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Ramsey-Optimal Fiscal Spending and Reserve Accumulation Policies under Volatile Aid

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ABSTRACT: This paper examines Ramsey-optimal policies related to fiscal spending and international reserve accumulation in response to volatile aid flows in Low-Income Countries (LICs). We develop a real Dynamic Stochastic General Equilibrium (DSGE) model of a small open economy, incorporating government transfers and public investment as fiscal spending components, along with two prominent characteristics of LICs: Dutch disease (DD) externalities and financially constrained households. Driven by considerations of precautionary saving, Ramsey-optimal policies involve partial reserve accumulation and partial fiscal spending of aid. Stronger DD externalities necessitate greater reserve ac- cumulation to stabilize future output, thereby mitigating consumption volatility. While transfers directly support private consumption smoothing, public investment also con- tributes to this goal by sustaining future income through gradual capital accumulation. Higher aid volatility calls for increased public investment, underscoring the role of public capital accumulation as a precautionary saving instrument, beyond its developmental role discussed in the literature.

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WORKING PAPERS

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1 Introduction

Foreign aid is an important but volatile revenue source for lower-income developing countries. Using data from 63 recipient countries from 1969 to 1995, Pallage and Robe (2001) report that aid volatility is about two to three times that of recipient countries' output. Bulíř and Hamann (2003, 2008) find that aid is generally more volatile than tax revenues, especially in highly aid-dependent countries. Using more recent data, we find that 78 percent of low-income countries (LICs) and 93 percent of lower middle-income countries (LMICs) experience higher volatility in aid than in tax revenues.[1]

Aid volatility complicates government decisions on aid allocation. While the literature has studied the macroeconomic implications of different policies in the context of aid scaling ups (e.g., Adam et al., 2009; Buffie et al., 2008; Berg et al., 2010a, 2015), it has paid comparatively less attention to the optimal configuration of policies to manage volatile aid. In this paper, we fill this gap by jointly considering fiscal spending and foreign exchange reserve accumulation policies, undertaking a fully Ramsey-optimal policy analysis.

We conduct our optimal policy analysis considering two prevalent features in LICs. The first one is Dutch disease (DD), a significant concern for policy makers during aid surges and extensively explored and documented in the literature (see, e.g, Torvik, 2001; Berg et al., 2007; Rajan and Subramanian, 2011; Lama and Medina, 2012). The second feature is financially constrained consumers, which is particularly relevant under aid volatility due to its implications for private consumption fluctuations. As highlighted in Pallage and Robe (2003), Pallage et al. (2006), and Arellano et al. (2009), LIC households often struggle to manage consumption fluctuations as they lack access to financial and capital markets, including international markets.[2]

To extract key insights regarding optimal policies amid volatile aid, we first use a simple two-

[1] Appendix A provides data and calculation details.

[2] Beyond the fluctuations in consumption, the literature has extensively documented additional adverse effects of aid volatility. Lensink and Morrissey (2000) and Hudson (2015) find that uncertain aid reduces its effectiveness in its contribution to economic growth, health, and social development. Furthermore, aid volatility has been linked to reduced private investment and government expenditure (Hudson and Mosley, 2008), while unpredictability in aid disbursements can impede LICs from making productive investments to stimulate growth (Celasum and Walliser, 2008; Agénor and Aizenman, 2010). Consequently, the literature also emphasizes the significance of donors' behavior in coordinating aid disbursement and enhancing the predictability of aid flows (e.g., Bulíř and Hamann, 2003; Arellano et al., 2009).

period model that incorporates Dutch disease (DD) effects and financially constrained consumers. Households are of the hand-to-mouth type, forced to consume their current available income and unable to smooth consumption on their own. DD effects are captured by a growth externality, whereby spending of aid in the first period (akin to a decrease in net exports) negatively impacts output in the second period. We refer to this as a DD externality. Regarding policies, the government can accumulate aid as international reserves or transfer it to hand-to-mouth households. Additionally, we assume these transfers can be productive by increasing output in the second period.

The analytical results from the simple model reveal that the Ramsey-optimal policies for fiscal spending and reserve accumulation can be described as rules incorporating a forward-looking component. Instead of solely reacting to current aid flows—a *contemporaneous* rule—, the optimal rules also take into account expected future aid, a forward-looking component, which is reminiscent of the permanent income hypothesis (Friedman, 1957). This departure from the simple contemporaneous policy rules, which are often discussed in the literature and policy circles (such as in Buffie et al., 2008; Berg et al., 2010b, 2015), suggests that such contemporaneous rules are unlikely to be optimal. Furthermore, in line with the Lucas critique (Lucas, 1976), the responsiveness of the optimal rules to current and future aid flows depends on structural parameters, including those associated with the DD externality and productivity of transfers.

Optimal policies in the simple model involve a combination of partial fiscal spending and reserve accumulation of aid. In the baseline, where DD externalities are absent and transfers are not productive, partial reserve accumulation arises from precautionary saving motives in the face of volatile aid. Meanwhile, fiscal transfers help smooth consumption for hand-to-mouth consumers. With increased aid volatility, the government opts to accumulate more reserves and allocates less aid to households in the current period.

DD externalities and productive transfers influence the precautionary savings motive, thereby impacting optimal policies. With DD externalities, optimal policies require greater reserve accumulation and reduced fiscal spending in the current period relative to the baseline scenario. This adjustment arises from recognizing that current aid spending negatively affects future output through the DD externality. By accumulating more aid in reserves and spending less, governments can counteract this externality, support future output, and mitigate the impact of volatile aid on private consumption. Conversely, when transfers are productive, optimal policies require less reserve accumulation and increased fiscal spending in the current period. In this scenario, higher spending and reduced reserve accumulation bolster future output and mitigate the negative effects of volatile aid on private consumption.

While our simplified model offers crucial insights into optimal policy decisions, it has limitations. Firstly, it assumes that all households face financial constraints. Secondly, the model relies on foreign reserves as the sole asset for consumption smoothing, with transfers serving a dual purpose: facilitating this smoothing and simultaneously promoting future output growth. Thirdly, the framework is built around a single production sector, thereby overlooking the impact of aid shocks on relative prices, such as the real exchange rate. Last, it lacks dynamics to a significant extent.

To address these limitations, we develop a richer quantitative real dynamic stochastic general equilibrium (DSGE) model, calibrate it to an average LIC, and solve for the Ramsey-optimal fiscal spending and reserve accumulation policies. The model is a two-sector framework of a small open economy with several LIC-specific features, including a large share of financially constrained hand-to-mouth households and learning-by-doing externalities à la van Wijnbergen (1984), which capture DD effects because real exchange rate appreciation can harm productivity growth of the traded goods sector. Government policies comprise three instruments: reserve accumulation, transfers, and public investment. Considering the accumulation of public capital through public investment is a critical policy due to its significant role in development and as an alternative form of savings, alongside international reserves.

In the quantitative model, optimal policies still involve partial spending and reserve accumulation of aid inflows. However, the results underscore the importance of directing some government spending towards public investment, in addition to transfers. This approach increases welfare compared to the case when the government solely transfers some of the aid and saves the rest in reserves. While transfers directly support consumption of hand-to-mouth households, public investment spending and the subsequent buildup of public capital indirectly smooth consumption by sustaining production and income over time. In this context, the accumulation of public capital serves as a saving instrument, similar to reserve accumulation, to address aid volatility.

In the face of increased aid volatility, both public investment and reserve accumulation rise, thereby supporting higher future transfers. Partial reserve accumulation helps sustain higher future government spending through two channels. First, it mitigates the real exchange rate appreciation associated with aid spending, thereby limiting the contraction in traded goods output and boosting overall tax revenues in the future. Second, reserve savings generate interest income, adding to overall government revenues. Meanwhile, public investment directly bolsters output by increasing public capital and indirectly by crowding in private investment, which raises private capital and, through this channel, output. This crowding-in effect also helps increase government revenues through taxation. These macroeconomic effects of public investment and reserve accumulation are crucial to understanding the welfare results under aid volatility, as they generate more income and output over time, supporting consumption in the medium term.

From a welfare perspective, higher aid volatility calls for increased public investment, combined with transfers and reserve accumulation. This is because aid volatility influences both the variance and mean effects on consumption in the welfare analysis. On one hand, higher aid volatility increases the variance of consumption, negatively impacting welfare. On the other hand, precautionary savings associated with reserve accumulation and public investment (public capital accumulation) lead to an increase in the mean of consumption, positively affecting welfare. In our analysis, the positive mean effect dominates, resulting in an overall increase in welfare. Specifically, in our calibrated model, a 50-percent increase in aid volatility raises savers' welfare to 0.79 percent of steady-state consumption from 0.35 percent, and hand-to-mouth households' welfare to 1.11 percent of steady-state consumption from 0.49 percent, provided that the government responds by accumulating more reserves and increasing public investment.[3]

Sensitivity analysis further highlights how structural characteristics of the economy affect the relative roles of the three policy instruments. A smaller share of hand-to-mouth households implies that the government should reorient spending toward public investment and reduce the accumulation of foreign reserves, as the need for the government to smooth consumption is reduced relative to the baseline scenario. More persistent DD externalities (implying more severe DD effects) result in a bigger decline in traded goods production and overall less government resources to allocate.

[3]This assumes, of course, that under increased aid volatility, the government is still able to accumulate more reserves and increase public investment, which may not be the case due to other frictions not present in the model. Moreover, as mentioned in footnote 2, there are other considerations that highlight the adverse effects of aid volatility.

This also leads to a slightly larger accumulation of reserves than under the baseline scenario, to help reduce the currency appreciation and mitigate ensuing DD effects. Lastly, a less productive public capital induces a reallocation from public investment to transfers, and an increase in reserve accumulation as a savings device that helps smooth consumption over time.

1.1 Related Literature

Our results contribute to the literature on policy responses to manage foreign aid, which overlooks aid volatility. Berg et al. (2010a) and Berg et al. (2015) focus on *contemporaneous* partial fiscal spending and reserve accumulation rules, examining welfare implications. Buffie et al. (2008) and Adam et al. (2009) study sterilized reserve accumulation policies and their effect on inflation and real exchange rate fluctuations. Excluding reserve accumulation, Kimbrough (1986) finds that the optimal response to an aid surge involves lowering tax rates and increasing government spending. Focusing on long-term outcomes and also omitting reserve accumulation, Gong et al. (2008) show that optimal policies require lowering inflation and the income tax rate while increasing public spending to encourage private capital accumulation. An exception in the literature is Prati and Tressel (2006), which uses a two-period model to demonstrate that reserve management can mitigate the negative effects of aid volatility. Our paper echoes this finding but highlights the crucial and alternative role of public investment by comprehensively solving the full Ramsey problem for both optimal reserve accumulation and fiscal spending policies.

Our paper relates to the literature on aid-financed public investment and its implications for development. The influential narrative of economic development posits that poor countries are ensnared in poverty traps, necessitating a substantial increase in aid-financed public investment—a Big Push—to catalyze a rise in per capita income (see, e.g., Sachs, 2005). Several works, including Adam and Bevan (2006), Agénor et al. (2008), Cerra et al. (2009), Chatterjee and Turnovsky (2007), and Zanna et al. (2019) have studied the macroeconomic effects, including on growth, of aid surges that finance public capital accumulation. Relative to this literature, our paper shows that, in addition to developmental considerations, public capital accumulation can also serve as a precautionary saving instrument in the context of volatile aid.

There is a burgeoning literature on the macroeconomic implications, as well as the costs and benefits, of sterilized foreign exchange interventions (FXI) in advanced and emerging market economies (see, e.g., Adler et al., 2019; Adrian et al., 2022; Alla et al., 2020; Basu et al., 2020; Benes et al., 2015; Cavallino, 2019; Fanelli and Straub, 2021; Faltermeier et al., 2022; Itskhoki and Mukhin, 2023). Some papers in this literature also explore the implications of combining FXI with monetary policy, capital flow measures, and macroprudential policies (see e.g., Adrian et al., 2022; Ghosh et al., 2018), including from an optimal perspective (Basu et al., 2020). Relative to this literature, our paper focuses on one of the most important sources of external financing of LICs—foreign aid—and considers the role of key characteristics of these countries—such as DD externalities and financially constrained households—in determining the optimal combination of fiscal spending, including public investment, alongside reserve accumulation (FXI).

Our results are also relevant to the literature on optimal natural resource allocation.[4] The conventional policy prescription suggests that resource-rich countries should smooth spending of natural resource revenues (see, e.g., Davis et al., 2001; Barnett and Ossowski, 2003; Bems and de Carvalho Filho, 2011), consistent with the permanent income hypothesis. However, when accounting for the capital scarcity of LICs and the potential growth effects arising from public infrastructure investment, a more immediate and sizable spending of resource revenues on public investment can be appropriate or optimal (see, e.g., Takizawa et al., 2004; van der Ploeg and Venables, 2011; International Monetary Fund, 2012a,b; Berg et al., 2013; Araujo et al., 2016).[5] By considering a detailed specification of fiscal spending policies and accounting for aid volatility, our paper emphasizes precautionary motives in optimal saving decisions, while also recognizing the savings role of public capital accumulation and its potential growth and welfare enhancing effects in the longer run.

The remainder of the paper is organized as follows: Section 2 outlines the simple analytical model that provides insights into the key results of the paper. Section 3 presents a richer quantitative model. Section 4 describes the solution method and calibration. Section 5 presents the main optimal policy results of the quantitative model, and Section 6 provides some sensitivity analysis.

[4] Aid inflows are not dissimilar to external natural resource revenues, which are volatile due to fluctuations in world market prices.

[5] To cope with DD effects, van der Ploeg and Venables (2013) suggest that the optimal management of external income involves investing in the nontraded goods sector while slowly increasing consumption. However, their analysis treats the income rise as a one-time, unanticipated surprise and does not consider reserve accumulation as a policy instrument.

The final section concludes.

2 Analysis with a Simple Model

To develop some insights about optimal fiscal spending and reserve accumulation polices under volatile aid, we first consider a two-period real model of a small open economy. The model has only traded goods and a fixed endowment in period 1, which can grow or decline in period 2, depending on some externalities. The setting incorporates several features and frictions of LICs: (i) financially constrained hand-to-mouth households that cannot smooth consumption over time and are forced to consume their current income and government transfers received each period; (ii) a government that receives foreign aid, accumulates international reserves, and makes transfers to households, which can be productive as an externality; (iii) an externality associated with net exports, which induce Dutch disease (DD) effects; and (iv) uncertain and volatile aid in period 2.

2.1 The Model Setup

Consider a two-period model of a small open economy that has a representative hand-to-mouth household and a government, which receives aid from abroad in both periods.

2.1.1 Stochastic Process for Aid

Aid in period 1, a_1^*, is known with certainty. Aid in period 2 is stochastic and follows a binomial distribution:

$$a_2^* = \begin{cases} a_{2L}^* & \text{with probability } \mathfrak{p} \\ a_{2H}^* & \text{with probability } 1-\mathfrak{p}, \end{cases} \tag{2.1}$$

where $a_{2H}^* > a_{2L}^*$ and $E(a_2^*) = a_1^*$.

2.1.2 Government

In period 1, the government decides how much aid to accumulate in reserves (res_1^*) and how much to spend as transfers to households (z_1), respecting the budget constraint

$$a_1^* = z_1 + res_1^* - (1+r^*)res_0^*, \tag{2.2}$$

where the initial stock of reserves, res_0^*, and the foreign real interest rate, r^*, are given exogenously.

In period 2, $res_2^* = 0$ by construction, and the government budget constraint depends on the realization of aid: if aid is low (a_{2L}^*),

$$a_{2L}^* = z_{2L} - (1 + r^*)res_1^*; \tag{2.3}$$

and if aid is high (a_{2H}^*),

$$a_{2H}^* = z_{2H} - (1 + r^*)res_1^*. \tag{2.4}$$

Depending on the period-2 level of aid, transfers to households can be low (z_{2L}) or high (z_{2H}).

2.1.3 Households

Hand-to-mouth households, denoted by the superscript h, have the following preferences:

$$\mathbb{U} = \frac{\left(c_1^h\right)^{1-\sigma}}{1-\sigma} + \beta \left[\mathfrak{p} \frac{\left(c_{2L}^h\right)^{1-\sigma}}{1-\sigma} + (1-\mathfrak{p}) \frac{\left(c_{2H}^h\right)^{1-\sigma}}{1-\sigma} \right], \tag{2.5}$$

where c_1^h denotes consumption in period 1, c_{2L}^h (c_{2H}^h) denotes consumption in period 2 when aid is low (high), $\beta < 1$ is the discount factor, and σ is the relative risk aversion coefficient.

Households do not have access to assets. In every period, they consume output (y_1, y_2) and transfers from the government. We assume that period-1 output is constant: $y_1 = y$. Given these assumptions, the household budget constraint in period 1 corresponds to:

$$c_1^h = y + z_1. \tag{2.6}$$

Again with uncertain aid in period 2, there are two possible household budget constraints:

$$c_{2L}^h = y_2 + z_{2L} \tag{2.7}$$

and

$$c_{2H}^h = y_2 + z_{2H}. \tag{2.8}$$

Output in the second period (y_2) is affected by two externalities. Following Choi and Taylor (2017), the first externality assumes that net exports in period 1 ($y - c_1^h$) induce a growth externality for traded goods output in period 2.[6] For our purpose, this captures some negative consequences similar to DD effects: lower net exports cause a decrease in future traded goods output.[7] The second externality captures the possibility that fiscal spending may be productive. To allow for this externality, we assume that transfers in period 1 (z_1) can increase output in period 2.[8] Putting together these two externalities in a simple form yields

$$y_2 = y\left[1 + \xi\left(\frac{y - c_1^h}{y}\right) + \alpha^G\left(\frac{z_1}{y}\right)\right], \tag{2.9}$$

where $\xi > 0$ and $\alpha^G > 0$ capture the intensity of the two externalities.

Note that transfers in period 1, z_1, have a dual impact on consumption over time: they can directly support consumption in period 1 as reflected by equation (2.6), and they can also indirectly support consumption in period 2, by affecting output in that period, as reflected by equations (2.7), (2.8), and (2.9).

2.1.4 Aggregate Constraints

Combining the government and household budget constraints, equations (2.2)–(2.4) and (2.6)–(2.8) yields the following aggregate constraints for the economy:

$$res_1^* = (1 + r^*)res_0^* + a_1^* + y - c_1^h, \tag{2.10}$$

$$0 = (1 + r^*)res_1^* + a_{2L}^* + y_2 - c_{2L}^h, \tag{2.11}$$

and

$$0 = (1 + r^*)res_1^* + a_{2H}^* + y_2 - c_{2H}^h. \tag{2.12}$$

[6] This is a standard simplification of the literature, as discussed in Korinek and Servén (2016).

[7] In the quantitative model that has traded and nontraded goods, this externality is captured by a learning-by-doing mechanism, which affects the TFP of traded goods output, as in van Wijnbergen (1984).

[8] In the quantitative model, we model public investment explicitly, separately from transfers.

Equations (2.10)–(2.12) represent the balance of payments equations for both periods and aid outcomes, whereby the accumulation of net foreign assets (or reserve accumulation) equals the difference between income, including aid, and spending.

2.2 The Ramsey Solution

We derive the optimal fiscal spending and reserve accumulation decisions of the Ramsey planner. Given res_0^*, a_1^*, a_{2L}^*, and a_{2H}^*, the planner in period 1 chooses c_1^h, c_{2L}^h, c_{2H}^h, res_1^*, and z_1 to maximize equation (2.5), subject to equations (2.6), (2.9), (2.10), (2.11), and (2.12). Once we solve for c_{2L}^h and c_{2H}^h, we can use equations (2.7) and (2.8) to retrieve z_{2L} and z_{2H}. For this problem, the dynamic equilibrium equations of the model consist of the Euler equation for consumption,

$$(c_1^h)^{-\sigma} = \beta(1 + r^* + \xi - \alpha^G) \left[\mathfrak{p} \left(c_{2L}^h \right)^{-\sigma} + (1 - \mathfrak{p}) \left(c_{2H}^h \right)^{-\sigma} \right], \tag{2.13}$$

and equations (2.6)–(2.12).

These equilibrium equations highlight some trade-offs regarding reserve accumulation and fiscal spending. In principle, given the path for aid (a_1^*, a_{2L}^*, and a_{2H}^*), the Ramsey planner optimally accumulates reserves and, via transfers, helps hand-to-mouth households smooth consumption over the two periods. However, there are trade-offs. On the one hand, accumulating reserves is akin to have higher exports which helps increase output and, therefore, consumption in period 2—see equations (2.7), (2.8), (2.9) and (2.10). On the other hand, accumulating reserves means less transfers in period 1, which implies lower output (when transfers are productive) and, therefore, consumption in period 2—see equations (2.2), (2.7), (2.8), and (2.9).

2.2.1 Optimal Policies without Aid Volatility

To derive the optimal policies, we first consider an even simpler case in which there is no aid volatility in period 2—i.e., $a_{2L}^* = a_{2H}^* = a_2^*$. This allows us to derive analytical results for optimal transfers and reserve accumulation in period 1, as summarized in the following proposition.

Proposition 1. *Assume that $a_{2L}^* = a_{2H}^* = a_2^*$ and $res_0^* = 0$. Then, in period 1, the optimal* *consumption is*

$$c_1^h = \left(\frac{1+r^*}{\mathfrak{m}}\right)\left(a_1^* + \frac{a_2^*}{1+r^*} + y_p\right), \tag{2.14}$$

and the optimal fiscal spending and reserve accumulation policies are given by the rules:

$$z_1 = \underbrace{\left(\frac{1+r^*}{\mathfrak{m}}\right)}_{\equiv\gamma} a_1^* + \left(\frac{1}{\mathfrak{m}}\right) a_2^* + \left[\left(\frac{1+r^*}{\mathfrak{m}}\right) y_p - y\right] \tag{2.15}$$

and

$$res_1^* = \underbrace{\left(\frac{\mathfrak{n}+\xi-\alpha^G}{\mathfrak{m}}\right)}_{\equiv\omega} a_1^* - \left(\frac{1}{\mathfrak{m}}\right) a_2^* - \left[\left(\frac{1+r^*}{\mathfrak{m}}\right) y_p - y\right], \tag{2.16}$$

where $\mathfrak{n} \equiv \left[\beta(1+r^*+\xi-\alpha^G)\right]^{1/\sigma}$, $\mathfrak{m} \equiv 1+r^*+\mathfrak{n}+\xi-\alpha^G$, *and* $y_p \equiv y + \left(\frac{1+\xi-\alpha^G}{1+r^*}\right) y$.

Proof. Use the assumptions $a_{2L}^* = a_{2H}^* = a_2^*$ and $res_0^* = 0$ with equations (2.10)–(2.12) to derive the *intertemporal* budget constraint for the economy. Combine this constraint with the Euler equation, equation (2.13), to solve for optimal consumption in period 1, c_1^h. Then use this solution and the household budget constraint, equation (2.6), to derive the optimal fiscal spending rule (2.15). Next, replace this spending rule into the government budget constraint, equation (2.2), to obtain the optimal reserve accumulation rule, equation (2.16). □

The results of Proposition 1 can be framed in the context of the literature on fiscal spending and reserve accumulation rules in aid-dependent LICs, including Buffie et al. (2008), Berg et al. (2010b), Berg et al. (2015). This literature features *contemporaneous rules*, whereby the policy instrument in period t responds to aid flows in the same period, as they capture the crux of discussions among policymakers: how much of current aid should the government spend and/or accumulate in international reserves? Translated into our simple model, these rules can be characterized as:

$$z_1 = \gamma a_1^* \qquad \text{and} \qquad res_1^* = \omega a_1^*, \qquad \text{where } \gamma, \omega \in [0,1]. \tag{2.17}$$

Proposition 1, however, states that in contrast to these contemporaneous rules, optimal policy setting should have forward-looking components: equations (2.15) and (2.16) respond to *both* current and future aid flows, a_1^* and a_2^*. As a result, it is unlikely that the simple, contemporaneous policy rules typically analyzed in the literature would be able to replicate the Ramsey-optimal outcomes.

The presence of the forward-looking components in our Ramsey-optimal rules is not surprising and can be explained by the permanent income hypothesis (PIH) of Friedman (1957). Under further assumptions—i.e., quadratic preferences, $\xi = \alpha^G = y = 0$, and $\beta(1+r^*) = 1$—, it can be shown that the Ramsey planner uses transfers to *fully* smooth consumption of the hand-to-mouth consumers across periods. In this case, equations (2.14) and (2.15) reduce to $c_1^h = c_2^h = z_1 = \frac{1+r^*}{2+r^*}\left(a_1^* + \frac{a_2^*}{1+r^*}\right)$, which is the two-period simple version of the PIH, when the only source of income is aid. But the same guiding principles extend to the results in Proposition 1 of the more general setting: to smooth consumption of hand-to-mouth households, implementing optimal policies requires the government to be forward-looking when accumulating aid as reserves and providing transfers to households.

Proposition 1 also shows that optimal policy dictates that the implied response coefficients of the rules to current aid—the coefficients γ and ω in equations (2.15) and (2.16)—depend on structural conditions of the economy, such as DD externalities and productive fiscal spending. As a result, changes in these conditions imply different policy responses to contemporaneous aid flows. The following corollary summarizes this result by focusing *only* on the response coefficients to current aid.

Corollary 1. *Consider the optimal fiscal spending and reserve accumulation policies in equations* (2.15) *and* (2.16)*, and assume a positive net return to the accumulation of reserves (*$1+r^*+\xi-\alpha^G > 0$*), then the response coefficients of the rules to current aid satisfy the following conditions:*

a) $\gamma \in (0,1)$*,* $\frac{\partial\gamma}{\partial\xi} < 0$ *and* $\frac{\partial\gamma}{\partial\alpha^G} > 0$*;*

b) $\omega \in (0,1)$*,* $\frac{\partial\omega}{\partial\xi} > 0$ *and* $\frac{\partial\omega}{\partial\alpha^G} < 0$*.*

Proof. Consider the expressions of γ and ω in equations (2.15) and (2.16) and take the respective derivatives. □

The corollary reveals several results. First, it is optimal to spend some of the current aid flows and accumulate the rest as reserves—$\gamma \in (0,1)$ and $\omega \in (0,1)$. Second, optimal policy rules should not be invariant to changes in structural parameters, a manifestation of the Lucas critique (Lucas, 1976). Ceteris paribus, higher DD externalities—higher ξ—implies more reserve accumulation out of aid $\left(\frac{\partial \omega}{\partial \xi} > 0\right)$ and, consequently, less transfers to households in period 1 $\left(\frac{\partial \gamma}{\partial \xi} < 0\right)$. This is because the DD externality implies that reserve accumulation today leads to higher output and, therefore, consumption tomorrow. Moreover, higher spending productivity—higher α^G—favors more transfers today $\left(\frac{\partial \gamma}{\partial \alpha^G} > 0\right)$ and reduces the incentive to accumulate current aid in reserves $\left(\frac{\partial \omega}{\partial \alpha^G} < 0\right)$. Recall that, in this simple model, government spending in period 1 can translate into higher output in period 2 and, via this, higher consumption in period 2. Hence, more aid should be spent in period 1.

At the core of the consumption smoothing results lies how the *net* return of accumulating reserves $(1 + r^* + \xi - \alpha^G)$, the discount factor β, and the intertermporal elasticity of substitution $1/\sigma$ affect consumption decisions in the Euler equation (2.13). When there is no aid volatility, this equation reduces to $\frac{1}{\beta}\left(\frac{c_2^h}{c_1^h}\right)^{\sigma} = 1 + r^* + \xi - \alpha^G$, highlighting how the discussed changes in the structural parameters induce some tilting of the optimal consumption path, which can be achieved by the appropriate optimal reserve accumulation of aid and associated transfers. This tilting is driven by intertemporal motives. As we study next, reintroducing aid volatility highlights other considerations for optimal consumption decisions, associated with precautionary motives.

2.2.2 Optimal Policies under Volatile Aid

Aid volatility has implications for optimal policy. To see this, we reintroduce the stochastic structure of aid described in equation (2.1). Despite the simplicity of the model, it is not possible to solve analytically for the optimal policy rules. Hence, we rely on model simulations. A formal calibration will be presented later for the quantitative model, but here we provide a quarterly parameterization that is to a great extent consistent with that calibration. We set $\beta = 0.98$, $\sigma = 2$, and $r^* = 0.04$. For period 1, we normalize output $(y = 1)$ and set the initial stock of reserves and aid flows as $R_0^* = 0.13$ and $a_1^* = 0.05$, which are roughly consistent with GDP-relative average ratios of these variables in LICs. For the baseline case, we abstract from some of the discussed externalities, that is, $\xi = \alpha^G = 0$. For alternative cases, we consider different values for ξ and α^G. We also assume $\mathfrak{p} = 0.5$ and ensure that the mean of the aid process is always preserved—$E(a_2^*) = a_1^*$, for $a_{2H}^* > a_{2L}^*$ and $a_{2L}^*, a_{2H}^* \geq 0$, where E is the mathematical expectation.

The simulation results for the optimal reserve accumulation and government spending policies are presented in Figure 1. The first row corresponds to the baseline case and is represented by solid lines, excluding DD externalities and productive fiscla spending (ξ=$\alpha^G = 0$); the second row plots the case with DD externalities ($\xi = 0.05$, dashed lines); and the third row plots the case of productive fiscal spending ($\alpha^G = 0.02$). In each plot of the alternative cases (in rows 2 and 3), the baseline optimal policies are also included for comparison. All the optimal policies are plotted against a measure of volatility/dispersion of aid in period 2, given by $a^*_{2H} - a^*_{2L}$. The left column plots the optimal reserve accumulation response in period 1 (res^*_1), whereas the right column presents the optimal government transfers in period 1 (z_1).

Consistent with the precautionary saving motive or prudence, higher aid volatility calls for more reserve accumulation and less government transfers to households in period 1 across all the cases examined. To understand this, it is helpful to do a Taylor approximation of equation (2.13) around c^h_1, and decompose the expected consumption growth into two factors that explain how saving, by building up reserves, is driven by both intertemporal and precautionary motives:

$$E\left(\frac{c^h_2 - c^h_1}{c^h_1}\right) = \underbrace{\frac{\beta(1 + r^* + \xi - \alpha^G) - 1}{\sigma\beta(1 + r^* + \xi - \alpha^G)}}_{\text{Intertemporal Motive}} + \underbrace{\frac{1}{2}(1 + \sigma)E\left(\frac{c^h_2 - c^h_1}{c^h_1}\right)^2}_{\text{Precautionary Motive}}. \tag{2.18}$$

Beyond intertemporal considerations, this equation shows that an increase in the variance of consumption, denoted by the term $E\left(\frac{c^h_2 - c^h_1}{c^h_1}\right)^2$, results in an increase in the expected growth of consumption. This increase aligns with a decrease in consumption during period 1, which can be attributed to precautionary savings. In other words, the government is forced to hedge against the risk of a low level of aid in period 2, by saving prudently in reserves and, therefore, lowering consumption in period 1.

Relative to the baseline, structural factors affect the optimal policy response similarly in the context of volatile aid as they do without aid volatility. Conditional on a given degree of aid volatility, DD externalities ($\xi = 0.05$) lead to higher reserve accumulation and lower transfers in period 1 (the dashed lines in the second row in Figure 1). On the other hand, productive spending ($\alpha^G = 0.02$) induces a reduction in reserve accumulation and an increase in fiscal spending in period 1 (the dashed lines in the third row in Figure 1). To understand these results, it is helpful to analyze the roles of the DD externality and the productivity of public spending in terms of their impacts on future output and expected consumption growth.

Figure 2 depicts how output in period 2 and optimal policies vary with the parameter ξ, which governs the degree of the DD externality. A higher externality is linked to a more negative impact on future output, due to current aid spending. To counteract this negative impact, optimal policy suggests increasing reserve accumulation (the higher the externality) and reducing government spending. This approach, in turn, helps mitigate the effects of volatile aid on private consumption. More formally, note that this higher optimal reserve accumulation is driven by how the DD externality influences both the intertemporal and precautionary motives as captured by equation (2.18). In our simple model, a stronger externality effectively translates into a higher interest rate, further stimulating savings due to the intertemporal motive (Figure 2). Simultaneously, a stronger DD externality raises the volatility of future consumption by negatively impacting future output, thereby necessitating more precautionary savings (reserve accumulation).

Similarly Figure 3 shows how output in period 2 and optimal policies vary with the productivity of government spending (transfers) α^G. The higher this productivity, the more positive the effects of these transfers on future output and consumption. Therefore as transfers productivity increases, greater spending bolster future output, helping mitigate the negative effects of volatile aid on private consumption. Like with the DD externality, this spending productivity shapes both the intertemporal and precautionary motives behind reserve accumulation policies. But in this case, an increase in this productivity effectively lowers the return on reserves (the sole saving instrument in this simplified model), thereby encouraging higher consumption in the present due to intertemporal considerations. Furthermore, an enhancement in transfer productivity boosts future output, which mitigates the volatility of future consumption by helping maintain future income. Consequently, the need for precautionary savings is reduced.

3 A Quantitative Model

We now introduce a more complex, infinite-horizon model, capturing important features of LICs, to evaluate the nature of optimal fiscal spending and reserve accumulation policies in the face of volatile aid. Crucially, some fiscal spending, specifically public investment, will not only be productive as before but will also help accumulate public capital, an asset.

The model includes heterogeneous households, two production sectors (producing nontraded and traded goods), and the government which has a richer set of policy options. The basic model structure largely follows the real side of the DSGE models in Berg et al. (2010a) and Shen et al. (2018) for LICs. Superscripts N and T indicate variables that are associated with the nontraded and traded goods production sectors, respectively. Superscript $*$ indicates a variable in units of foreign goods, and a variable without a time subscript indicates its deterministic steady-state value.

3.1 Households

Households are of two types: a fraction η are savers (denoted by superscript a) and the remaining $1-\eta$ are hand-to-mouth households (denoted by superscript h). Only savers have access to asset and capital markets, while the hand-to-mouth are liquidity constrained.

3.1.1 Savers

Savers have access to capital markets and hold a portfolio of assets, including private capital, domestic government bonds, and foreign assets. These households are able to adjust savings to smooth consumption in response to shocks.

The representative saver chooses consumption (c_t^a), labor (l_t^a), sector-specific investment ($i_t^{N,a}$ and $i_t^{T,a}$) and private capital ($k_t^{N,a}$ and $k_t^{T,a}$), domestic government bonds (b_t^a), and foreign assets $\left(b_t^{*,a}\right)$, to maximize expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \underbrace{\left[\frac{(c_t^a)^{1-\sigma}}{1-\sigma} - \frac{(l_t^a)^{1+\psi}}{1+\psi} \right]}_{\equiv \mathbb{U}_t^a}, \tag{3.1}$$

subject to the budget constraint

$$\begin{aligned} & c_t^a + i_t^{N,a} + i_t^{T,a} + b_t^a + s_t b_t^{*,a} + ac_t^{i,a} + s_t ac_t^{*,a} \\ = & (1-\tau)\left(w_t l_t^a + r_t^N k_{t-1}^{N,a} + r_t^T k_{t-1}^{T,a} \right) + r_{t-1} b_{t-1}^a + s_t r^* b_{t-1}^{*,a} + s_t r m^* + \Pi_t^a + z_t, \end{aligned} \tag{3.2}$$

where E_t is the mathematical expectation conditional on information available at time t, β is the discount factor, and σ and ψ are inverses of the intertemporal elasticity of substitution for consumption and the labor Frisch elasticity, respectively. Savers receive labor and capital rental income taxed at a constant rate τ, where w_t is the real wage rate and r_t^N and r_t^T are the rental prices of capital in the two production sectors. They also receive dividends from firms, Π_t^a, government transfers, z_t, and foreign remittances, rm^*. Since our focus is not on tax policy and remittance flows, they are assumed to be constant. The real exchange rate, s_t, is in units of domestic consumption per unit of foreign goods.

Domestic government bonds pay a gross real interest rate r_t at $t+1$, while foreign assets pay a constant gross interest rate r^* and are subject to portfolio adjustment costs given by $ac_t^{*,a} \equiv \frac{\upsilon}{2}(b_t^{*,a} - b^{*,a})^2$, where parameter υ governs the degree of capital account openness.[9] We assume that investment in physical capital is subject to sector-specific adjustment costs, totaled up to $ac_t^{i,a} \equiv \frac{\kappa}{2}\left[\left(\frac{i_t^{N,a}}{k_{t-1}^{N,a}} - \delta\right)^2 k_{t-1}^{N,a} + \left(\frac{i_t^{T,a}}{k_{t-1}^{T,a}} - \delta\right)^2 k_{t-1}^{T,a}\right]$. The law of motion for capital is

$$k_t^{j,a} = (1-\delta)k_{t-1}^{j,a} + i_t^{j,a}, \quad j \in \{N, T\}, \tag{3.3}$$

where δ is the capital depreciation rate. Total investment by savers is then $i_t^a = i_t^{N,a} + i_t^{T,a}$. The household's optimal choices and all other equilibrium conditions are provided in Appendix B.

Consumption and investment are constant-elasticity-of-substitution (CES) aggregates of non-traded and traded goods, with the intratemporal elasticity of substitution χ and the degree of home bias φ,

$$c_t = \left[\varphi^{\frac{1}{\chi}}\left(c_t^N\right)^{\frac{\chi-1}{\chi}} + (1-\varphi)^{\frac{1}{\chi}}\left(c_t^T\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}. \tag{3.4}$$

The normalized price of the consumption basket is an index of the relative prices of nontraded and traded goods, p_t^N and s_t,[10]

$$1 = \left[\varphi\left(p_t^N\right)^{1-\chi} + (1-\varphi)(s_t)^{1-\chi}\right]^{\frac{1}{1-\chi}}. \tag{3.5}$$

The nontraded consumption bundle, c_t^N, is assumed to be a CES aggregate of a continuum of

[9] Such portfolio adjustment costs help close the small open economy model, as discussed in Schmitt-Grohé and Uribe (2003).

[10] Assuming that the law of one price holds for traded goods, the real exchange rate is then the relative price of traded goods.

nontraded goods varieties, $c_t^N(i)$, produced by monopolistically competitive firms indexed by $i \in [0,1]$,

$$c_t^N = \left[\int_0^1 c_t^N(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{3.6}$$

where θ is the elasticity of substitution between varieties.

Households supply labor to both production sectors, with savers' total labor supply given by

$$l_t^a = \left[(\varphi^l)^{-\frac{1}{\chi^l}} \left(l_t^{a,N}\right)^{\frac{1+\chi^l}{\chi^l}} + (1-\varphi^l)^{-\frac{1}{\chi^l}} \left(l_t^{a,T}\right)^{\frac{1+\chi^l}{\chi^l}}\right]^{\frac{\chi^l}{1+\chi^l}}, \tag{3.7}$$

where φ^l is the steady-state share of labor in the nontraded good sector, and $\chi^l > 0$ is the elasticity of substitution between the two types of labor. The implied aggregate real wage index is

$$w_t = \left[\varphi^l \left(w_t^N\right)^{1+\chi^l} + (1-\varphi^l)\left(w_t^T\right)^{1+\chi^l}\right]^{\frac{1}{1+\chi^l}}, \tag{3.8}$$

where w_t^N and w_t^T are the wage rates in the two sectors.

3.1.2 Hand-to-Mouth Households

Hand-to-mouth households also derive utility from consumption, $\left(c_t^h\right)$, and disutility from inelastically supplied labor, $\left(l^h\right)$, according to the periodic utility function

$$\mathbb{U}_t^h = \left[\frac{\left(c_t^h\right)^{1-\sigma}}{1-\sigma} - \frac{\left(l^h\right)^{1+\psi}}{1+\psi}\right]. \tag{3.9}$$

Unlike savers, hand-to-mouth households do not have access to capital markets. They are meant to capture relatively poor households in the economy, who do not have asset income. Their level of consumption each period is determined by after-tax labor income, foreign remittances, and lump-sum transfers received, as described in their budget constraint,

$$c_t^h = (1-\tau)\, w_t l^h + s_t r m^* + z_t. \tag{3.10}$$

3.2 Firms

The two production sectors have different market structures. Nontraded goods firms are monopolistically competitive, as nontraded goods can only be produced domestically. Since manufacturing in LICs is often concentrated in resource-based and low-technology production, traded goods firms are assumed to be perfectly competitive.[11]

3.2.1 Nontraded Goods Sector

The monopolistically competitive firm $i \in [0,1]$ in the nontraded goods sector uses labor, $l_t^N(i)$, private capital, $k_{t-1}^N(i)$, and public capital, k_{t-1}^G, to produce goods using the technology

$$y_t^N(i) = z^N \left[k_{t-1}^N(i)\right]^{1-\alpha^N} \left[l_t^N(i)\right]^{\alpha^N} \left(k_{t-1}^G\right)^{\alpha^G}, \tag{3.11}$$

where z^N is a constant TFP term, specific to nontraded goods production, α^N corresponds to the labor share in nontraded goods output, and α^G is the elasticity of output with respect to public capital. The differentiated nontraded goods are aggregated into the nontraded good bundle via the CES aggregator, $y_t^N = \left[\int_0^1 y_t^N(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$. The associated demand function for each good i is

$$y_t^N(i) = \left[\frac{p_t^N(i)}{p_t^N}\right]^{-\theta} y_t^N. \tag{3.12}$$

The firm chooses the price, labor, and capital to maximize profits

$$\Pi_t(i) = (1-\iota)\, p_t^N(i) y_t^N(i) - w_t^N l_t^N(i) - r_t^N k_{t-1}^N(i) + \iota p_t^N y_t^N, \tag{3.13}$$

subject to the production function, equation (3.11), and the demand constraint, equation (3.12). We introduce an implicit tax (cost) ι, which discourages firms from producing at a higher level. This is a short-cut to rationalize why in LICs, given the high marginal return to capital implied by capital scarcity, we do not observe a higher investment-to-output ratio. Unlike income taxes,

[11] Natural resources take the largest share of African exports, accounting for 56 percent of Africa's total exports in 2016 (UNCTAD, 2019). Their prices are largely driven by international commodity markets.

the revenues collected via this implicit tax do not enter the government budget but remain in the private sector. For simplicity, we assume they are rebated back to firms in a lump-sum fashion.

Aggregating across firms, total nontraded goods output and profits are given by $y_t^N = \int_0^1 y_t^N(i)di$ and $\Pi_t = \int_0^1 \Pi_t(i)di$. Profits accrue to savers (the firms' owners) as dividends.

3.2.2 Traded Goods Sector

Firms in the traded goods sector are perfectly competitive. The representative firm i uses labor $l_t^T(i)$, private capital $k_{t-1}^T(i)$, and public capital k_{t-1}^G to produce goods using the technology

$$y(i)_t^T = z_t^T \left[k(i)_{t-1}^T\right]^{1-\alpha^T} \left[l(i)_t^T\right]^{\alpha^T} \left(k_{t-1}^G\right)^{\alpha^G}. \tag{3.14}$$

The TFP term z_t^T is assumed to be time-varying, following a stationary but endogenous process,

$$\log\left(\frac{z_t^T}{z^T}\right) = \rho_{z^T} \log\left(\frac{z_{t-1}^T}{z^T}\right) + \xi \log\left(\frac{y_{t-1}^T}{y^T}\right), \tag{3.15}$$

where $\rho_{z^T} \in (0,1)$, $\xi > 0$, and $y_t^T = \int_0^1 y_t^T(i)di$ is total traded goods output. This specification implies learning-by-doing externalities capturing Dutch disease (DD) effects as in van Wijnbergen (1984): when traded goods output falls, it reduces traded-sector productivity with some persistence.

Firms choose labor and capital to maximize period-t profits given by

$$(1-\iota)\, s_t y(i)_t^T - w_t^T l(i)_t^T - r_t^T k(i)_{t-1}^T + \iota y_t^T, \tag{3.16}$$

where the tax rate ι has the same calibration role as in equation (3.13).

Total output produced in the economy in period t is $y_t = p_t^N y_t^N + s_t y_t^T$.

3.3 The Public Sector

Each period, the government receives income tax revenues and foreign aid (a_t^*) and issues a constant amount of domestic debt ($b_t = b\ \forall t$).[12] Expenditures include government consumption

[12] As our focus is on the allocation of aid, we do not explore government financing with debt. For simplicity, we assume domestic debt to be constant. Regarding external debt, note that for most LICs this type of debt is concessional and effectively considered aid.

(g_t^C), public investment in productive capital (g_t^I), lump-sum transfers to households (z_t), and debt services. The government also has a stock of foreign exchange reserves (res_t^*), which earn the gross foreign rate of interest r^*. The government's flow budget constraint is

$$p_t^G \left(g_t^C + g_t^I\right) + z_t + s_t res_t^* + s_t ac_t^{res} = tax_t + (1 - r_{t-1})\, b + s_t r^* res_{t-1}^* + s_t a_t^*, \tag{3.17}$$

where $tax_t = \tau \left(w_t l_t + r_t^N k_{t-1}^N + r_t^T k_{t-1}^T\right)$ represents overall tax revenues, with l_t, k_{t-1}^N, and k_{t-1}^T being the aggregate labor and capital stocks in the economy.

We assume that foreign reserves are subject to adjustment costs, $ac_t^{res} \equiv \frac{\upsilon^{res}}{2}(res_t^* - res^*)^2$. Technically, this assumption avoids the random walk in reserves that would otherwise emerge under the Ramsey-optimal policy problem. Under the benchmark calibration, these adjustment costs are very low, essentially placing little restriction on the use of reserves as a policy instrument.[13]

In terms of policy choices, our focus is on the optimal setting of fiscal spending and reserve accumulation in the face of volatile aid. In LICs, government transfers are an important fiscal policy, as they have a direct effect on the consumption and welfare of poorer households, while public investment can substantially affect production and economic development over the longer run. Also, the analysis from our simple model suggests that productive government spending can play an important role in the government's optimal spending and savings decisions. Thus, we jointly choose reserves, transfers, and public investment as the optimal saving and spending policy instruments. Government consumption is set at its steady-state level, $\left(g_t^C = g^C\right)$.[14]

The total government goods purchase, $g_t = g^C + g_t^I$, is a CES aggregate of traded and nontraded goods, with elasticity of substitution χ and degree of home bias φ^G, typically different from that of private households (φ),

$$g_t = \left[\left(\varphi^G\right)^{\frac{1}{\chi}} \left(g_t^N\right)^{\frac{\chi-1}{\chi}} + (1-\varphi^G)^{\frac{1}{\chi}} \left(g_t^T\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}, \tag{3.18}$$

[13] The adjustment costs of reserve accumulation can also be seen as reduced-form modeling of some costs of reserve accumulation discussed in the literature, such as foregone domestic investment (Feldstein, 1999) or the spreads between holding low-yield external financial assets by a central bank and the interest rates of short-term borrowing by the private sector (Rodrik, 2006).

[14] Alternatively, one could consider government consumption, g_t^C, as public goods provision from which households derive utility. That specification would yield similar results to those we have here from transfers' provision. In our model, we allow for a wasteful but constant level of government consumption, g^C, to help with the model calibration.

and the implied relative price of this bundle is $p_t^G = \left[\varphi^G \left(p_t^N\right)^{(1-\chi)} + \left(1-\varphi^G\right)(s_t)^{1-\chi}\right]^{\frac{1}{1-\chi}}$.

The public capital accumulation is given by

$$k_t^G = \left(1-\delta^G\right) k_{t-1}^G + \epsilon g_t^I, \tag{3.19}$$

where δ^G is the depreciation rate of public capital. To capture the low efficiency of public investment in LICs, we introduce the investment efficiency parameter, $\epsilon \in (0,1)$, whereby one dollar of public investment expenditure delivers less than one dollar of public capital.[15]

Finally, aid follows an exogenous AR(1) process

$$\log\left(\frac{a_t^*}{a^*}\right) = \rho_a \log\left(\frac{a_{t-1}^*}{a^*}\right) + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0, \sigma_a^2\right), \tag{3.20}$$

with persistence ρ_a and standard deviation of the shock σ_a.[16]

3.4 Aggregation and Market Clearing

With two types of households, aggregate per-capita consumption and labor are computed as

$$x_t = \eta x_t^a + (1-\eta) x_t^h, \qquad x \in \left\{c, c^N, c^T, l, l^N, l^T\right\}. \tag{3.21}$$

Since only savers have access to asset and capital markets, investment, capital, debt, dividends, and adjustment costs are determined as

$$x_t = \eta x_t^a, \quad x \in \left\{i^N, i^T, k^N, k^T, b, b^*, \Pi, ac^i, ac^*\right\}. \tag{3.22}$$

Remittances are identical for all households, hence, $rm_t^{*,a} = rm_t^{*,h} = rm_t^*$, and so are government transfers, z_t.

[15] For an analysis of some misconceptions about public investment efficiency and growth see Berg et al. (2019).

[16] In reality, aid is often procyclical (e.g., Bertoli et al., 2007; Arellano et al., 2009; Dang et al., 2013) such that aid shocks can be correlated with other macroeconomic shocks in recipient countries. Assuming that aid follows an exogenous, stochastic process allows us to focus more cleanly on the implications of volatility in external government receipts.

Finally, the market clearing condition of nontraded goods is

$$y_t^N = (p_t^N)^{-\chi} \left[\varphi \left(c_t + i_t + ac_t^i\right) + \varphi^G (p_t^G)^{\chi} g_t\right] \tag{3.23}$$

and the balance of payment condition is

$$\begin{aligned} & c_t + i_t + p_t^G \left(g^C + g_t^I\right) + ac_t^i - y_t + s_t \left[ac_t^* + ac_t^{res} - rm^* - (r^* - 1)\left(res_{t-1}^* + b_{t-1}^*\right)\right] \\ & = s_t \left[a_t^* - \left(b_t^* - b_{t-1}^*\right) - \left(res_t^* - res_{t-1}^*\right)\right]. \end{aligned} \tag{3.24}$$

4 Calibration and Solution

Tables 1 and 2 list the key parameter values and the aggregate data ratios used in the calibration. A number of parameter values are set based on existing estimates from the literature, while others are calibrated to match data moments. We also conduct sensitivity analysis with respect to a set of these parameters. We use the algorithm of Schmitt-Grohé and Uribe (2004) to obtain a second-order accurate solution of the model.

4.1 Calibration of Structural Parameters

The model is at a quarterly frequency. The discount factor, $\beta = 0.98$, implies an annual real interest rate of 8 percent, consistent with the data for 12 HIPC (heavily indebted poor countries) economies in Fedelino and Kudina (2003). The risk aversion parameter σ is set to 2, which is the typical value assumed in the literature. For the Frisch labor supply elasticity, we set $\frac{1}{\psi} = 2$ for savers using the average estimate from developed economies based on macro level data (see, e.g., Chetty et al., 2011; Peterman, 2016). Together with hand-to-mouth households' inelastic labor supply, the average Frisch labor supply elasticity is 0.4.[17]

We assume savers in our economy are a quarter of the population, $\eta = 0.25$. Based on data collected in 2011, Demirguc-Kunt and Klapper (2012) report that on average about 24 percent of adults in Sub-Saharan Africa (SSA) have an account in a formal financial institution (the mid-range

[17] Goldberg (2016) estimates that the labor supply elasticity in a daily labor market in rural Malawi is 0.15–0.17. The concept of her estimated elasticity—the elasticity of working with respect to a change in daily working wages—is different from the Frisch labor elasticity though.

between a higher estimate of 45 percent in the richest quintile of SSA countries and 12 percent in the poorest quintile). The intratemporal elasticity of substitution between labor of the two sectors is set to $\chi^l = 0.6$ following Shen et al. (2018).[18] The elasticity of substitution between varieties of goods is set to $\theta = 6$, so a steady-state markup in the goods market is 20 percent, as calibrated in Galí and Monacelli (2005) for a small open economy.

The degree of home bias in private consumption and investment is set to $\varphi = 0.6$, while it is $\varphi^G = 0.7$ for government purchases. Since distribution costs can be high in rural Africa, we assume a slightly higher share than the typical value of 0.5 (Burstein et al., 2005). We follow the convention to assume a higher degree of home bias in government purchases because a large part of government spending goes to pay for civil services.

For the elasticity of substitution between traded and nontraded goods, we set $\chi = 0.44$, following Stockman and Tesar's (1995) estimate based on a sample of 30 countries including developing and developed countries. The labor income shares in nontraded and traded production are set to $\alpha^N = 0.45$ and $\alpha^T = 0.6$, following Buffie et al. (2012) for calibrating an average African economy. The depreciation rate of private capital is set to $\delta = 0.025$, corresponding to an annual depreciation rate of 10 percent, while that of public capital is set to 8 percent annually, implying $\delta^G = 0.02$. The investment adjustment cost parameter is set to $\kappa = 1.4$, based on the only estimate we could locate for a developing country with the same specification (for Mexico in Aguiar and Gopinath, 2007). To calibrate the public investment efficiency ϵ, we refer to estimates in the literature: when the TFP growth rate is assumed to be zero, Pritchett (2000) estimates that the public investment efficiency is 0.49 for SSA economies, while Hurlin and Arestoff (2010) obtain a value of 0.4 for Colombia and Mexico. Our benchmark calibration assumes $\epsilon = 0.4$.[19] The parameters defining the learning-by-doing externality and affecting the magnitude of DD effects, ρ_{zT} and ξ, are both set at the low value of 0.1, but alternative values are explored in sensitivity analysis.

[18] Horvath (2000) estimates this elasticity to be 1 using the U.S. sectoral data. Artuc et al. (2015) estimate that on average labor mobility costs are 4.26 times of annual wages in SSA countries, and only 2.41 times in developed countries. Thus, the model assumes less labor mobility relative to developed countries.

[19] The low public investment efficiency calibrated here is also consistent with an average low public investment and management index (PIMI) for LICs (Dabla-Norris et al., 2012), which broadly assesses various components in public investment implementation, including appraisal, selection, budgeting, etc.

4.2 Calibration of Steady-State Ratios

The remaining parameters and a number of steady-state variables are calibrated to match data moments, summarized in Table 2. We use the 2012-2019 data, excluding the Covid-19 pandemic period, of simple country averages for SSA LICs and lower-middle income countries in World Development Indicators (World Bank, 2020) to calibrate various output ratios in the steady state. These include the output ratios of private consumption, private investment, public debt, foreign exchange reserves, aid, and external private debt—proxied by the negative of portfolio investment as a share of GDP in US dollars. For the transfers-to-output ratio and the public debt-to-annual output ratio, we use the data of the World Economic Output Database (International Monetary Fund, 2020), while for the public investment-to-output ratio, we use the Investment and Capital Stock Dataset (International Monetary Fund, 2019). The implicit tax rate $\iota = 0.18$ allows the model to match the private investment to GDP data ratio. Together with the calibrated shares of private consumption and investment to GDP, the model implies that about 64 percent of labor works in the nontraded goods sector $\left(\varphi^l = 0.64\right)$, and the value added by traded goods output in steady state is 26 percent of GDP, given a calibrated TFP value in that sector $z^T = 1.85$.

The output elasticity with respect to public capital $\alpha^G = 0.39$ is such that the public investment-to-output ratio matches the data average of 9.2 percent. Although slightly on the high side, this value falls well within the range of estimates in Bom and Lighthart (2014). In sensitivity analysis, we consider a lower value of this parameter. We calibrate the steady-state value of public transfers z to the data average of 2.6 percent of GDP, while the income tax rate τ of 19 percent helps match the data on tax revenues-to-GDP ratio of 14 percent. The government consumption-to-output ratio, $\frac{g^C}{y}$, is then determined from the government budget constraint, given the values of other fiscal variables, while foreign remittances as a share of GDP, $\frac{s \cdot rem^*}{y}$, are such that the balance of payments condition holds, given the other calibrated aggregate ratios.

To calibrate the aid process, we use the available data for 23 LICs to estimate an AR(1) process for each country (see Appendix A for the sample description). The persistence and standard deviation parameters, ρ_a and σ_a, are then calibrated to the means of the two statistics across 23 countries: $\rho_a = 0.72$ and $\sigma_a = 0.098$.

Lastly, we set the values for reserves adjustment costs parameter, υ^{res}, and the households' portfolio adjustment cost parameter, υ. Following the approach in Schmitt-Grohé and Uribe (2003), we calibrate υ^{res} to match the volatility of the reserves-to-GDP ratio, as measured in the data. This yields a very low value $\upsilon^{res} = 0.0006$. Given this value and the rest of the calibration, the remaining parameter υ could be set to match the volatility of the current account-to-GDP ratio. Simulations indicate that the value of υ has little bearing on this volatility. Hence, for the benchmark calibration we choose $\upsilon = 500$, the mid-point of the range examined in Shen et al. (2018) for simulating government spending effects in LICs and implying a fairly limited capital account mobility.

Given the uncertainty in several parameters, alternative values are used in sensitivity analysis to help gauge the relative importance of reserve accumulation in the government's optimal policy response.

5 Analysis with the Quantitative Model

We derive and analyze optimal fiscal spending and reserve accumulation policies in response to volatile foreign aid flows. The policy maker chooses policy instruments to maximize the households' welfare, subject to the equilibrium conditions of the economy, equations (B.1)–(B.32) in Appendix B. Following Leith et al. (2015), the optimal policy problem is set up in terms of a Lagrangian, as

$$L_0 = \max_{\mathbf{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t [\mathbb{U}(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) - \lambda_t \mathbb{F}(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t)], \tag{5.1}$$

where $\mathbf{y}_t$ and $\mathbf{u}_t$ are vectors of the endogenous and exogenous variables. In this Lagrangian, $\mathbb{U}(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = \eta \mathbb{U}_t^a + (1-\eta)\, \mathbb{U}_t^h$ is an aggregate measure of utility capturing the welfare of the two types of households, where $\mathbb{U}_t^a$ and $\mathbb{U}_t^h$ were defined in equations (3.1) and (3.9), respectively, $\mathbb{F}(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = 0$ consists of the constraints—the model's equilibrium conditions—to satisfy, and λ_t is a vector of Lagrange multipliers associated with these constraints. The policy instruments correspond to government transfers, z_t, public investment spending, g_t^I, and reserves, res_t^*.

The optimization implies the following first order conditions:

$$E_t\left[\frac{\partial \mathbb{U}(.)}{\partial \mathbf{y}_t}+\beta F\frac{\partial \mathbb{U}(.)}{\partial \mathbf{y}_{t-1}}+\beta^{-1}\lambda_{t-1}F_{-1}\frac{\partial \mathbb{F}(.)}{\partial \mathbf{y}_{t+1}}+\lambda_t\frac{\partial \mathbb{F}(.)}{\partial \mathbf{y}_t}+\beta\lambda_{t+1}F\frac{\partial \mathbb{F}(.)}{\partial \mathbf{y}_{t-1}}\right]=0, \tag{5.2}$$

where F is the lead operator, such that F_{-1} is a one-period lag. A second-order accurate solution to the optimal policy involves solving these first-order conditions in combination with the non-linear equilibrium conditions of the model, $\mathbb{F}(\mathbf{y}_{s+1}, \mathbf{y}_s, \mathbf{y}_{s-1}, \mathbf{u}_s) = 0$, using the perturbation methods of Schmitt-Grohé and Uribe (2004).

5.1 The Baseline Scenario

In the baseline scenario, the policy instruments are government transfers, z_t, public investment spending, g_t^I, and reserves, res_t^*. We examine the optimal policy response to an exogenous 1 percent increase in aid inflows above steady-state levels and its macroeconomic implications under the benchmark calibration (Figure 4). An increase in aid flows represents additional revenues that the government can use to raise spending and saving in the form of foreign reserves. Consistent with the implications from the simple model, the optimal policy in the quantitative model has an interior solution to the aid allocation: reserves, government transfers, and public investment spending rise following the aid shock to help smooth consumption and mitigate DD effects.[20]

Given the large share of hand-to-mouth consumers, the policy maker places significant emphasis on achieving consumption smoothing for these households, which are unable to self-insure. Increased government transfers directly impact the income and consumption of hand-to-mouth households. Meanwhile, higher spending on public investment enhances the economy's production capacity, thereby supporting increased output and consumption in the long term. The expanded stock of public capital directly boosts goods production and indirectly raises the marginal product of private capital and labor, stimulating higher private investment and consumption. Increased income further drives up consumption across all households. Initially, a slight reduction in public investment can be seen as an optimal response, facilitating higher transfers to immediately support

[20] Besides mitigating DD externalities, there may be other reasons that motivate reserve accumulation. For instance, Aguiar and Amador (2011) develop a model in which the government can incentivize its own citizenry to invest in domestic productive capital by accumulating reserves. This accumulation of external assets reduces the probability of expropriating domestic capital to repay foreign creditors.

the consumption of poorer households.

In addition to spending on transfers and public investment, the Ramsey planner saves the rest of the foreign aid inflow as foreign reserves, primarily to mitigate DD effects. This additional accumulation of reserves also provides longer-run funds for government spending through interest earnings and reserve de-cumulation over time. Despite saving part of the aid inflows as reserves, the real exchange rate still appreciates (a decline in s_t), mainly because some of the aid is spent domestically. The exchange rate appreciation has a negative effect on the income and consumption of hand-to-mouth consumers, as it reduces the domestic-currency value of the foreign remittances they receive. Additionally, an appreciated real exchange rate lowers the competitiveness of traded goods production in international markets, decreasing traded goods output.

At the aggregate level, however, higher private consumption and public investment spending represent an increase in demand for goods, with a preponderance for domestic nontraded goods, partly due to home bias effects and partly to substitution effects driven by the real exchange rate appreciation. Nontraded goods firms expand production accordingly, leading to an overall increase in total output. The contraction in traded goods production is somewhat persistent, as DD effects are transmitted through a long-lasting reduction in sectoral productivity (z_t^T), as specified in equation (3.15). However, capital accumulation (both private and public) eventually offsets this negative productivity effect, raising production in both sectors over the longer run. The aggregate labor input falls, mainly driven by the wealth effect from savers' higher income.

As in the simple model, productive government spending enhances future output and consumption. But here it takes the form of public investment which, by building up capital, serves as a saving instrument to generate long-lasting positive macroeconomic effects. This is in line with the proposals of the development literature as well as with the literature on managing natural resource revenues, which proposes that investing natural resource windfalls on building productive public capital can be an optimal way of managing these revenues (e.g., van der Ploeg and Venables, 2011; Berg et al., 2013; Araujo et al., 2016).

5.2 The Alternative Scenarios

To assess the relative roles of the three policy instruments (transfers, public investment and reserve accumulation), we consider two alternative policy scenarios. In alternative scenario I, the Ramsey planner uses only transfers and reserve accumulation to allocate aid inflows, keeping public investment constant at its deterministic steady-state level. In alternative scenario II, she uses only fiscal instruments by fully spending additional aid inflows on transfers and public investment.

5.2.1 Alternative Scenario I: Excluding Public Investment

Figure 5 compares the impulse responses to a 1 percent aid shock under two scenarios: the alternative scenario using transfers and reserves as policy instruments (dashed red lines) and the baseline scenario (solid blue lines). The optimal allocation in the alternative scenario shows a larger increase in transfers and reserve accumulation compared to the baseline. With higher transfers, the consumption of poorer (hand-to-mouth) households increases more in the short-to-medium run. However, without the additional investment in public capital, the overall economic expansion is smaller. Beyond the initial years, output, consumption, and private investment stay significantly below their levels in the baseline scenario. Initially, savers invest more to build up private capital and smooth consumption. However, due to the lack of public capital increase, the marginal product of labor does not rise as much, resulting in a smaller increase in real wages.

This alternative scenario highlights the dual role of public investment as both a spending and saving instrument. When public investment does not increase to absorb aid inflows, other types of government spending, such as transfers, increase more to support private consumption. However, these effects are relatively short-lived compared to the benchmark case, where some aid is also spent on public investment. Additionally, although reserve accumulation increases more than in the baseline scenario, this type of savings cannot substitute for the saving role of public investment and capital accumulation, which are essential for supporting longer-term growth and consumption. The significant difference in output between the two scenarios in Figure 5 underscores the importance of public investment as a crucial saving instrument for managing aid inflows.

5.2.2 Alternative Scenario II: Excluding Reserve Accumulation

Alternative scenario II examines the policy mix in which reserve accumulation is largely shut down. The comparison to the baseline scenario allows for the isolation of the role played by reserve accumulation. We do so by assuming very high reserves adjustment costs—with $\upsilon^{res} = 1000$, the reserves stock is held virtually constant in response to the aid shock. The Ramsey planner responds by adjusting fiscal instruments only, via transfers and public investment. The results are illustrated by the dash-dot impulse responses in Figure 6.

Since adjusting reserves is now very costly, the aid inflow is channeled into government transfers and public investment spending, which increase by significantly more in the short run relative to the baseline scenario (solid lines). Associated with this much larger goods' demand, nontraded goods production increases by much more, and strong labor demand leads to a surge in the real wage. At the same time, more public capital accumulation generates a private investment boom. The higher households' income that arises, including transfers from the government, supports higher consumption for both types of households.

Relative to the baseline scenario, the same aid inflow becomes much more expansionary. Without reserve accumulation, however, the initial increase in fiscal spending is short-lived, largely following the aid inflow persistence. The simultaneous surge in government spending and real output implies a higher degree of fiscal policy procyclicality than in the baseline scenario. Our impulse responses are plotted for a positive aid shock. In the case of a negative aid shock, the optimal policy mix without reserve accumulation would imply a sudden decline in government spending followed by a severe economic contraction. Since aid inflows are volatile in reality, the policy combination without reserve accumulation can become a source of worsening macroeconomic instability in LICs, if volatile aid is spent each period.

Aside from the macroeconomic stability concern, another undesirable consequence of excluding reserve accumulation is the emergence of stronger Dutch disease effects. Relative to the baseline scenario, the domestic currency appreciates by more and traded-goods output falls sharply in the short run, although the rebound is more quickly than in the baseline scenario. A highly appreciated real exchange rate reduces competitiveness of traded goods production substantially in the perfectly competitive international market. As the full spending of aid is much more expansionary, the built-up of public capital and strong goods demand soon reverse the decrease in traded goods output, keeping total output above the path of the baseline scenario.

Taken together, the two alternative scenarios highlight that, although public investment and reserve accumulation both play a savings role, they cannot fully substitute each other. Saving via reserves remains important, as it mitigates strong demand effects from full spending and dampens the harmful Dutch disease effects. One caveat to note is that the long-lasting expansionary effect under policy mixes that include public investment depends on the high productivity of public capital assumed under the benchmark calibration. In sensitivity analysis, we explore a lower value of α^G.

5.3 The Welfare Implications of Aid Volatility

We first rank the three policy scenarios (the baseline and the two alternatives) based on their welfare implications. Next, we examine the impact of aid volatility on optimal policies and welfare, under the baseline scenario using all three policy instruments.

5.3.1 Welfare of Various Policy Scenarios

We compute *unconditional* welfare as the unconditional expectation of discounted lifetime utility for the two types of households: $\mathbb{W}^a = E\sum_{t=0}^{\infty}\beta^t\mathbb{U}_t^a$ and $\mathbb{W}^h = E\sum_{t=0}^{\infty}\beta^t\mathbb{U}_t^h$. The aggregate welfare is then $\mathbb{W} = \eta\mathbb{W}^a + (1-\eta)\,\mathbb{W}^h$. Table 3 reports the welfare values for the three policy scenarios analyzed above.

For both types of households, welfare is higher when some of the fiscal spending is allocated to public investment; see columns (1) and (3) of Table 3. The highest aggregate welfare is achieved when the Ramsey planner uses all three policy instruments. This is consistent with the impulse response analysis, which shows that consumption for both savers and hand-to-mouth households is best smoothed under the baseline policy scenario.

Although the baseline scenario has the highest aggregate welfare, savers are indifferent between the baseline and alternative scenario II that excludes reserve accumulation. As savers own private capital, the welfare loss from a more unstable economy with more procyclical fiscal policy under scenario II is offset by the welfare gains from higher capital income. Between the two alternatives, scenario II generates higher welfare than scenario I that excludes public investment, for all households. This illustrates the importance of public investment in the optimal policy mix to enhance welfare, not only for savers who are capital owners but also for the poorer hand-to-mouth households, as the benefits of higher capital trickle down via more income over time.

5.3.2 Welfare under Aid Volatility: Mean vs. Variance Effects

The analysis so far was conditional on a certain level of aid volatility, as given by the benchmark calibration of the aid process in equation (3.20). We now consider different degrees of aid volatility by varying the standard deviation of the aid process, σ_a, under the baseline policy configuration with all three instruments.

Figure 7 plots the standard deviations of key macroeconomic variables for σ_a ranging from 0.08 to 0.16. Higher aid volatility translates into higher volatility in key macroeconomic variables, including consumption, output, and labor. The increased consumption variability amplifies precautionary saving motives such that optimizing households and the Ramsey planner have a stronger incentive to save. More saving involves more public capital and reserve accumulation, while savers reinforce this by accumulating more private capital. As a result, the average levels of these variables (expressed in deviations from the deterministic steady state) increase as aid volatility rises, as shown in Figure 8. The higher capital stocks result in higher average output and consumption. Meanwhile, the higher output generates more tax revenues, which in turn increase average government spending.

A higher degree of aid volatility thus creates two opposing effects on welfare. First, it leads to higher consumption volatility, which is welfare reducing. Second, through enhanced precautionary saving motives and, hence, asset accumulation, it gives rise to higher long-run consumption levels, which are welfare improving. To isolate these two effects, we decompose the overall welfare changes into two components that reflect the mean changes in consumption and hours worked ("mean effects") and the variance changes of these variables ("variance effects").[21]

[21] Similar decompositions are conducted in Kollmann (2002), Moldovan (2010), and Kim and Kim (2018). We note that, for hand-to-mouth households, changes in utility and welfare are only driven by changes in consumption, given their constant labor supply.

We compute an overall welfare measure for each type of household $j = \{a, h\}$, expressed in terms of equivalent compensation in steady-state consumption, ζ^j, as in

$$\mathbb{W}^j \equiv E \sum_{t=0}^{\infty} \beta^t \mathbb{U}\left(c_t^j, l_t^j\right) = \frac{1}{1-\beta} \mathbb{U}\left[\left(1+\zeta^j\right) c^j, l^j\right], \tag{5.3}$$

where $\mathbb{W}^j$ is the discounted lifetime utility of households type j. Then, using a second-order approximation for utility, we have

$$\begin{aligned} \mathbb{W}^j \cong & \frac{\mathbb{U}^j}{1-\beta} + E \sum_{t=0}^{\infty} \beta^t \left[\left(c^j\right)^{-\sigma}\left(\tilde{c}_t^j\right) - \left(l^j\right)^{\psi}\left(\tilde{l}_t^j\right)\right] \\ & + E \sum_{t=0}^{\infty} \beta^t \frac{1}{2}\left\{\left[(-\sigma)\left(c^j\right)^{-\sigma-1}\right]\left(\tilde{c}_t^j\right)^2 + \left[(-\psi)\left(l^j\right)^{\psi-1}\right]\left(\tilde{l}_t^j\right)^2\right\}. \end{aligned} \tag{5.4}$$

where $\tilde{x}_t = x_t - x$ represents the deviation of x_t from its non-stochastic steady-state value, x. Denoting the welfare measure due to mean effects by $\mathbb{W}^{j,M}$ and that due to variance effects by $\mathbb{W}^{j,V}$, the two welfare components are calculated as

$$\mathbb{W}^{j,M} = \frac{\mathbb{U}^j}{1-\beta} + \frac{1}{1-\beta}\left[\left(c^j\right)^{-\sigma} E\left(\tilde{c}_t^j\right) - \left(l^j\right)^{\psi} E\left(\tilde{l}_t^j\right)\right] \tag{5.5}$$

and

$$\mathbb{W}^{j,V} = \frac{\mathbb{U}^j}{1-\beta} + \frac{1}{1-\beta}\frac{1}{2}\left\{\left[(-\sigma)\left(c^j\right)^{-\sigma-1}\right] E\left(\tilde{c}_t^j\right)^2 + \left[(-\psi)\left(l^j\right)^{\psi-1}\right] E\left(\tilde{l}_t^j\right)^2\right\}. \tag{5.6}$$

The corresponding steady-state consumption compensation terms, $\zeta^{j,M}$ and $\zeta^{j,V}$, are then computed as

$$\mathbb{W}^{j,M} = \frac{1}{1-\beta}\mathbb{U}\left[\left(1+\zeta^{j,M}\right) c^j, l^j\right] \quad \text{and} \quad \mathbb{W}^{j,V} = \frac{1}{1-\beta}\mathbb{U}\left[\left(1+\zeta^{j,V}\right) c^j, l^j\right]. \tag{5.7}$$

Rows (1)-(6) of Table 4 report the welfare implications of volatility, including the mean and variance effects, for the two types of households under three values of σ_a. Row (7) gives the aggregate measure of welfare. The variance effects ($\zeta^{a,V}$ and $\zeta^{h,V}$) are negative, reflecting the reduction in welfare due to consumption volatility in a stochastic economy, while the mean effects

($\zeta^{a,M}$ and $\zeta^{h,M}$) are positive, reflecting the welfare enhancing effects of higher average consumption levels.

Both the mean and variance effects are amplified when aid becomes more volatile, as shown by the values in Table 4, for higher σ_a's relative to the benchmark value of 0.098. As shown in Figure 7, higher aid volatility leads to higher consumption volatility; thus, $\zeta^{a,V}$ and $\zeta^{h,V}$ become more negative as σ_a rises. Instead, enhanced precautionary saving motives give rise to higher mean consumption levels; thus, $\zeta^{a,M}$ and $\zeta^{h,M}$ increase as σ_a increases. Overall, the mean effects are stronger and dominate in total welfare. In terms of magnitudes, a 50 percent higher aid volatility (σ_a from 0.098 to 0.147) is associated with higher welfare for both types of households: savers' (hand-to-mouth households') welfare increases to 0.79 percent (1.11 percent) of steady-state consumption from 0.35 percent (0.49 percent).[22]

These results highlight that, despite the negative effects of increased volatility, higher uncertainty can improve welfare in environments where precautionary savings motives lead to a strong accumulation of capital and sufficiently higher long-run consumption. Similar results have been obtained in the literature—see, for example, Kollmann (2002), Moldovan (2010), and the discussions in Lester et al. (2014) and Cho et al. (2015).[23]

6 Sensitivity Analysis

We explore the robustness of our results to alternative values for a set of parameters that reflect important characteristics of LICs: (i) a smaller share of hand-to-mouth households, (ii) stronger DD externality of traded goods production, and (iii) less productive public capital. As in the baseline scenario, the Ramsey planner sets policy optimally, using all three policy instruments.

[22] The model simulations account for the log-linear specification of the aid process, where the average log-deviation of aid is unaffected by the variance of the process. However, we also considered an alternative specification of the aid process as AR(1) in levels, $a_t^* = (1-\rho_a)a^* + \rho_a a_{t-1}^* + \varepsilon_t$, and qualitatively we obtained the same welfare implications of higher aid volatility. These results are available upon request.

[23] Cho et al. (2015) suggest that, for uncertainty to increase welfare, the shocks under consideration should be multiplicative to endogenous choices. In our model, the optimal allocation of volatile aid to productive public capital serves as a multiplicative factor in production, in turn affecting the households' optimal choices of private capital and labor.

6.1 Share of Hand-to-Mouth Households

Our benchmark assumes a relatively large share of hand-to-mouth households ($1 - \eta = 0.75$). Figure 9 compares the optimal policy settings and impulse responses under the benchmark calibration with $\eta = 0.25$ (solid blue lines) to those with $\eta = 0.45$ (dashed red lines), a value consistent with the evidence from richer SSA economies, as reported in Demirguc-Kunt and Klapper (2012).

When the economy has a smaller share of hand-to-mouth households, the Ramsey planner accumulates less of the aid inflow as reserves, and allocates more spending on public investment and less on transfers. The initial stronger public investment spending, which also attracts more private investment, results in a relatively larger output expansion and higher wages, supporting the consumption of both types of households. The smaller increase in reserve accumulation, under a smaller share of hand-to-mouth households, reflects a reduced need for precautionary savings to help smooth consumption over the longer run, as more households have access to financial markets and can self-insure against aid shocks.

6.2 Dutch Disease (DD) Effects

The quantitative model captures DD effects through a growth externality on traded goods production as specified in equation (3.15). Figure 10 compares the impulse responses under two values of ρ_{zT}, which govern the TFP process of traded goods production: $\rho_{zT} = 0.1$ (benchmark calibration, solid lines) versus $\rho_{zT} = 0.3$ (stronger externality, dashed lines). With a stronger DD externality, the productivity of the traded goods sector declines by a larger magnitude in the short-to-medium run but also recovers faster later on. This is because a higher ρ_{zT} amplifies both the initial decline and subsequent recovery when traded output changes.

As in the simple model, under a stronger DD externality, the quantitative model shows slightly more reserve accumulation and less spending, particularly less public investment spending. The larger productivity decline in the traded goods sector also reduces the magnitude of the private investment increase. Weaker public and private demand then lowers aggregate goods demand, resulting in a smaller increase in output under $\rho_{zT} = 0.3$. A smaller buildup in public and private capital also leads to a smaller increase in the marginal product of labor, hence the real wage rate does not rise as much. To compensate for weaker income and consumption growth, the optimal spending strategy indicates a slightly higher increase in transfers in the shorter run, relative to the benchmark calibration of $\rho_{z^T} = 0.1$.

In equilibrium, the real exchange rate appreciates slightly less under a more persistent DD externality. This reflects the dampened demand for domestic goods, stemming from the relatively lower spending of aid inflows, and aligns with slightly more reserve accumulation in the optimal policy mix under $\rho_{z^T} = 0.3$.

6.3 Public Capital Productivity

Finally, we consider the sensitivity of our results with respect to public capital productivity, captured by α^G in equations (3.11) and (3.14). This parameter is relevant for both the spending/saving decision of the policy maker, as well as for the allocation of spending between transfers and public investment. Figure 11 compares the impulse responses to the aid shock under the benchmark calibration of $\alpha^G = 0.39$ (solid lines) with those under a lower $\alpha^G = 0.187$, based on the estimate in Gupta et al. (2014) for LICs (dashed lines).

With a lower α^G, there is a stronger accumulation of reserves and a reduction in public investment spending in response to the aid shock. This result is akin to that of the simple model, where lower spending productivity implied more reserve accumulation and less spending of aid. The relative reduction in public investment spending results in a lower increase in aggregate demand, leading to less growth in private investment and labor demand, and a smaller increase in the real wage compared to the benchmark calibration with more productive public capital. A less productive public capital also diminishes the role of investment spending as a saving instrument over the long run. With reduced accumulation of public and private capital, output is markedly lower when public capital is less productive. However, increased savings in the form of larger reserves help sustain transfers to households over time—beyond the initial periods, transfers increase more relative to the benchmark scenario. Despite the increased transfer payments and a less negative labor response under $\alpha^G = 0.187$, private consumption increases less than under the benchmark calibration, primarily due to the much smaller increase in real wages.

We also conducted a sensitivity analysis regarding the efficiency of public investment, denoted by ϵ in equation (3.19).[24] The results are somewhat analogous to those observed for public capital productivity. They are consistent with the intuition that decreasing efficiency of spending leads to reduced allocation of aid towards public investment (and thus capital accumulation) and more savings as reserves. Associated with less capital accumulation, we observe a weaker output expansion following the aid shock and a smaller increase in consumption. Note that, following the discussion in Berg et al. (2019), the analysis that we conducted corresponds to decreasing the efficiency of public investment over time, which can mimic public investment adjustment costs that arise due to absorptive capacity constraints.

7 Conclusion

We analyze Ramsey-optimal policies regarding the spending and saving of volatile aid flows using a DSGE model that captures key features of a typical LIC. Our model incorporates heterogeneous agents—forward-looking savers and financially constrained hand-to-mouth households—alongside DD externalities in traded goods production and a detailed fiscal policy specification. The policy spectrum includes transfers to households, public investment, and reserve accumulation.

Our findings underscore the importance of directing some aid spending towards public investment, alongside transfers and reserve accumulation. This approach enhances welfare compared to allocating aid solely to transfers and reserve accumulation. While transfers directly support hand-to-mouth consumption, investing in public infrastructure—and thereby building public capital—indirectly stabilizes consumption by sustaining production and income over the long term. From this perspective, public investment functions as a precautionary saving instrument, akin to reserve accumulation, amidst volatile aid flows. Sensitivity analyses further highlight the critical role of key features specific to LICs in affecting optimal policy decisions. For example, a higher proportion of financially constrained consumers, intensified DD externalities, or lower productivity of public capital call for reduced allocation of aid to public investment and more reserve accumulation.

Our model-based welfare analysis indicates that higher aid volatility may require increased public investment, along with transfers and reserve accumulation. Although higher aid volatility is associated with more negative variance effects on welfare—due, for instance, to higher consump-

[24] Results are available upon request.

tion volatility—, it generates more positive mean effects as precautionary saving motives drive up savings in both public investment and reserve accumulation, supporting higher consumption over the medium term.

Our analysis of Ramsey-optimal policies in the context of volatile aid may have broader implications for managing other types of volatile external income, such as natural resource revenues. Specifically, our insights into spending and saving through public investment provide a foundation for policy recommendations on managing volatile resource revenues in developing countries, consistent with suggestions made by international organizations such as International Monetary Fund (2012a,b). Additionally, our findings on the optimal combination of reserve accumulation with fiscal policy, including public investment, contribute to the existing literature and policy discussions on combining various policies—e.g., monetary, fiscal, and foreign exchange intervention—in response to external shocks, such as capital flows (e.g., Basu et al., 2020; Tobias et al., 2024). Further research is needed, however, to fully explore the implications of public investment as a fiscal instrument in this context.

Tables and Figures

Parameters	Values	
Literature-based values		
β	0.98	Discount factor
σ	2	Inverse of intertemporal elasticity of substitution for consumption
ψ	0.5	Inverse of Frisch labor elasticity, savers
φ	0.6	Degree of home bias in private consumption and investment
φ^G	0.7	Degree of home bias in government consumption and investment
χ	0.44	Elasticity of substitution between traded and nontraded goods
θ	6	Elasticity of substitution among nontraded goods
χ^l	0.6	Elasticity of substitution between labor of two sectors
η	0.25	Fraction of savers
κ	1.4	Investment adjustment cost parameter
δ	0.025	Depreciation rate of private capital
δ^G	0.02	Depreciation rate of public capital
ϵ	0.4	Public investment efficiency
z^N	1	TFP in nontraded sector, normalization
ρ_{z^T}	0.1	AR(1) parameter in traded sector for TFP
ξ	0.1	Learning-by-doing parameter
α^N	0.45	Labor income share in nontraded goods output
α^T	0.60	Labor income share in traded goods output
Calibrated parameters		
α^G	0.39	Output elasticity with respect to public capital
ι	0.18	Implicit production cost parameter
φ^l	0.64	Steady-state labor share in nontraded good sector
τ	0.19	Income tax rate
z^T	1.85	TFP in traded sector
υ	500	Capital account openness parameter
υ^G	0.0006	Adjustment costs parameter, foreign exchange reserves
ρ_a	0.72	Foreign aid persistence
σ_a	0.098	Foreign aid standard deviation

Table 1: **Benchmark calibration: the quantitative model**

c/y	88%	Private consumption to output ratio
i/y	16.1%	Private investment to output ratio
g^I/y	9.2%	Public investment to output ratio
z/y	2.6%	Government transfers to output ratio
tax/y	14%	Tax revenues to output ratio
$\frac{b}{4y}$	51.7%	Government debt to output ratio (annual)
$\frac{s \cdot b^*}{4y}$	-0.66%	Private external debt (annual)
$\frac{s \cdot res^*}{4y}$	13%	Foreign reserves to output ratio (annual)
$\frac{s \cdot a^*}{y}$	8.4%	Foreign aid to output ratio (annual)
$std\left(\frac{s \cdot b^*}{4y}\right)$	7.1%	Standard deviation of private external debt/gdp
$std\left(\frac{s \cdot res^*}{4y}\right)$	4.2%	Standard deviation of foreign reserves/gdp

Table 2: **Data moments.** See Section 4.2 for data description.

	(1) Baseline Scenario	(2) Alternative Scenario I	(3) Alternative Scenario II
$\mathbb{W}^a$	−8.892	−8.916	−8.892
$\mathbb{W}^h$	−43.895	−44.028	−43.916
Aggregate welfare, $\mathbb{W}$	−35.145	−35.251	−35.161

Table 3: **Welfare of three policy scenarios**. $\mathbb{W}^a$ and $\mathbb{W}^h$ are unconditional welfare for savers and hand-to-mouth households. $\mathbb{W} = \eta \mathbb{W}^a + (1-\eta)\,\mathbb{W}^h$. The baseline scenario has transfers, public investment, and reserves as policy instruments. Alternative scenario I has transfers and reserves as policy instruments. Alternative scenario II has transfers and public investment as policy instruments.

	$\sigma_a = 0.098$ (benchmark)	$\sigma_a = 0.12$	$\sigma_a = 0.147$
(1) Savers $(\zeta^a, \%)$	0.353	0.531	0.798
(2) Mean effect $\left(\zeta^{a,M}, \%\right)$	0.376	0.565	0.850
(3) Variance effect $\left(\zeta^{a,V}, \%\right)$	−0.023	−0.034	−0.051
(4) Hand-to-mouth $\left(\zeta^h, \%\right)$	0.491	0.738	1.111
(5) Mean effect $\left(\zeta^{h,M}, \%\right)$	0.547	0.822	1.239
(6) Variance effect $\left(\zeta^{h,V}, \%\right)$	−0.055	−0.083	−0.124
(7) Aggregate welfare: $\mathbb{W}$	−35.145	−35.062	−34.937

Table 4: **Welfare of different aid volatility: mean vs. variance effects.** The numbers are equivalent compensation in percent of steady-state consumption for each type of household.

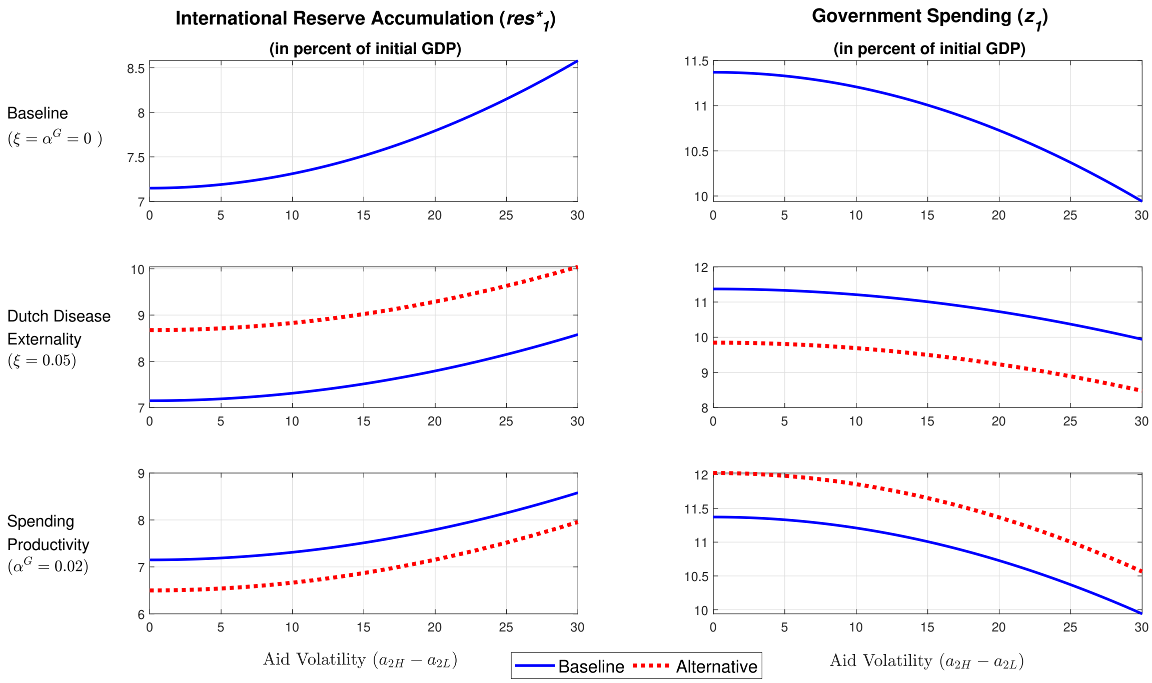

Figure 1: **The role of aid volatility in optimal reserve accumulation and government spending policies.** The baseline versus alternative simulations with Dutch disease externalities and productive government spending.

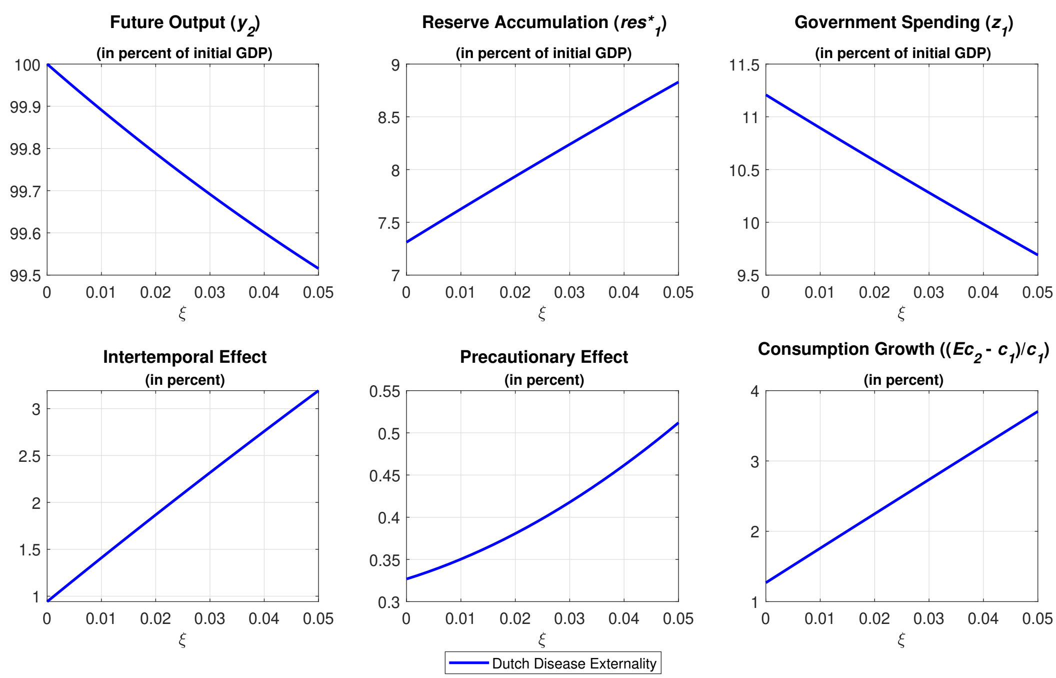

Figure 2: **The role of the Dutch disease (DD) externality on optimal reserve accumulation and government spending policies.** The degree of the DD externality is measured by ξ.

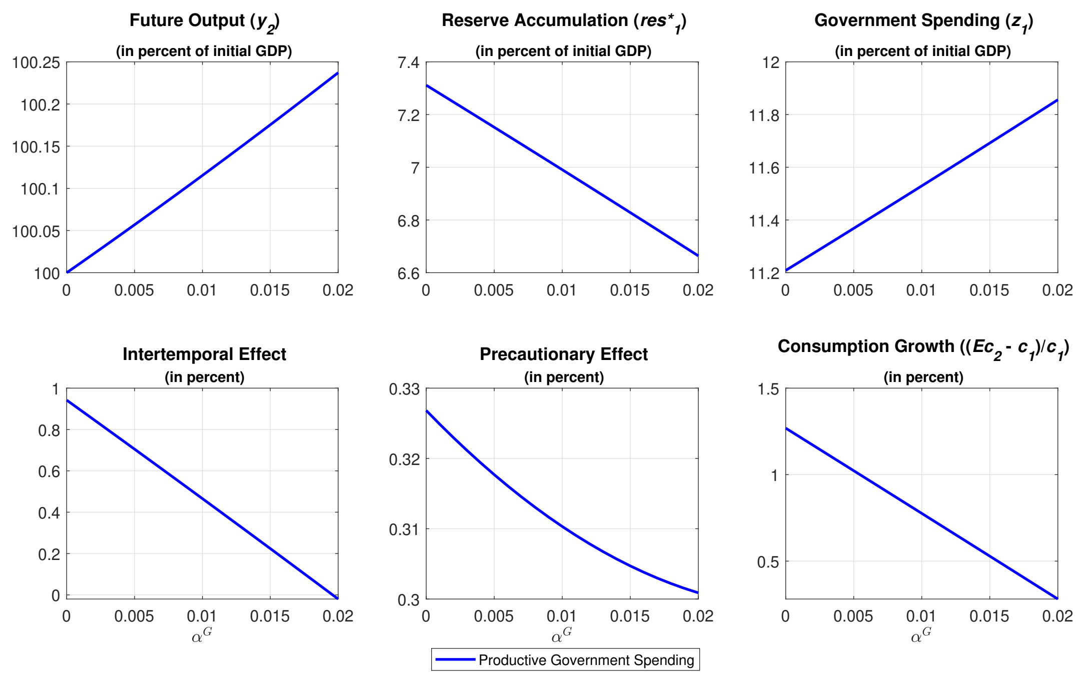

Figure 3: **The role of the productivity of government spending on optimal reserve accumulation and government spending policies.** The degree of productivity is measured by α^G.

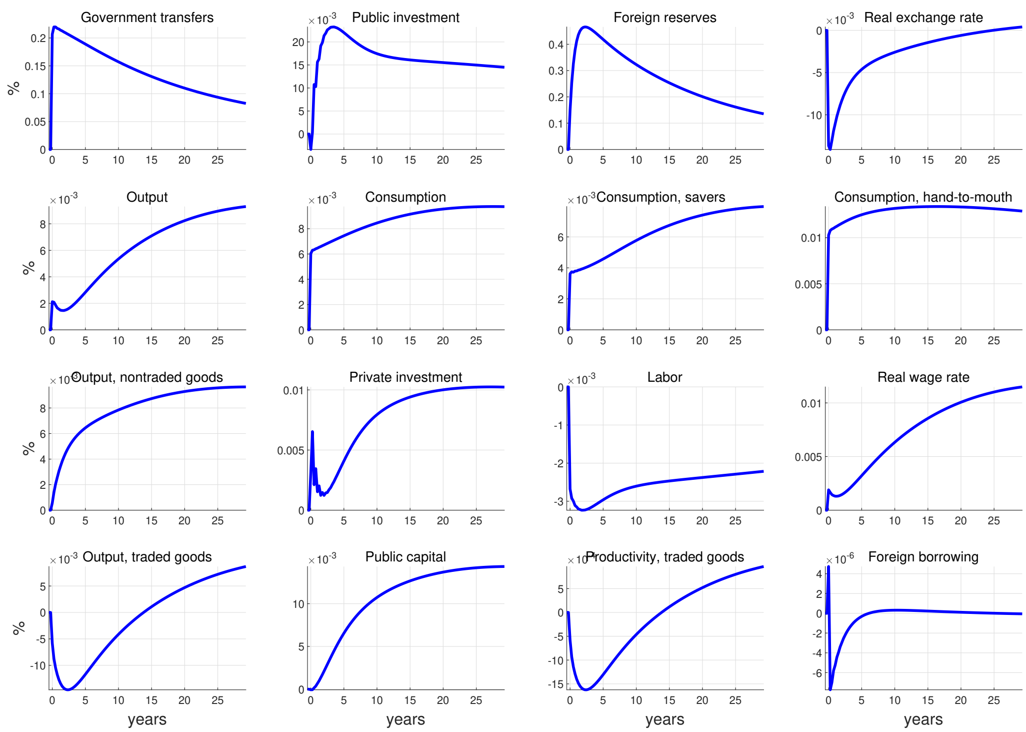

Figure 4: **Impulse responses to a 1 percent aid shock: the baseline scenario.** The baseline scenario has all three policy instruments: 1) transfers, 2) public investment, and 3) reserve accumulation. The y-axes are in percent deviation from the steady state.

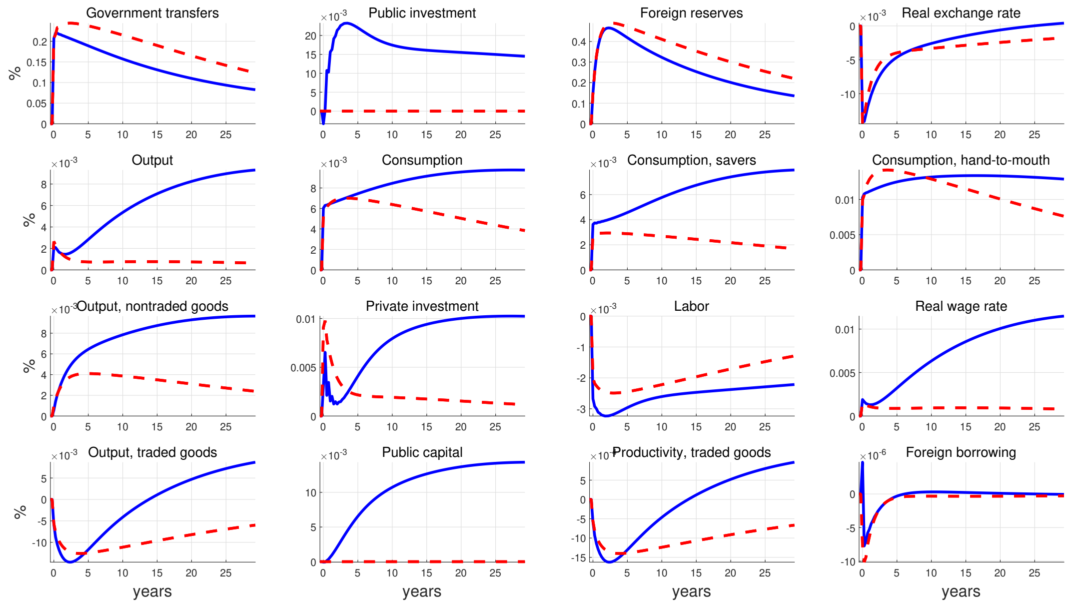

Figure 5: **Impulse responses to a 1 percent aid shock: alternative scenario I—excluding public investment.** Alternative scenario I (dash red lines) has transfers and reserve accumulation as policy instruments. See Figure 4 for the baseline scenario (solid blue lines).

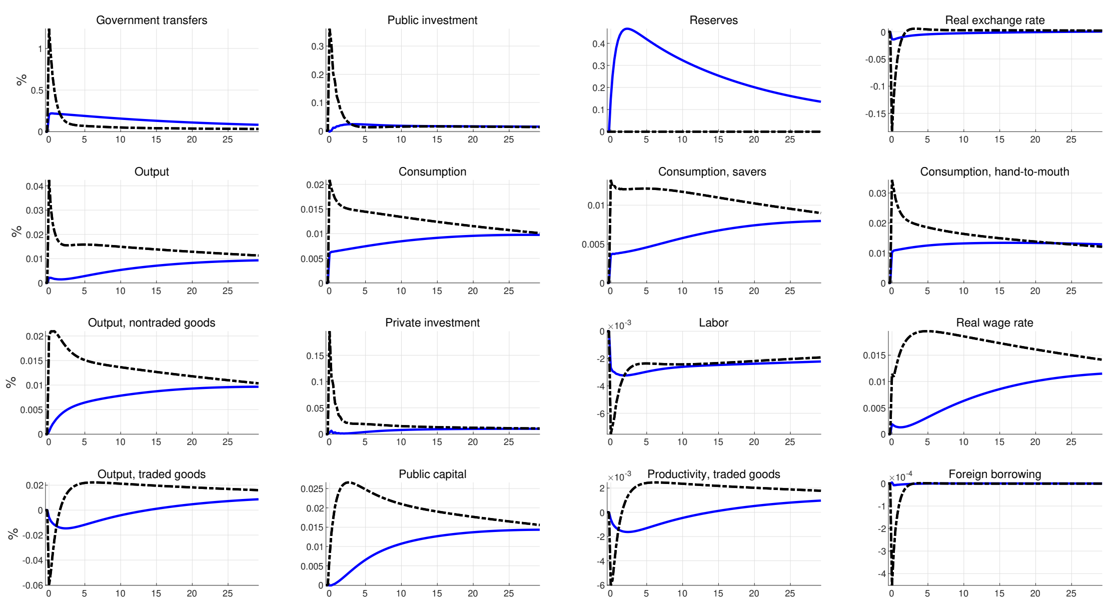

Figure 6: **Impulse responses to a 1 percent aid shock: alternative scenario II—excluding reserve accumulation.** Alternative scenario II (dash dot black lines) has transfers and public investment as policy instruments. See Figure 4 for the baseline scenario (solid blue lines).

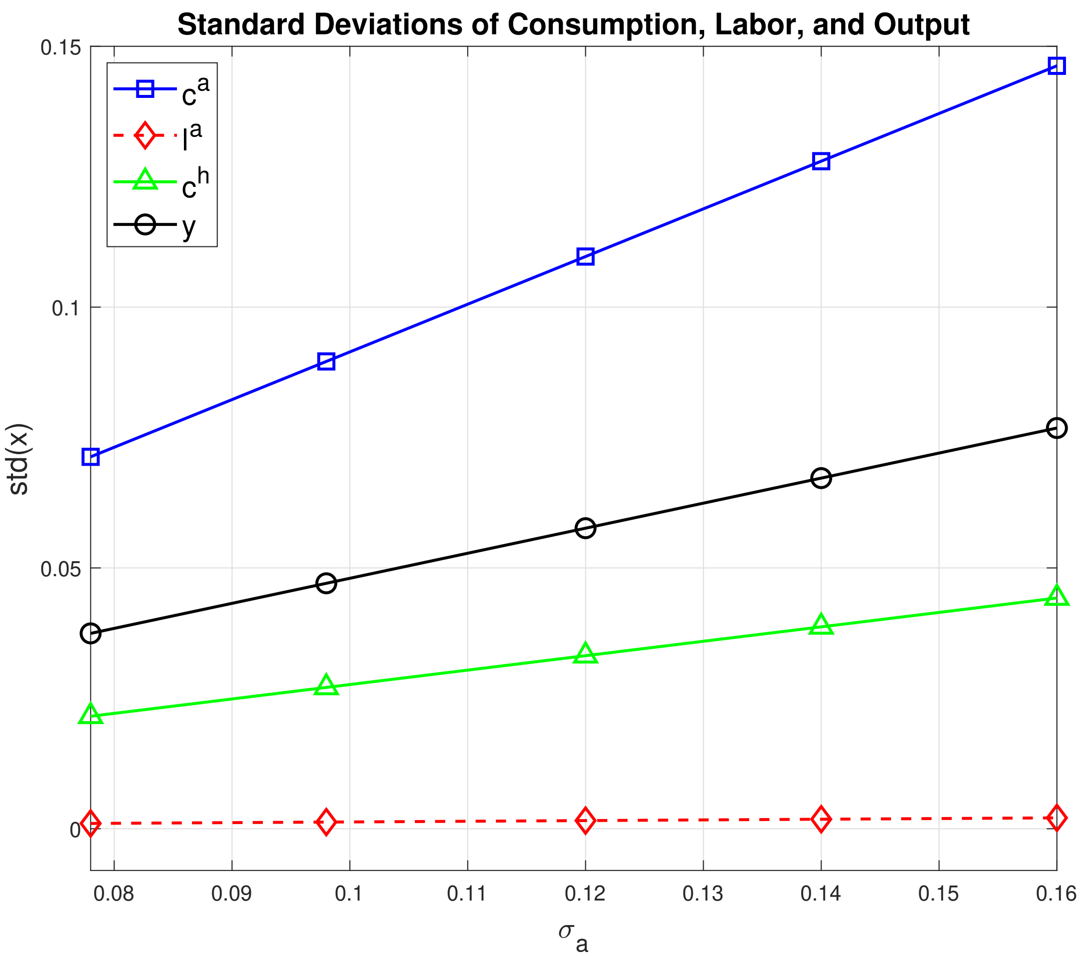

Figure 7: **Standard deviations of consumption (c^a, c^h), labor (l^a), and output (y), for increasing aid volatility (σ_a)**, expressed in percentages.

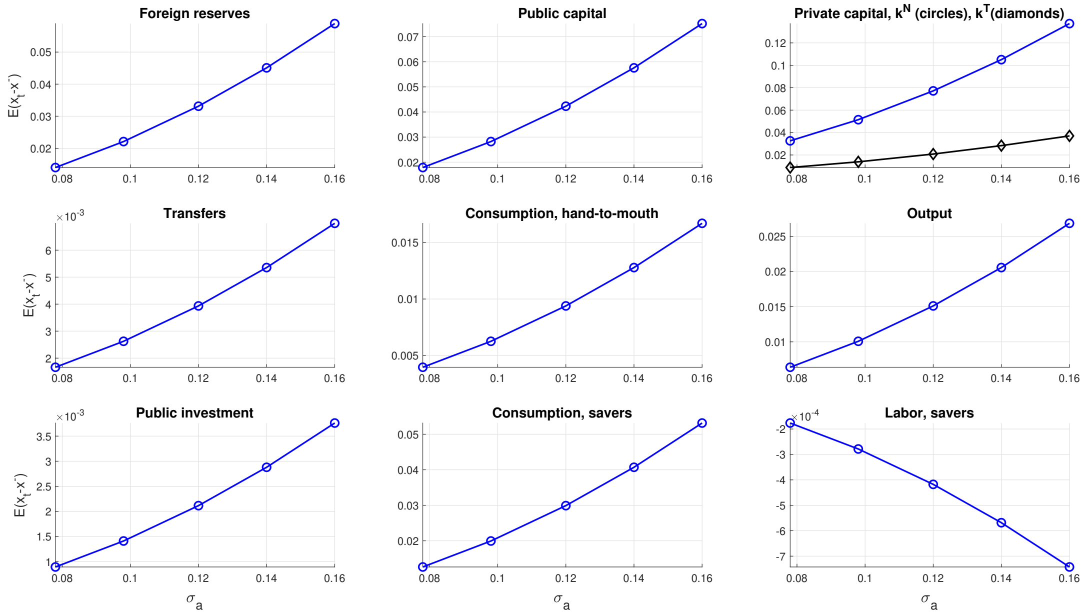

Figure 8: **Average levels of selected variables under different degrees of aid volatility.** The y-axes are in deviations from the deterministic steady state.

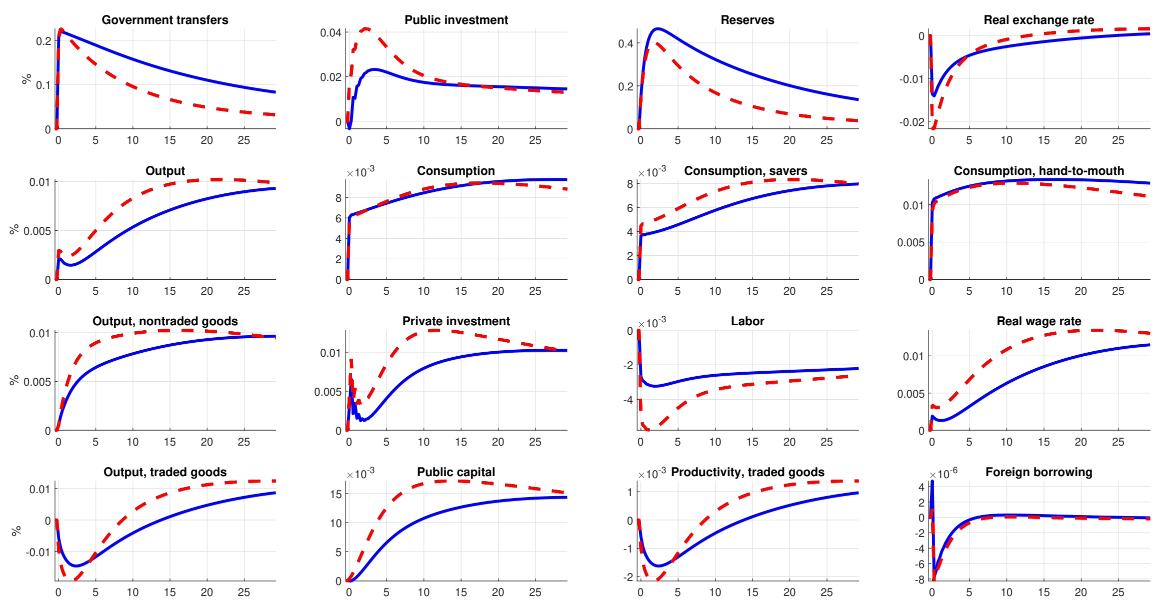

Figure 9: **Sensitivity analysis: different shares of hand-to-mouth households.** Solid blue lines are for $\eta = 0.25$ (the benchmark calibration), and dashed red lines are for $\eta = 0.45$.

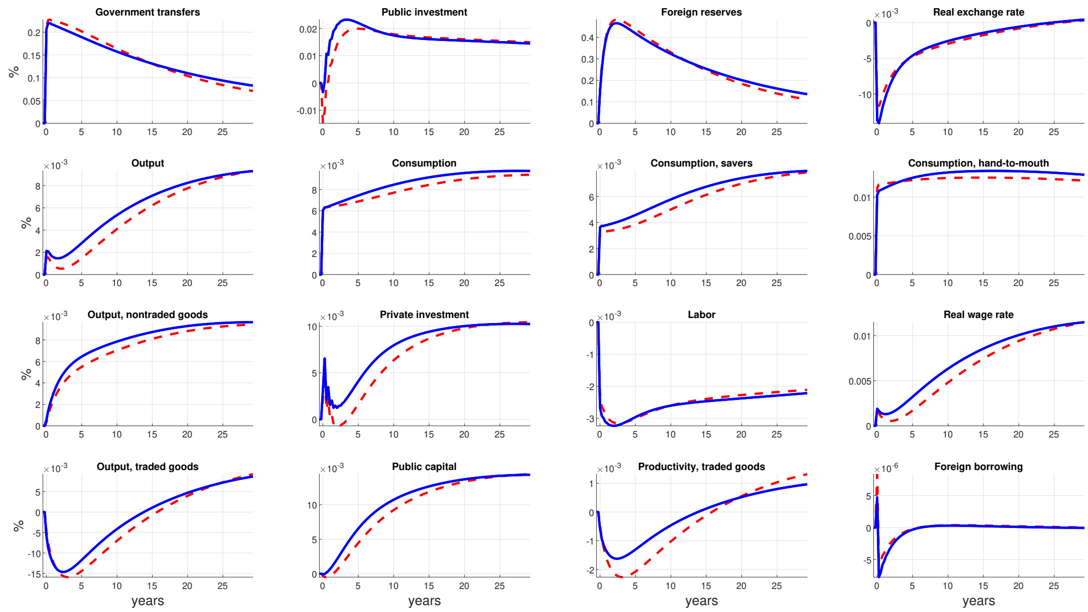

Figure 10: **Sensitivity analysis: different Dutch Disease externality in traded goods production.** Solid blue lines are for $\rho_{z^T} = 0.1$ (the benchmark calibration), and dash-red lines are for $\rho_{z^T} = 0.3$.

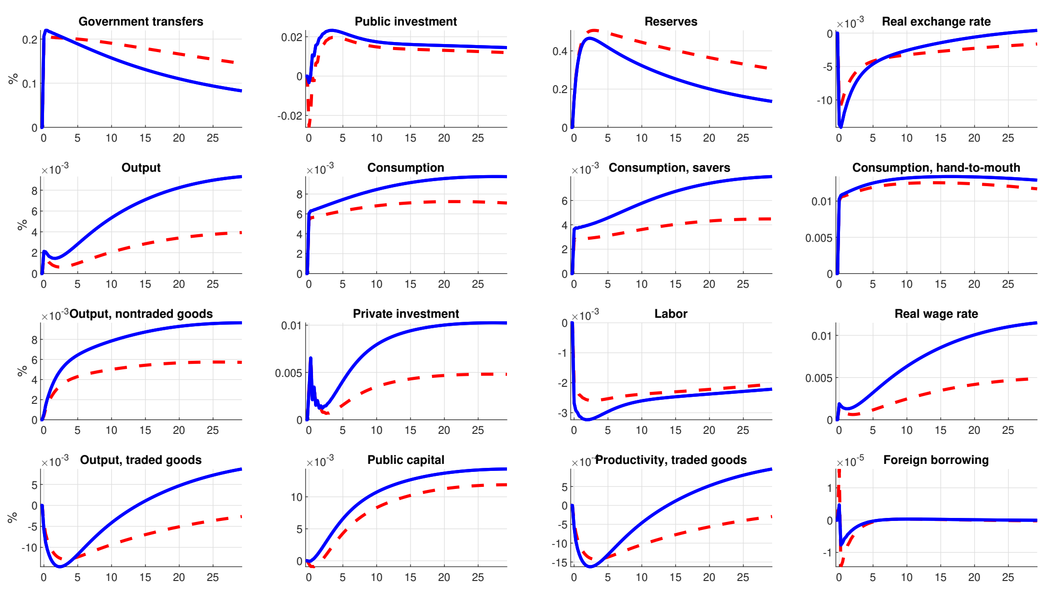

Figure 11: **Sensitivity analysis: different degrees of public capital productivity.** Solid blue lines are for $\alpha^G = 0.39$ (the benchmark calibration), and dash-red lines are for $\alpha^G = 0.187$.

Appendices

Appendix A Aid vs. Tax Revenue Volatility in the Data

The sample consists of 23 low income countries (LICs) and 53 lower middle income countries (LMICs) based on World Bank's classification of 2021.[25] Aid is measured by the net official development assistance (ODA) received as percent of GDP, where ODA is in current US dollars (Organisation for Economic Co-Operation and Development, 2022), and GDP is in current US dollars (World Bank, 2022). The tax revenue data are general government tax revenues as percent of GDP from the World Development Indicators database (World Bank, 2022). We set the overall sample period from 1980 to 2019, but data availability varies across countries and variables, so the sample period for each country is the period for which both aid and tax revenues have no missing values.

Table 5 reports the proportion of countries in an income group for which aid volatility is higher than tax revenues' volatility, as measured by the standard deviation of detrended series as a share of GDP. Three methods of de-trending have been used: first difference, HP filter (with a smoothing parameter of 6.25 for annual data Ravn and Uhlig, 2002), and a linear trend.

	LICs	**LMICs**
First difference series	78.3%	92.5%
HP filter	82.6%	94.3%
Linear trend	78.3%	92.5%

Table 5: **Volatility comparison: aid vs. tax revenues.**

Appendix B Equilibrium Conditions

When solving savers' maximization problem, let λ_t^a be the Lagrangian multiplier for the budget constraint and Q_t^N and Q_t^T for the laws of motion of capital in the two production sectors. Tobin's Q for k_t^N and k_t^T are $q_t^N \equiv \frac{Q_t^N}{\lambda_t^a}$ and $q_t^T \equiv \frac{Q_t^T}{\lambda_t^a}$.

[25] Sample periods of each country are available upon request.

First order condition (FOC) for private consumption savers, c_t^a:

$$\lambda_t^a = (c_t^a)^{-\sigma} \tag{B.1}$$

Labor supply, savers, l_t^a

$$(l_t^a)^{\psi} = \lambda_t^a w_t (1-\tau) \tag{B.2}$$

Euler equation, domestic bonds:

$$\lambda_t^a = \beta E_t \left(\lambda_{t+1}^a r_t\right) \tag{B.3}$$

Holdings of foreign assets:

$$\lambda_t^a s_t \left[1 + \upsilon \left(b_t^{*,a} - b^{*,a}\right)\right] = \beta E_t \left(\lambda_{t+1}^a s_{t+1}\right) r^* \tag{B.4}$$

Hand-to-mouth households' budget constraint:

$$c_t^h = (1-\tau) w_t l^h + s_t r m^* + z_t \tag{B.5}$$

FOC for capital nontraded sector, k_t^N:

$$q_t^N = \beta E_t \frac{\lambda_{t+1}^a}{\lambda_t^a} \left[(1-\tau) r_{t+1}^N - \frac{\kappa}{2}\left(\frac{i_{t+1}^N}{k_t^N} - \delta\right)^2 + \kappa \left(\frac{i_{t+1}^N}{k_t^N} - \delta\right)\left(\frac{i_{t+1}^N}{k_t^N}\right) + q_{t+1}^N (1-\delta)\right] \tag{B.6}$$

FOC for capital traded sector, k_t^T:

$$q_t^T = \beta E_t \frac{\lambda_{t+1}^a}{\lambda_t^a} \left[(1-\tau) r_{t+1}^T - \frac{\kappa}{2}\left(\frac{i_{t+1}^T}{k_t^T} - \delta\right)^2 + \kappa \left(\frac{i_{t+1}^T}{k_t^T} - \delta\right)\left(\frac{i_{t+1}^T}{k_t^T}\right) + q_{t+1}^T (1-\delta)\right] \tag{B.7}$$

FOC for investment nontraded sector, i_t^N

$$q_t^N = 1 + \kappa \left(\frac{i_t^N}{k_{t-1}^N} - \delta\right) \tag{B.8}$$

FOC for investment traded sector, i_t^T

$$q_t^T = 1 + \kappa \left(\frac{i_t^T}{k_{t-1}^T} - \delta\right) \tag{B.9}$$

Capital accumulation nontraded sector, k_t^N

$$k_t^N = (1-\delta) k_{t-1}^N + i_t^N \tag{B.10}$$

Capital accumulation traded sector, k_t^T

$$k_t^T = (1-\delta) k_{t-1}^T + i_t^T \tag{B.11}$$

Labor supplied to the nontraded goods sector:

$$l_t^N = \varphi^l \left(\frac{w_t^N}{w_t}\right)^{\chi^l} l_t \tag{B.12}$$

Labor supplied to the traded goods sector:

$$l_t^T = \left(1 - \varphi^l\right) \left(\frac{w_t^T}{w_t}\right)^{\chi^l} l_t \tag{B.13}$$

Production of nontraded goods:

$$y_t^N = z^N \left(l_t^N\right)^{\alpha^N} \left(k_{t-1}^N\right)^{1-\alpha^N} \left(k_{t-1}^G\right)^{\alpha^G} \tag{B.14}$$

Pricing condition:

$$(1-\iota)\, p_t^N = \frac{\theta}{\theta - 1} mc_t^N \tag{B.15}$$

where $mc_t^N = \left[\left(1-\alpha^N\right)^{-\left(1-\alpha^N\right)} \left(\alpha^N\right)^{-\alpha^N}\right] \left(r_t^N\right)^{1-\alpha^N} \left(w_t^N\right)^{\alpha^N} \left[\left(z^N\right)^{-1} \left(k_{t-1}^G\right)^{-\alpha^G}\right]$.
Factors' share in nontraded goods production

$$(1-\alpha^N) w_t^N l_t^N = \alpha^N r_t^N k_{t-1}^N \tag{B.16}$$

Production of traded goods:

$$y_t^T = z_t^T \left(l_t^T\right)^{\alpha^T} \left(k_{t-1}^T\right)^{1-\alpha^T} \left(k_{t-1}^G\right)^{\alpha^G} \tag{B.17}$$

Labor demand in the traded goods sector, l_t^T:

$$w_t^T l_t^T = (1-\iota)\, s_t \alpha^T y_t^T \tag{B.18}$$

Capital demand in the traded goods sector k_t^T:

$$r_t^T k_{t-1}^T = (1-\iota)\, s_t (1-\alpha^T) y_t^T \tag{B.19}$$

TFP process

$$\log\left(\frac{z_t^T}{z^T}\right) = \rho_{zT} \log\left(\frac{z_{t-1}^T}{z^T}\right) + \varkappa \log\left(\frac{y_{t-1}^T}{y^T}\right) \tag{B.20}$$

Government budget constraint:

$$p_t^G \left(g^C + g_t^I\right) + z_t + s_t res_t^* + s_t ac_t^{res} = tax_t + (1 - r_{t-1})\, b + s_t r^* res_{t-1}^* + s_t a_t^* \tag{B.21}$$

where $ac_t^{res} \equiv \frac{\upsilon^{res}}{2}(res_t^* - res^*)^2$ and $tax_t = \tau\left(w_t l_t + r_t^N k_{t-1}^N + r_t^T k_{t-1}^T\right)$.
Foreign aid process:

$$\log\left(\frac{a_t^*}{a^*}\right) = \rho_a \log\left(\frac{a_{t-1}^*}{a^*}\right) + \varepsilon_t \tag{B.22}$$

Law of motion for public capital:

$$k_t^G = \left(1-\delta^G\right) k_{t-1}^G + \epsilon g_t^I \tag{B.23}$$

Aggregate consumption:

$$c_t = \eta c_t^a + (1-\eta) c_t^h \tag{B.24}$$

Aggregate labor:

$$l_t = \eta l_t^a + (1-\eta) l^h \tag{B.25}$$

Real wage index:

$$w_t l_t = w_t^N l_t^N + w_t^T l_t^T \tag{B.26}$$

Total private investment:

$$i_t = i_t^N + i_t^T \tag{B.27}$$

Aggregate output:

$$y_t = p_t^N y_t^N + s_t y_t^T \tag{B.28}$$

Market clearing condition for nontraded goods:

$$y_t^N = \left(p_t^N\right)^{-\chi} d_t^N, \qquad \text{where } d_t^N = \varphi\left(c_t + i_t + ac_t^i\right) + \varphi^G (p_t^G)^{\chi} g_t, \tag{B.29}$$

with $ac_t^i = \frac{\kappa}{2}\left[\left(\frac{i_t^N}{k_{t-1}^N} - \delta\right)^2 k_{t-1}^N + \left(\frac{i_t^T}{k_{t-1}^T} - \delta\right)^2 k_{t-1}^T\right]$.
Balance of payments:

$$\begin{aligned} & c_t + i_t + p_t^G\left(g^C + g_t^I\right) + ac_t^i - y_t + s_t\left[ac_t^* + ac_t^{res} - rm^* - (r^*-1)\left(res_{t-1}^* + b_{t-1}^*\right)\right] \\ & = s_t\left[a_t^* - \left(b_t^* - b_{t-1}^*\right) - \left(res_t^* - res_{t-1}^*\right)\right]. \end{aligned} \tag{B.30}$$

where $ac_t^* \equiv \frac{\upsilon}{2}(b_t^* - b^*)^2/\eta$ and $ac_t^{res} \equiv \frac{\upsilon^{res}}{2}(res_t^* - res^*)^2$.
The relative price of consumption goods:

$$1 = \left[\varphi\left(p_t^N\right)^{1-\chi} + (1-\varphi)(s_t)^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{B.31}$$

The relative price of government purchases:

$$p_t^G = \left[\varphi^G\left(p_t^N\right)^{(1-\chi)} + \left(1-\varphi^G\right)(s_t)^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{B.32}$$

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