta}{\partial\, i_t^{RL}} = f_{\hat{\delta}}\left(\frac{E_t - e^m\, RWA_t}{L_t^R}\right)\left(-\frac{e^m}{L_t^R}\frac{\partial\, RWA_t}{\partial\, i_t^{RL}} + \frac{E_t - e^m\, RWA_t}{\left(L_t^R\right)^2}\varepsilon^{RL}\frac{L_t^R}{i_t^{RL}}\right)$$

With all of these partial derivatives ready, we can easily derive the first order conditions (FOCs), starting with the equilibrium interest rate (yield) on securities:

$$F_\delta \frac{\partial\, ROE_t}{\partial\, i_t^S} + \frac{\partial\, F_\delta}{\partial\, i_t^S} ROE_t = F_\delta \frac{\begin{pmatrix} \frac{\partial\, IR_t}{\partial\, i_t^S} - F_\sigma\, F_c\left(i_t^D(1-c)+i_t^R c\right)\frac{\partial\, TA_t}{\partial\, i_t^S} - (1-F_\sigma\, F_c)\left(i_t^R+\chi\right)\frac{\partial\, TA_t}{\partial\, i_t^S} \\ -F_\sigma\left[\left(i_t^D(1-c)+i_t^R c\right)(TA_t-E_t)-\left(i_t^R-i_t^D\right)(1-c)R_t^X - \left(i_t^R+\chi\right)(TA_t-E_t)\right]\frac{\partial\, F_c}{\partial\, i_t^S}\end{pmatrix}}{E_t}(1-\tau) +$$
$$f_{\hat{\delta}}\left(\frac{E_t - e^m\, RWA_t}{L_t^R}\right)\left(-\frac{e^m}{L_t^R}\right)\frac{\partial\, RWA_t}{\partial\, i_t^S}\, ROE_t = 0$$

Substituting the partial derivatives from above:

$$F_{\delta}\frac{\left(\begin{array}{c}(1-\varepsilon^S)\,S_t+F_{\sigma}\,F_c\left[\left(i_t^D\,(1-c)+i_t^R\,c\,\right)\varepsilon^S\frac{S_t}{i_t^S}\right]+(1-F_{\sigma}\,F_c)\left[\left(i_t^R+\chi\,\right)\varepsilon^S\frac{S_t}{i_t^S}\right]\\ +F_{\sigma}\left[\left(i_t^D\,(1-c)+i_t^R\,c\,\right)(TA_t-E_t)-\left(i_t^R-i_t^D\right)(1-c)R_t^X-\left[\left(i_t^R+\chi\,\right)(TA_t-E_t)\right]\right]\\ \left(\frac{1}{1-rr}\,f_{\hat{c}}\left(\frac{CB_t-rr(TA_t-E_t)+(1-rr)R_t^X}{(1-rr)(TA_t-E_t)+(1-rr)R_t^X}\right)\left(\eta^S\,\varepsilon^S\frac{S_t}{i_t^S}\frac{1}{TA_t-E_t+R_t^X}-\varepsilon^S\frac{S_t}{i_t^S}\frac{CB_t+R_t^X}{\left(TA_t-E_t+R_t^X\right)^2}\right)\right)\end{array}\right)}{K}(1-\tau)+$$

$$f_{\hat{\delta}}\left(\frac{E_t-e^m\,RWA_t}{L_t^R}\right)\left(-\frac{e^m}{L_t^R}\right)\,\left(-\omega^S\,\varepsilon^S\,\frac{S_t}{i_t^S}\right)\,ROE_t\;=0$$

Multiplying both sides by $\frac{E_t\,i_t^S}{(1-\tau)\,S_t}$ and denoting $f_c\equiv f_{\hat{c}}\left(\frac{CB_t-rr(TA_t-E_t)+(1-rr)R_t^X}{(1-rr)(TA_t-E_t)+(1-rr)R_t^X}\right)$ and $f_{\delta}\equiv f_{\hat{\delta}}\left(\frac{E_t-e^m\,RWA_t}{L_t^R}\right)$, we get:

$$F_{\delta}\,\varepsilon^S\left(\begin{array}{c}F_{\sigma}\,F_c\,(i_t^D\,(1-c)+i_t^R c)+(1-F_{\sigma}\,F_c)(i_t^R+\chi\,)\\ +F_{\sigma}\Big((i_t^D\,(1-c)+i_t^R c\,)(TA_t-E_t)-(i_t^R-i_t^D)(1-c)R_t^X-[(i_t^R+\chi\,)(TA_t-E_t)]\Big)\\ \left(\frac{1}{1-rr}\,f_c\left(\eta^S\,\frac{1}{TA_t-E_t+R_t^X}-\frac{CB_t+R_t^X}{\left(TA_t-E_t+R_t^X\right)^2}\right)\right)\end{array}\right)+$$

$$\varepsilon^S\,f_{\delta}\,\omega^S\,e^m\,\frac{E_t}{L_t^R}\frac{ROE_t}{1-\tau}\;=F_{\delta}(\varepsilon^S-1)\,i_t^S$$

Dividing both sides by $F_{\delta}\,(\varepsilon^S-1)$ and rearranging:

$$i_t^S=\frac{\varepsilon^S}{\varepsilon^S-1}\Big(\,F_{\sigma}\,F_c\,(1-c)\,i_t^D+\big(1-F_{\sigma}\,F_c\,(1-c)\big)\,i_t^R+(1-F_{\sigma}\,F_c)\,\chi+\,\frac{1}{1-rr}\,f_c\,F_{\sigma}\,\left(\frac{CB_t+R_t^X}{TA_t-E_t+R_t^X}-\eta^S\right)\,\left[(i_t^R-i_t^D)\,(1-c)+\chi\,\frac{TA_t-E_t}{TA_t-E_t+R_t^X}\right]+\,\frac{f_{\delta}}{F_{\delta}}\,\omega^S\,e^m\,\frac{E_t}{L_t^R}\,\frac{ROE_t}{1-\tau}\Big)$$

We have similar equations for other interest rates set by banks.

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