## Options for Strengthening Ireland's Fiscal Framework

Rossen Rozenov, Raphael Lam, Yang Yang, and Yinjie Yu

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ABSTRACT: Ireland's reliance on corporate income tax (CIT) receipts from multinational enterprises, concentrated in a small number of companies, presents significant risks to the budget. The uncertain nature of this revenue calls for a robust fiscal framework to safeguard public finances. This paper proposes strengthening the national fiscal framework by establishing a prudent medium-term debt anchor and an expenditure rule to guide the annual budget process. We first establish a prudent debt anchor for Ireland by calibrating CIT shocks and simulating possible debt trajectories. Second, we propose an operational rule based on multi-year expenditure ceilings. The ceilings are calibrated such as to stabilize debt at the anchor level while accounting for the economy's cyclical position. Although tailored to Ireland, the methodology employed has broader applicability for designing effective fiscal rules.

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## **WORKING PAPERS**

# Options for Strengthening Ireland's Fiscal Framework

Prepared by Rossen Rozenov, Rapahel Lam, Yang Yang and Yinjie Yu<sup>1</sup>

<sup>&</sup>lt;sup>1</sup> The authors would like to thank Santiago Previde for excellent research assistance, and seminar participants in Dublin and Washington, D.C. for their helpful comments and suggestions.

## **Contents**

EXECUTIVE SUMMARY	3
A. Introduction	4
B. Calibrating a Prudent Debt Anchor	5
C. Calibrating an Operational Rule	
D. Conclusion	15
FIGURES	
1. Developments of CIT Revenues	4
2. CIT and FDI Variation	7
3. Simulated Macro Variables	8
4. Simulated Debt Fan Charts: FDI Risks	9
5. Simulated Debt Fan Charts: Firm Risks	10
6. Debt Trajectories Under Different Expenditure Paths	
7. Optimal Trajectories	
8. Revenue Ratio and Expenditure Growth	
9. Revenue Shock Scenario	
REFERENCES	17
APPENDIX	
I. Technical Appendix	18
○ A. FDI Risks	18
B. Concentration Risks     C. Fiscal Reaction Function	22 25

## **EXECUTIVE SUMMARY**

The paper makes a case for a strengthened fiscal framework for Ireland in view of the challenges from rising global uncertainty, concentration of corporate tax revenues and future spending pressures. The methodology employed, however, has broader applicability and can be used to inform the design of fiscal rules in both advanced and developing economies, conditional on data availability.

Over the past decade, corporate income tax (CIT) receipts in Ireland have grown substantially, driven by the activities of a small number of large multinational enterprises (MNEs). While this has supported strong fiscal outcomes in recent years in terms of headline numbers, it has also increased Ireland's exposure to unpredictable revenue swings driven by external shocks or policy shifts. Moreover, Ireland faces significant spending needs to alleviate supply constraints, notably in housing and infrastructure, and address long-term demographic pressures.

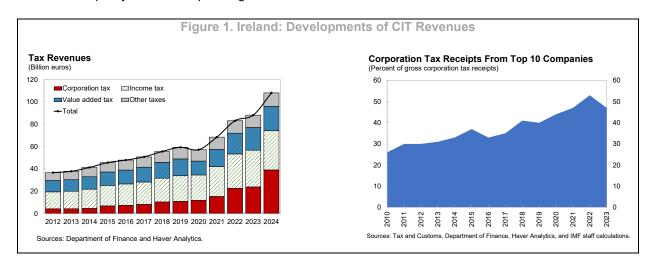
To enhance fiscal resilience, an approach to strengthening the national fiscal framework is proposed, designed to complement Ireland's obligations under the EU fiscal rules, while taking into consideration country-specific circumstances, notably the use of appropriate debt concept and denominator. The cornerstone of the proposed framework is adoption of a medium-term debt anchor. A risk-based framework is used to calibrate the debt anchor, accounting for potential shocks to revenue stemming from volatile MNE operations. Quantification of the risks is done independently through analysis of cross-country FDI (top-down approach) and firm-level data (bottom-up approach). The anchor is determined such as to ensure with high probability that debt will not reach a level that could trigger a negative market reaction. Both approaches suggest that a ratio of net debt to Modified Gross National Income (GNI\*) of around 40 percent would provide a prudent anchor for fiscal policy.

In addition, the paper recommends an operational fiscal rule in the form of multi-year expenditure ceilings. A debt anchor alone would generally be insufficient for effective fiscal management as it lacks a mechanism to guide short-term policy decisions. Thus, it needs to be complemented with an operational rule to ensure that the debt objective is achieved and maintained, while accounting for economic conditions. Expenditure-based rules are considered more effective in terms of reducing fiscal policy procyclicality and have been adopted by a number of countries. Fiscal reaction functions (FRFs) provide a useful tool to guide the calibration of expenditure rules. Various approaches to calibrating FRFs are explored which, under plausible assumptions about key economic parameters and policy preferences, point to an average nominal expenditure growth of around 5–6 percent in the medium term.

A well-designed fiscal framework is key for safeguarding long-term sustainability, while maintaining economic flexibility. For the framework to be effective, it needs to include several additional elements, including an escape clause and triggering conditions, a correction mechanism, independent oversight, periodic reviews and links to annual budgets and the medium-term fiscal and structural plan. A stronger national fiscal rule would help Ireland balance competing priorities and contribute to policy certainty, predictability and transparency which are conducive to confidence and investment.

## A. Introduction

Tax revenues in Ireland are vulnerable to external shocks that can be disproportionate to the size of the country's domestic economy. Corporate income tax (CIT) from multinational enterprises (MNEs), the second largest revenue source, exhibits a high degree of concentration and is subject to substantial uncertainty. In addition to their direct impact on CIT, MNEs also contribute significantly to employment and wages, boosting personal income tax and VAT in Ireland. Firm- and sector-specific shocks, as well as global trade and tax policy shifts could pose significant risks to Ireland's fiscal outlook.



The uncertain nature of CIT revenue calls for a strong national fiscal framework to safeguard Ireland's public finances. While Ireland currently enjoys a relatively comfortable fiscal position thanks to large windfall CIT revenues, the situation can change quickly, as past GFC experience with the sudden unwinding of property-related receipts has shown. The previous government introduced a spending rule stipulating that the annual growth of nominal expenditure net of discretionary revenue measures would not exceed 5 percent during 2021–25.¹ This rule, essentially linking expenditure growth to the potential growth of the economy, could have provided a reasonable guide for fiscal policy had it been adhered to. However, it was repeatedly breached without corrective actions following noncompliance. Moreover, it covered only the central government, was not legislated, and has now lapsed. Although Ireland has adopted the EU fiscal framework, Ireland-specific factors limit its effectiveness. In particular, 1) the EU framework relies on GDP as a denominator when the Modified Gross National Income (GNI\*)² is a more appropriate measure of the domestic economy; and 2) it does not capture the disproportionately large revenue risks specific to Ireland. Against this backdrop, a national fiscal rule that ensures macroeconomic stabilization and debt sustainability and enhances the credibility of fiscal policy would help strengthen Ireland's fiscal framework.

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<sup>&</sup>lt;sup>1</sup> Summer Economic Statement July 2021. Department of Finance.

<sup>&</sup>lt;sup>2</sup> Modified Gross National Income (GNI\*) is an indicator designed by the Central Statistical Office (CSO) to measure the size of the Irish economy by excluding MNEs' globalized operations: Modified GNI - CSO - Central Statistics Office.

At the same time, Ireland faces significant spending needs to alleviate supply constraints and address long-term pressures. In the absence of adequate fiscal buffers, the sudden reversal of property-related fortunes during 2009–2012 contributed to significant shortfalls in housing and infrastructure investments thereafter. Ireland needs substantial public investment to upgrade infrastructure, maintain public services, and increase housing supply for a growing population. Addressing these needs while staying within a sound fiscal framework would ensure that additional spending does not just lead to higher costs but delivers real output. A stable fiscal framework would also ensure certainty that is needed for both public and private investment. In addition, long-term pressures related to climate mitigation/adaptation, population aging, defense, and digital transformation also present important risks to the fiscal outlook. Therefore, it is important to take a long-term perspective when designing the framework to safeguard the well-being of future generations by building adequate buffers.

This paper proposes to strengthen Ireland's national fiscal framework with a prudent medium-term fiscal anchor and consistent expenditure ceilings. Although the focus of the study is specifically on Ireland, the methodologies employed can be applied more broadly to inform the design of fiscal rules, provided that enough data are available to quantify pertinent risks. A fiscal anchor based on debt to output ratios can guide fiscal policy towards its objectives. It is necessary because operational rules such as expenditure limits alone do not by themselves guarantee fiscal sustainability: depending on the starting point, they may lock in perpetual structural surpluses or deficits (IFAC 2021). Under the sustainability and stabilization objectives of fiscal policy, an effective fiscal rule should meet several criteria, including simplicity, operational guidance, resilience, and ease of monitoring and enforcement (IMF, 2018). The paper abstracts from discretionary revenue measures in its analysis, and defines expenditure limits as net of such measures, consistent with the definition of the previous government's net spending rule. Although beyond the scope of this paper, broadening the tax base will allow for higher investment spending while reducing Ireland's reliance on CIT and staying within net expenditure limits under the enhanced fiscal framework.

## B. Calibrating a Prudent Debt Anchor

A risk-based framework is applied to calibrate an appropriate medium-term debt anchor for Ireland (IMF, 2018). Customized shocks are modelled and simulated to quantify MNE-related risks, and debt trajectories are simulated with calibrated shocks to determine a prudent debt anchor over the medium term. Specifically, the quantitative approach aims to account for potentially volatile and persistent CIT shocks in simulating a debt "fan chart" for Ireland.<sup>3</sup> A debt anchor can be derived within this framework for given risk tolerance and an exogenously imposed debt benchmark. However, backward-looking statistical models may not fully capture certain tail-risks such as a mass relocation of MNEs. In addition, not all risks can be mitigated, insured, or provisioned for through contingency funds in the budget. It is therefore crucial to create sufficient buffers from the debt benchmark when designing anchors.

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<sup>&</sup>lt;sup>3</sup> The EU fiscal framework and the IMF's DSF also rely on risk-based approaches to gauging medium-term debt sustainability. However, those frameworks do not fully account for the large uncertainty of Ireland's MNE sector. Casey and Cronin (2023) explore how the proposed new framework might look were it better tailored to Ireland. They substitute GNI\* for GDP and adjust for excess corporation tax receipts when determining the stress tests.

Two quantitative approaches (top-down and bottom-up) are employed to derive debt anchors. The top-down approach uses a cross-country FDI panel dataset to gauge risks associated with MNE operations. This approach relies on the historical behavior of FDI in Ireland and other countries to calibrate the FDI shocks Ireland may face in the future. The bottom-up approach uses firm-level data to estimate firm or sector specific risks that could have a significant impact on Ireland's highly concentrated CIT. The two approaches aim to quantify the underlying sources of CIT volatility from two perspectives and can be compared for cross-checking and robustness.

Debt anchors can be specified such that they will encourage the buildup of fiscal buffers and limit the probability that debt will reach high levels that could lead to adverse consequences. Recent evidence suggests that fiscal buffers needed to manage shocks may be larger than previously thought (Caselli et al., 2022). The European Union's 60 percent debt to GDP benchmark, a centerpiece of the EU fiscal framework, serves as a useful upper threshold. The 60 percent benchmark is not a strict upper bound, and the maximum debt carrying capacity for Ireland, while very uncertain, is likely to be higher. However, countries with debt exceeding 60 percent of GDP run the risk of entering the Excessive Deficit Procedure (EDP)<sup>4</sup> and potentially triggering negative market reactions. Therefore, this paper considers that a prudent anchor should be set to keep the debt-to-output ratio below 60 percent with a sufficiently high probability to preserve strong credit ratings associated with low borrowing costs. The difference between the debt benchmark and the estimated debt anchor is the safety buffer. Simulations based on the calibration of MNE shocks form a debt distribution that provides a probabilistic assessment of how likely debt will exceed the benchmark within a given horizon. This approach can ensure that public debt stays around sufficiently prudent levels to build up fiscal buffers (Brunnermeier, Merkel, and Sannikov 2022).

Setting a prudent debt anchor should consider measurement choices for output (GDP vs. GNI\*) and debt (gross vs. net debt). While GDP is commonly used and reported as a denominator that is consistent with cross-country comparison, in the case of Ireland it is not the appropriate metric due to distortions caused by MNE operations. Importantly, a significant share of GDP is generated by MNE activities that may be of transient nature and do not necessarily reflect the domestic economy (IFAC 2021). GNI\*, on the other hand, is a measure of the taxation capacity of domestic national output excluding the MNEs' globalized operations, hence a more appropriate measure to capture the debt servicing capacity of the domestic economy, when excess CIT revenues become unavailable. For example, one study by IFAC finds that GNI\* is statistically better able to explain historical movements in taxes, making it a superior measure for predicting future taxes and assessing public finances (IFAC, 2021). With GNI\* as the denominator, it is essential to account for the part of CIT revenues that is saved as government's liquid assets, by using a net debt concept (gross debt – cash balance). The following sections consider both measures in calibrating debt anchors: gross debt to GDP as used in the EU fiscal rules, and net debt to GNI\*.

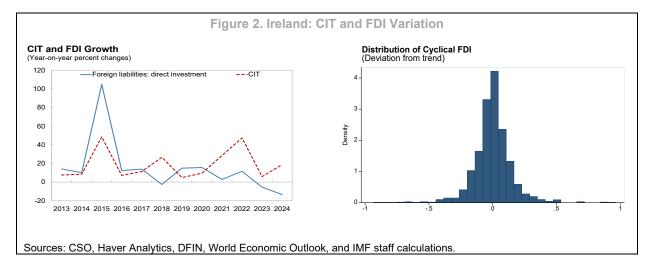
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<sup>&</sup>lt;sup>4</sup> Once the EDP is launched, the European Commission will adopt a recommendation for the member state concerned to take the necessary action within six months to address excessive deficits and restore debt sustainability. If, by the deadline, no effective action has been taken, or the member state does not comply with the recommendation, the Council may impose sanctions.

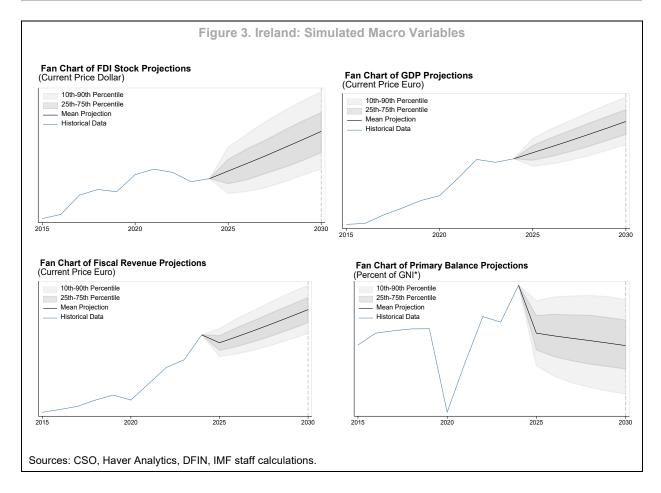
#### Top-down FDI Risk Approach

#### Risks to MNE activities are quantified using historical FDI data of similar small and open economies.

Conceptually, changes in FDI capture a variety of shocks that affect both MNE output and CIT, including global factors, prospects of firm and industry profitability, and domestic and international tax policy. While FDI does not translate one-for-one into CIT in the short run, FDI should determine the levels of MNE activities and profits, and hence CIT, in the steady state. Empirically, Ireland's time series data, characterized by country-specific structural breaks, may not allow for assessing the full extent of potential FDI-related risks that Ireland may face in the future. Therefore, for the purpose of this analysis, FDI risks are calibrated by exploring the historical distribution of FDI among 58 small open economies (see Appendix I). This way, the analysis adds to the standard macro-fiscal shocks based on Ireland's historical data by using variations in other countries' FDI to simulate risks that Ireland may face in the future.



Fan charts for key macro variables with FDI shocks are simulated to illustrate the potential uncertainty around Ireland's future debt paths. While the panel data approach mitigates the effect of the structural breaks in Ireland's data such as the large shift of intangible assets in 2015, past international capital flows were also affected by common trends including the rapid globalization. However, the variation of FDI around the trend should still be able to capture the substantial risks around any baseline projections. For this reason, assumptions on trend growth are made to link FDI, GDP or GNI\*, revenue, and debt in the central scenario over the projection period. CIT is assumed to grow in line with FDI, while other revenues are assumed to grow in line with the domestic economy. In the baseline, both FDI and the domestic economy are assumed to grow by 4.25 percent in nominal terms over the medium-term (Department of Finance, 2023; Central Bank of Ireland, 2024), while nominal expenditure is assumed to grow by 5 percent annually, in line with the previous government's rule. Variations around the central scenario capture upside and downside risks, which are largely symmetric.

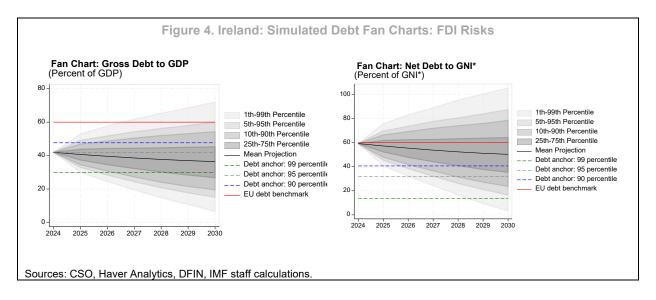


Simulations suggest that debt anchors ranging from 30 to 40 percent of GDP or GNI\* would put Ireland on a comfortable path over the medium term. If gross debt starts at the level of about 40 percent of GDP as of end 2024, there would be 5 percent probability of it exceeding 60 percent of GDP by 2030 under the output

and expenditure growth path discussed earlier. With 40 percent of GDP as the medium-term anchor, the authorities' fiscal consolidation plan laid out in the 2025 budget and medium-term fiscal structural plan, in which the debt-to-GDP ratio is projected to continue to decline over the medium term, is consistent with maintaining debt at the anchor level.

Simulated Debt Anchors						
	Risk tolerance (percent)					
1 5 10						
Gross debt/GDP	31	42	48			
Net debt/GNI*	16	32	42			
Sources: CSO, DFIN, and IMF staff calculations.						

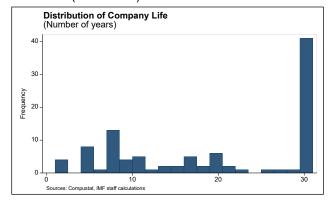
An anchor based on net debt to GNI\*, however, would imply a greater consolidation effort. This is because the size of GNI\* is significantly smaller than that of GDP as a denominator, and the same shock to revenues would have a larger impact on the debt ratio, making it more likely to exceed the 60 percent benchmark. With a risk tolerance of 10 percent, the GNI\*-based net debt anchor is about 40 percent which still provides a comfortable buffer relative to the 60 percent benchmark. The budget balance path in the medium-term fiscal-structural plan entails a gradual reduction in net debt-to-GNI\* ratio toward the debt anchor.



#### **Bottom-up Firm Risk Approach**

Firm and sector-specific shocks are calibrated to account for concentration risks. The top 10 corporate taxpayers account for more than 50 percent of Ireland's total CIT (IFAC 2023). Income shocks to or

operational changes by any of these large taxpayers can have a significant impact on CIT receipts (Department of Finance, 2022). The CIT vulnerability to sector- and firm-specific shocks is estimated using Compustat firm-level data. More specifically, the largest multinational companies in the chemical and IT sectors are selected to estimate a historical distribution of firm profits over the past 3 decades. The estimated distribution of profits is then employed to calibrate firm-specific



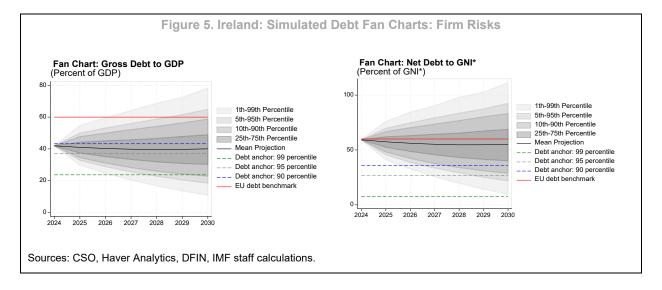
profitability shocks for the top 10 CIT payers in Ireland, and an exit rate of 5 percent per year is assumed to account for firm exits.

Simulation results from the bottom-up approach are broadly in line with the top-down approach. A gross debt to GDP ratio of 43 percent or below would ensure the probability of breaching the 60 percent

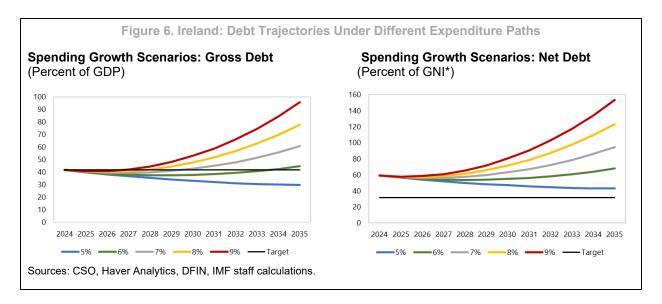
benchmark by 2030 is no more than 10 percent. Using the net debt to GNI\* ratio would again imply a lower debt anchor. To ensure net debt does not breach 60 percent of GNI\* by 2030 with 90 percent probability, the level of net debt would need to be brought down to around 36 percent of GNI\*, compared to the current level of about 60 percent. Overall, both macro panel

Simulated Debt Anchors							
	Risk tolerance (percent)						
	1 5 10						
Gross debt/GDP	24	37	43				
Net debt/GNI*	8	27	36				
Sources: CSO, DFIN, and IMF staff calculations.							

data and firm-specific approaches suggest that a debt anchor set at 40 percent of GNI\* (for net debt) would be prudent enough to ensure that the EU benchmark is not breached in most scenarios.



Annual spending growth targets should be consistent with a prudent debt anchor. The 5 percent annual nominal spending growth assumed in the baseline would ensure a downward path for debt over a ten-year period. A 6-percent expenditure growth annually could still anchor debt over the medium term, although the debt trajectory would already begin to trend up toward the end of the simulation horizon. With higher growth rates this happens sooner, implying that such spending trajectories may not be compatible with long-term fiscal sustainability. Furthermore, increasing expenditure at a constant rate may not be optimal as it fails to capture the economic stabilization role of fiscal policy. And in the very long term, even a 5-percent spending growth may not be sufficient for debt stabilization under the annual revenue growth assumption of 4.25 percent.



## C. Calibrating an Operational Rule

An operational rule is needed to guide near-term fiscal policy. A debt anchor alone would generally be insufficient for effective fiscal management as it lacks a mechanism to guide short-term policy decisions and ensure compliance over time. Thus, a debt anchor must be complemented by an operational rule—such as expenditure or fiscal balance rule—to provide a clear and actionable framework. Ideally, operational rules should be designed such as to help achieve and maintain the debt objective while considering the economic conditions and implementation constraints. Without an operational rule, there are higher risks of procyclical policies and delayed adjustments which could undermine the credibility and effectiveness of the debt anchor.

Expenditure rules are usually considered more effective than budget balance or revenue rules. They are easier to monitor and enforce (governments have direct control on spending) and allow automatic stabilizers to operate. They are also less vulnerable to overly optimistic revenue forecasts than balance-based rules, for example. Indeed, studies have found (Belu Manescu and Bova, 2020) that the procyclical bias of fiscal policy is lower in the presence of expenditure rules. The design matters, however, including coverage, legal basis, monitoring and consequences for non-compliance.

For Ireland, an operational rule in the form of multi-year expenditure ceilings seems appropriate. The ceilings should ideally cover all institutions under the general government classification. Frameworks involving multi-year limits on government spending have been in place in a number of countries. For example, in the Netherlands, after elections, the new government publishes a coalition agreement which, among other things, defines expenditure ceilings for the main budgetary areas (central government, social security and healthcare) for the next four years (Vierke and Masselink, 2017). Similarly, the Danish Budget Act sets four-year expenditure ceilings for the state, municipalities and regions. Ireland could benefit from a framework that provides a clear link between the debt anchor and the annual fiscal outcomes. Given a revenue path consistent with the medium-term macroeconomic projections, the multi-year expenditure ceilings can be informed by considerations for macroeconomic stability while ensuring debt sustainability. In times of positive revenue shocks, expenditure ceilings would help prevent procyclical spending.

Fiscal reaction functions (FRFs) could be a useful conceptual framework to guide the calibration of expenditure ceilings. A typical fiscal reaction function sets the primary balance as a function of the cyclical position of the economy, the level of public debt and the past value of the primary balance (PB):

(1) 
$$p_{t+1} = \beta_0 p_t + \beta_1 \gamma_t + \beta_2 b_t,$$

where  $p_t$  is the primary balance,  $\gamma_t$  is the output gap or some other measure of the economic cycle, and  $b_t$  denotes public debt. For a given revenue path, the above FRF determines uniquely the expenditure trajectory.

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<sup>&</sup>lt;sup>5</sup> See https://www.retsinformation.dk/eli/lta/2012/547 (in Danish).

The specification of an FRF requires information about the economy's cyclical position. A key difficulty comes from estimating the output gap which is not measured directly. While this is a demanding task in general, it is all the more challenging in the case of Ireland where GDP is distorted by activities of the large MNE sector. GNI\* could be used instead but it is available only at the annual frequency and data are published with a considerable delay. Another possible measure is modified domestic demand, but it only focuses on domestic activity and does not cover external operations. As an alternative to the output gap, one can consider the unemployment gap. While this does not circumvent the problem of estimating an unobservable quantity (natural unemployment), it offers the advantage that employment data are timely and subject to less revisions. For the purposes of this paper, natural unemployment is estimated based on a two-state Markov switching model (Appendix I).

FRFs can be either estimated from past data or calibrated based on theoretical frameworks. In an influential study, Bohn (1998) estimates a primary balance (PB) equation with U.S. data and finds strong evidence of a positive response of the PB to the debt ratio. This finding has been confirmed by many other studies. For example, Plödt and Reicher (2015) estimate a debt response coefficient between 0.05 and 0.08 for a panel of euro area economies and a cyclical response coefficient between 0.4 and 0.7. While the econometric approach provides an evidence-based assessment of fiscal behavior, it may be sensitive to data quality and structural changes. This is an important consideration in the case of Ireland, given the large volatility of times series around the global financial crisis and the structural break in 2015. Alternatively, FRFs can be calibrated using economic models that incorporate policy objectives, fiscal multipliers, and sustainability conditions. Calibration is particularly useful when historical data have limitations and the goal is to design forward-looking fiscal rules.

**Optimization techniques can be used to identify parameter values that best align with the policy objectives.** Based on a simple linear model of fiscal policy entailing unemployment gap, primary balance and debt equations, and a quadratic cost function penalizing deviations from target values of the variables of interest, an FRF in the form of (1) is obtained as the feedback solution to an intertemporal optimization problem. To explicitly account for uncertainty in the model (misspecification, shocks, measurement errors), robust control methods have been tried as well (see Appendix I for details).

Optimization-based results suggest that an average nominal expenditure growth around 5 percent over the next ten years, with somewhat higher growth initially, is consistent with sustainability.

Simulations are based on net debt and GNI\* using the 2024 estimates as presented in the Budget 2025 documentation as initial values (outcomes are similar when gross debt and GDP are used and are not reported here).<sup>6</sup> Under plausible assumptions about the

Fiscal Rea	ction Funct	ions	
	û	$\hat{p}$	$\hat{b}$
Baseline	-0.43	-0.57	0.15
Robust (minimax)	-0.73	-1.09	0.29
Robust (stabilizing)	-0.53	-0.89	0.28
Source: IMF staff calculations			

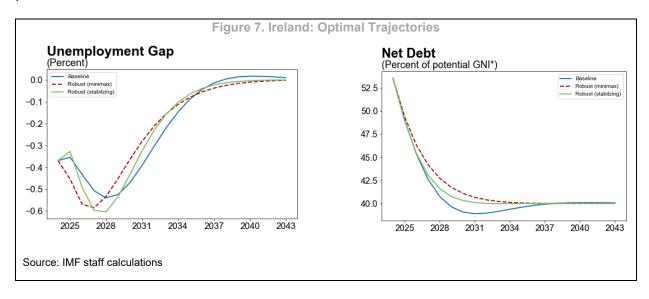
model parameters, the optimized FRFs are successful in stabilizing debt at the anchor (assumed at 40 percent), and the unemployment gap at zero (Figure 7). Different methodologies imply varying speeds of

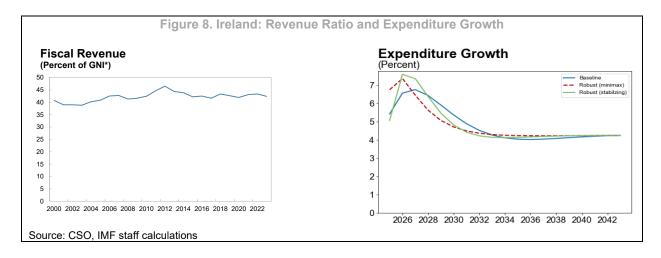
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<sup>&</sup>lt;sup>6</sup> See "Budget 2025: Economic and Fiscal Outlook", Department of Finance, October 2024

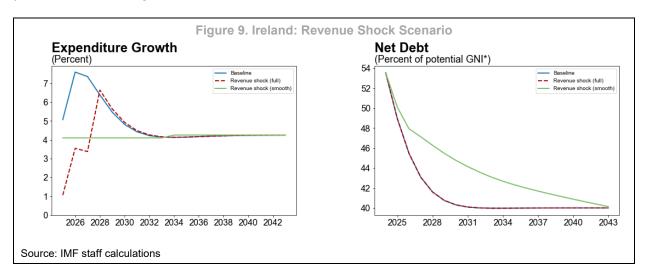
adjustment, reflecting differences in the FRF coefficients (text table). Qualitatively, however, they are similar and imply a gradual unwinding of the primary surplus which initially is associated with negative unemployment gaps. Given the model-based primary balance path and assuming a constant ratio of fiscal revenue to GNI\*, which is broadly supported by the data (Figure 8), one can recover an expenditure growth path consistent with the policy objectives.

While the time profile of spending growth varies, the average rates are similar across methods. They range between 6 ¼ and 6 ½ in the first 5 years and around 5½ percent in the first 10 years. After that, the FRFs prescribe that expenditure should grow at the same rate as nominal revenue. The higher spending growth in the initial years is primarily driven by the favorable current fiscal position. There is clearly a trade-off between the need to close the negative unemployment (positive output) gap on the one-hand, which implies tighter fiscal policy, and to reduce the substantial primary surplus on the other which requires higher expenditure. The model helps to resolve this tension based on the weights assigned to the two objectives. If cyclical considerations are of main concern, the weight on the unemployment gap can be made much larger relative to that of debt. In such a scenario, the closing of the unemployment gap would occur faster. The size of the fiscal multiplier plays a role for the speed of adjustment as well. In the illustrative simulations presented here, the weight on the cyclical position is higher than that of debt but not significantly higher, resulting in a more persistent gap. This, however, offers an opportunity to step up productive public investment in the initial period.





A permanent shock to revenue would require the use of fiscal buffers to smooth primary expenditures toward the new equilibrium. For illustrative purposes it is assumed that the CIT windfall estimated at about 4.5 percent of GNI\* disappears over the course of three years, resulting in a drop in the revenue ratio which falls to about 38 percent and remains at that level afterwards. Two scenarios for expenditure are presented in Figure 9—one where spending adjusts fully to achieve the primary balance prescribed by the model-based rule and one involving smoothing of expenditure. In the first case, nominal expenditure only increases between 1 and 3 ½ percent in the first three years but the growth rate catches up later<sup>7</sup>, while in the smoothing scenario it grows at about 4 percent per annum. This, however, comes at the cost of postponing the moment in time when debt reaches the anchor; it only happens toward the end of the simulation horizon. Intermediate scenarios are, of course, possible. It is important to note that the shock scenario presented above does not take into account the possible use of financial assets outside of the definition of net debt, such as those in the two savings funds, for example. Drawing on these assets would cushion the effect of the shock in the initial years while still allowing the debt anchor to be achieved within a reasonable timeframe.



<sup>&</sup>lt;sup>7</sup> In level terms though, expenditure remains lower – about 90 percent of that in the baseline.

INTERNATIONAL MONETARY FUND

## D. Conclusion

A well-designed fiscal framework is essential for maintaining long-term fiscal sustainability while allowing for economic flexibility. Specifically, a prudent debt anchor combined with a net spending rule as an operational target seems an appropriate choice in light of Ireland's circumstances. A debt anchor of about 40 percent provides a fiscal buffer to absorb economic shocks while maintaining a prudent level of indebtedness. In determining the appropriate debt metric, both gross debt to GDP and net debt to GNI can be considered. Given that GNI reflects the domestic economy's size more accurately, it may be a more suitable denominator for defining fiscal anchors. Setting the operational rule in terms of expenditure (net of discretionary revenue measures) has multiple advantages, including limiting procyclicality, enhancing transparency and facilitating monitoring and enforcement. In terms of design, expenditure ceilings, informed by cyclical conditions and debt levels, would provide a stable and predictable framework.

For the fiscal framework to be credible, flexible, and transparent, several additional elements need to be considered:

- Escape clause and triggering conditions: National fiscal rules should be given legislative status. An
  escape clause should allow for temporary deviations from fiscal rules in response to exceptional
  circumstances, such as severe economic downturns or natural disasters. Clear and objective conditions
  for activating the clause must be predefined to prevent misuse.
- Correction mechanism: A well-defined correction mechanism should specify remedial actions if fiscal rules
  are breached. This could include automatic spending adjustments or requirements for corrective budget
  plans.
- Independent fiscal oversight: The Irish Fiscal Advisory Council should play a critical role in monitoring
  compliance with fiscal rules, assessing budget forecasts, and ensuring that fiscal policy remains credible.
- Periodic review: Regular reviews should be conducted to ensure that fiscal rules remain well-calibrated to
  macroeconomic and fiscal conditions. Adjustments may be necessary as macro-fiscal conditions and
  structures evolve.
- Link to annual budgets and medium-term fiscal framework (MTFF): The fiscal rule should be embedded in
  the annual budgeting process and aligned with the MTFF to ensure consistency between short-term fiscal
  policies and long-term objectives. Satisfying the operational rule should imply that the fiscal anchor is
  achieved.
- The two savings funds established in 2024—the Future Ireland Fund and the Infrastructure, Climate and Nature Fund—should be integrated into this strengthened fiscal framework.

Implementation of multi-year expenditure ceilings entails various design choices. Some of these include:

- Nominal vs. real: Spending limits can be set in nominal or in real terms. Nominal ceilings offer a number of
  advantages such as simplicity and transparency but may fail in an unstable inflationary environment.
   Setting the limits in real terms ensures that expenditure retains purchasing power but can be more difficult
  to communicate and requires choice of an appropriate price index (e.g., HICP, CPI or GDP deflator).
- Levels vs growth rates. Defining the ceilings in terms of growth rates has the benefits of ease of
  communications and continuity, and with full compliance the two approaches are equivalent. However, if
  the ceiling is breached in the fiscal outturn of a given year, the growth rates in subsequent years would
  require an adjustment to stay on the same fiscal path, given the higher base.
- Treatment of cyclical expenditure: Automatic stabilizers, such as unemployment benefits, could be
  excluded from expenditure ceilings to prevent unintended constraints on countercyclical fiscal policy.
- Public investment: In times of fiscal consolidation, the burden often falls on capital outlays. For a country with significant investment needs like Ireland, this would not be an optimal outcome. Consideration could be given to an approach that defines an overall expenditure ceiling consistent with the debt anchor and separate subceilings for current and capital expenditure within the overall limit. This, along with strengthened public investment management, would help limit unintended crowding out of capital spending. Excluding public investment from the expenditure ceiling, e.g., a golden rule or a rule that covers only current expenditures is not recommended as it increases the risk of creative accounting, selection of projects with low social returns, and excessive borrowing, given the weakened link to debt sustainability (IMF, 2018). Instead, a slower adjustment toward the debt anchor could give policy space for upfront growth-enhancing investment in a transparent manner.
- Link to financial assets: Ireland has substantial financial assets (about 40 percent of GNI\*). Part of these assets is held in cash instruments which will be counted toward the net debt definition should a net debt anchor be adopted. Still, a significant part will remain outside of the scope of the fiscal rule and will serve as a buffer and financing source for future spending pressures, e.g., related to ageing or climate change.<sup>8</sup> In particular, the two savings funds should play an important role in an integrated fiscal strategy.

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<sup>&</sup>lt;sup>8</sup> A fiscal rule comprising explicit targets on net worth would create a number of operational and conceptual challenges. First, net worth is a volatile indicator due to frequent revisions of the discount rate used to value nonfinancial assets, making it difficult to assess confidently whether the rules are met or not. Second, the concept may not be straightforward to the public. Third, a net worth rule, on its own, would mean there are no limits on borrowing for investment and it would also risk gold plating, as the quality of investment would not be taken into account. Fourth, net worth is an imperfect indicator of fiscal sustainability since non-financial assets are mostly illiquid; they cannot realistically be sold to meet immediate financing needs. A large stock of assets that a government cannot sell will not translate into market confidence that the government can afford to borrow more without limit.

## References

- Belu Manescu, C. & Bova, E. (2020). National Expenditure Rules in the EU: An Analysis of Effectiveness and Compliance, *European Commission Discussion Paper* 124.
- Bohn, H. (1998). The Behavior of US Public Debt and Deficits, *The Quarterly Journal of Economics*, Vol. 113, No. 3, pp. 949–963.
- Boyd, S., Feron, E., Ghaoui, L. & Balakrishnan, V. (1994). Linear Matrix Inequalities in System and Control Theory, SIAM Studies in Applied and Numerical Mathematics, vol. 15.
- Brunnermeier, M.K., Merkel, S.A. and Sannikov, Y., 2022. Debt as safe asset (No. w29626). National Bureau of Economic Research.
- Caselli, F., Davoodi, H., Goncalves, C., Hong, G.H., Lagerborg, A., Medas, P., Nguyen, A. and Yoo, J. The Return to Fiscal Rules. IMF SDN/2022/002. 2022
- Central Bank of Ireland. Long-Term Growth Prospects for the Irish Economy. December 2024.
- Conefrey, T., Lawless, M. & Linehan, S. (2015). Developments in the Irish Labour Market during the Crisis: What Lessons for Policy?, *Journal of the Statistical and Social Inquiry Society of Ireland*, Vol. XLIV.
- Department of Finance. Summer Economic Statement. July 2021.
- Department of Finance. Horizon Scanning calibrating medium to long-term economic projections. October 2023.
- Eddie Casey and Brian Cronin. Ireland's spending rule and the third wave of the EU's fiscal rules (2023).
- Hansen, L. & Sargent, T. (2008). Robustness, Princeton University Press.
- Hatchondo, J. C., L. Martinez, & Roch, F. (2022a). Fiscal Rules and the Sovereign Debt Premium, *American Economic Journal: Macroeconomics*, Vol. 14. No 4.
- Hatchondo, J. C., L. Martinez, & Roch, F. (2022b). Numerical Fiscal Rules for Economic Unions: The Role of Sovereign Spreads, *Economic Letters*, Vol. 210.
- International Monetary Fund. How to Calibrate Fiscal Rules A Primer. 2018.
- Irish Fiscal Advisory Council. Fiscal Assessment Report June 2021. December 2021.
- Irish Fiscal Advisory Council. Fiscal Assessment Report June 2023. June 2023.
- Khlebnikov, M., (2011). Optimization of Linear Systems Subject to Bounded Exogenous Disturbances: The Invariant Ellipsoid Technique, *Automation and Remote Control*, Vol. 72, No. 11, pp. 2227–2275.
- Monacelli, T., Perotti, R. & Trigari, A. (2010). Unemployment Fiscal Multipliers, *Journal of Monetary Economics*, pp. 531–553.
- Plödt, M, & Reicher, C. (2015). Estimating Policy Reaction Functions: The Role of Model Specification, *Journal of Macroeconomics*, Vol. 46, pp. 113–128.
- Vierke, H. & Masselink, M. (2017). The Dutch Budgetary Framework and European Fiscal Rules, *European Economy Economic Briefs* 027, DG ECFIN, European Commission.

## **Appendix I. Technical Appendix**

## A. FDI Risks

#### Sample and Data

Ireland's relatively short historical data, coupled with a large financial crisis and one-off MNE asset relocations in 2015, may not provide a good guide for the risks the country will face in the future. Therefore, a cross-country panel dataset is used to capture the range of risks entailed in MNEs' FDI activities. The model assumes that each country has a fixed effect  $u_i$  drawn from a Gaussian distribution. The innovations  $\epsilon_{it}$  are allowed heteroskedasticity but are assumed to be independent and Gaussian, which can be validated by the histogram of the cyclical FDI component below. All countries are assumed to have the same autoregression matrix B.

A cross-country panel dataset covering 212 countries and regions from 1990 to 2023 is constructed for this exercise. Inward FDI stock data, expressed in nominal US dollars, is sourced from countries' official external sector statistics. Domestic macroeconomic and fiscal variables, including GDP, GNI (GNI\* for Ireland), GDP deflator, fiscal revenue, government spending, primary balance, and gross domestic debt are sourced from the IMF's WEO database. These variables are all measured in current prices in local currency. We estimate the model in real terms and deflate GNI of sample countries using their GDP deflators. Other variables such as revenues, spending and gross debt are only used for Ireland's debt simulations.

A sample of small and open economies is selected based on their pre-Covid GDP and openness. We select countries in the 30<sup>th</sup> to 90<sup>th</sup> percentiles for GDP size and the 50<sup>th</sup> to 99<sup>th</sup> percentiles for openness (measured by FDI-GDP ratio) distribution, resulting in a sample of 58 countries (Table 1). This sample selection step ensures that the countries used in the analysis are sufficiently comparable to Ireland and that Ireland may face similar FDI risks to those experienced historically by these countries.

Country	No. of obs	FDI/GDP	GDP (US bn)	Country	No. of obs	FDI/GDP	GDP (US bn)
Albania	16	0.6	23	Lithuania	27	0.5	78
Austria	18	0.6	516	Luxembourg	21	48.2	86
Bahrain	27	0.8	44	Malaysia	22	0.6	416
Belgium	28	1.4	632	Malta	24	13.6	18
Bosnia and Herzegovina	19	0.4	27	Morocco	21	0.5	141
Botswana	28	0.3	19	Mozambique	18	2.7	21
Bulgaria	25	0.7	102	Myanmar	17	0.4	69
Cambodia	27	1.2	42	Netherlands	27	4.5	1118
Chile	26	0.9	336	New Zealand	23	0.4	249
Colombia	28	0.7	364	Norway	24	0.5	486
Costa Rica	18	0.7	87	Panama	28	0.8	83
Croatia	25	0.5	83	Peru	28	0.5	267
Cyprus	20	14.3	32	Poland	28	0.5	812
Denmark	28	0.6	405	Portugal	27	0.8	287
Egypt	19	0.4	394	Romania	26	0.4	346
El Salvador	26	0.4	34	Serbia	15	0.8	75
Estonia	27	1.3	41	Singapore	22	5.2	501
Finland	28	0.5	300	Slovenia	28	0.4	68
Georgia	24	0.8	31	South Africa	27	0.3	381
Honduras	19	0.6	34	Sudan	18	0.8	35
Hong Kong	26	6.1	381	Sweden	28	1.0	593
Hungary	27	2.3	213	Switzerland	28	2.2	885
Iceland	26	0.4	31	Thailand	27	0.6	515
Ireland	18	3.4	546	Trinidad and Tobago	12	0.3	28
Israel	28	0.5	510	Tunisia	27	0.9	46
Jamaica	18	1.0	19	Uganda	23	0.3	48
Jordan	22	0.8	49	Ukraine	23	0.3	177
Kazakhstan	22	0.7	263	Uruguay	23	0.8	77
Latvia	28	0.7	44	Zambia	17	0.4	28

## **Model Specification**

**Public debt projections are generated using a two-stage approach.** The first stage is to estimate the following panel VAR (1) model for GNI and FDI stock. We assume that both variables grow along a deterministic trend in the medium run, but their growth is subject to shocks and might deviate from the trends, i.e., they are trend-stationary. We estimate a model of the cyclical components of GNI and FDI, denoted as  $y_{it}^*$  and  $i_{it}$ . We pass GNI and FDI to a HP-filter for each country, and then calculate  $y_{it}^*$  and  $i_{it}$  as the percentage deviation from the filtered trends.

$$\begin{bmatrix} y_{it}^* \\ i_{it} \end{bmatrix} = u_i + B \begin{bmatrix} y_{it-1}^* \\ i_{it-1} \end{bmatrix} + \epsilon_{it}$$

GNI and FDI series are assumed to be trend-stationary for all selected countries in the sample. The trend GNI growth and FDI growth vary across years due to domestic or external shocks, but the VAR model assumes that the impact of shocks diminishes over time. To test the validity of the VAR model in this context, we run the Im-Pesaran-Shin panel unit root test for the FDI data and the result rejects the null hypothesis that all countries' FDI time series have unit roots. The alternative hypothesis of the IPS test only suggests that at least one country's FDI is trend-stationary. For model selection's purpose, we also run a Levin-Lin-Chu (LLC) test with the alternative hypothesis being that all countries' FDI series are stationary. The test generates a p-value of 0.00, rejecting the null hypothesis that all countries have unit roots. In addition, Dickey-Fuller (DF) unit root tests are run for individual countries. The null hypothesis of unit root is rejected at the 95 percent confidence level for most countries.

While the trend-stationarity property of VAR models is suitable for many countries, it may not always be appropriate for the Irish economy. Shocks to the operations of MNEs and the associated CIT revenues in Ireland could be permanent. For example, the DF test fails to reject unit root for Ireland. This is likely to be driven by the turmoil the country experienced during the Global Financial Crisis period and a large one-off transfer of intangible assets in 2015, which the model and calibrations cannot fully account for. Therefore, it is necessary to revisit parameter values of this exercise when significant structural shifts occur.

The second stage of the model translates the process of FDI to GDP and CIT from MNEs, and further to the evolution of fiscal revenue and debt. We start by assuming that FDI will translate to capital in domestic production one-for-one, whether in the form of physical capital or intellectual property.

• The MNEs' production function takes the Cobb-Douglas form with  $\alpha$  being the capital income share, with the capital share close to one in the case of intellectual property.

$$Y_{it} = A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha}$$
.

Labor supply is assumed to be perfectly elastic at fixed wage and matches increases in capital as FDI
settles in Ireland. Changes of output are proportionate to the percentage changes of capital and labor as
shown below, where the lower-case letters refer to the logarithm of the upper-case variables.

$$\Delta y_{it} = \Delta k_{it} = \Delta l_{it}$$
.

• To translate changes in output to changes in taxable income, we assume that in a monopolistically competitive environment, firms' profits are in proportion to their output with their margins determined by the demand elasticity  $\eta > 1$ . Consequently, changes in taxable corporate income are proportionate to changes in capital. Further, percentage changes in CIT from MNEs are also proportionate to changes in FDI. Denoting profits of firm i by  $\Pi_{it}$  and its corporate income tax payment by  $T_{it}$ , we have

$$\Delta au_{it}^c = \Delta \pi_{it} = \frac{1}{\eta} \Delta y_{it} = \frac{1}{\eta} \Delta k_{it}$$

• Labor is assumed to match FDI growth, which will generate more tax revenues such as personal income tax ( $\Delta \tau_{it}^p = \Delta l_{it}$ ). This assumption of elastic labor might be strong in that we assume no crowding out effect to the domestic labor market. It could be partially justified by a segmented labor market whereby MNEs

reallocate human resources from abroad when they expand in Ireland instead of hiring on the local labor market. Relaxing this assumption with a less than one-for-one response of labor and output to FDI would not change the main conclusions from the analysis.

On aggregate, the percentage changes of corporate income taxes are the share-weighted sum of
percentage changes of domestic CIT and MNE CIT. The percentage change of MNE CIT is the shareweighted sum of CIT contribution of individual MNEs which is proportional to their individual FDI.

$$\Delta \tau_t^c = s^{c,dom} \Delta \tau_t^{c,dom} + s_i^c \Delta \tau_{it}^c = s^{dom} \Delta \tau_t^{c,dom} + \frac{1}{n} s_i^c \Delta k_{it} = s^{c,dom} \Delta \tau_t^{c,dom} + \frac{1}{n} s^{c,FDI} \Delta k_t$$

We denote the share of domestic firms in total CIT as  $s^{c,dom}$  and the share of MNEs as  $s^{c,FDI}$ . They sum up to one by construction. Similarly, we define  $s^{p,dom}$  and  $s^{p,FDI}$  as the share of the two sectors in personal income tax. The percentage changes of personal income taxes can be calculated using

$$\Delta \tau_t^p = s^{p,dom} \Delta \tau_t^{p,dom} + s_i^p \Delta \tau_{it}^p = s^{p,dom} \Delta \tau_t^{p,dom} + s^{p,FDI} \Delta k_t$$

We assume that domestic firm CIT and PIT track GNI star one for one  $\Delta \tau_t^{p,dom} = \Delta \tau_t^{c,dom} = \Delta y_t^*$ . Then, the total percentage change of fiscal revenue is

$$\Delta \tau_t = \mu \Delta \tau_t^c + (1 - \mu) \Delta \tau_t^p = (\mu s^{c,dom} + (1 - \mu) s^{p,dom}) \Delta y_t^* + \left(\frac{\mu}{\eta} s^{c,FDI} + (1 - \mu) s^{p,FDI}\right) \Delta k_t$$

where  $\mu$  is the share of tax revenue from corporate income.

The difference between GDP and GNI star is assumed to be the value added generated by MNEs' operations, thus its growth rate is assumed to track FDI growth, i.e.,

$$\Delta \ln(Y_t - Y_t^*) = \Delta k_t$$

· Finally, given the government's budget primary balance

$$PB_t = T_t - G_t$$

domestic debt evolves following

$$B_t = (1 + R_t)B_{t-1} - PB_t$$
.

#### **Parameter Assumptions for Simulations**

We simulate the cyclical component of FDI and GNI according to the VAR(1) estimates and add them to pre-determined trends that assumes a 2½ percent real growth rate for GDP, GNI\* and FDI stock. We also assume a trend growth of 3 percent for real government spending. The interest rate for government debt and inflation are both assumed to be 2 percent per annum.

## B. Concentration Risks

#### Sample and Data

Firm data is from CompStat. We use the North America Fundamental Annual dataset for corporate financial information and the Index Monthly Prices dataset for delisting related information. We selected the top 100 US firms in the tech and pharmaceutical industry (GIND code 502030, 352020, 451030, 452020, 452010) in 1990 according to their average revenue in 1990. For companies that got delisted during the sample period, we only regard companies as bankrupt if their delisting reason is "bankruptcy". We treat "merger and acquisition" as survival during the whole sample period. We also collected the top 10 MNEs' revenue in Ireland in the base year from local sources. For simulations, we assume a real log trend growth of 2¼ percent for all the MNEs. The rest of the assumptions remain the same as the FDI risk model.

Company	No. of obs Asse	ets (mn USD)	Company	No. of obs Ass	ets (mn USD)	Company	No. of obs Ass	ets (mn USD)
ADC TELECOMMUNICATIONS INC	21	1475	MAXTOR CORP	16	2178	WARNER-LAMBERT CO	10	11442
ALPHARMA INC -CL A	18	1288	MEASUREX CORP	7	337	WESTERN DIGITAL CORP	35	24188
AST RESEARCH INC	7	831	MENTOR GRAPHICS CORP	27	2261	XEROX HOLDINGS CORP	34	1000
ALLEN TELECOM INC	13	530	MERCK & CO	34	106675	NOVELL INC	21	222
AMDAHL CORP	7	1596	STREAMLOGIC CORP	6	180	GREATE BAY CASINO CORP	8	1
WYETH	19	44032	PHARMACIA CORP	13	18517	SUN MICROSYSTEMS INC	20	1123
ANDREW CORP	18	2351	MOTOROLA SOLUTIONS INC	34	13336	MICROSOFT CORP	35	51216
APPLE INC	35	364980	NCR VOYIX CORP	34	4990	ORACLE CORP	34	14097
APPLIED MAGNETICS CORP	9	300	PFIZER INC	34	226501	GENICOM CORP	9	23
AUTODESK INC	34	9912	AGILYSYS INC	16	762	KASPIEN HOLDINGS INC	33	4
BBN CORP	7	249	MINOLTA-QMS INC	10	151	SILICON GRAPHICS INC	19	41
BRISTOL-MYERS SQUIBB CO	34	95159	QUANTUM CORP	9	2484	MOSLER INC	10	17
COMPAQ COMPUTER CORP	12	23689	REYNOLDS & REYNOLDS -CL A	16	956	SEQUENT COMPUTER SYSTEMS INC	9	79
CA INC	28	13060	RHONE-POULENC RORER	7	8768	KOMAG INC	17	97
CRAY RESEARCH	6	978	VALEANT PHARMACEUTICALS -OLD	20	1305	CADENCE DESIGN SYSTEMS INC	34	566
DSC COMMUNICATIONS CORP	8	2440	SCHERER (R P)/DE	8	822	WIRELESS WEBCONNECT INC	8	8
DATA GENERAL CORP	9	1065	SCHERING-PLOUGH	19	28117	BORLAND SOFTWARE CORP	19	25
DIEBOLD NIXDORF INC	34	4162	SCIENTIFIC-ATLANTA INC	16	2590	CONNER PERIPHERALS	5	146
DIGITAL EQUIPMENT	8	9693	SEAGATE TECHNOLOGY-OLD	11	7167	DELL TECHNOLOGIES INC	34	8208
ACTERNA CORP	13	406	STERLING SOFTWARE INC	10	1230	ENTERASYS NETWORKS INC	15	29
EASTMAN KODAK CO	34	2355	STORAGE TECHNOLOGY CP	15	2408	ALLERGAN INC	25	1241
GENERAL INSTRUMENT CORP	9	2188	STRATUS COMPUTER INC	8	750	MEMOREX TELEX NV -ADR	6	26
SALIENT 3 COMMUN INC -CL A	10	125	TANDEM COMPUTERS INC	7	1745			
HP INC	35	39909	TELLABS INC	23	1638			
INTERGRAPH CORP	16	621	3COM CORP	19	1815			
JOHNSON & JOHNSON	34	167558	TYLER TECHNOLOGIES INC	34	4677			
LILLY (ELI) & CO	34	64006	FRANKFORT TOWER INDS INC	12	132			
LOTUS DEVELOPMENT CORP	5	904	PHARMACIA & UPJOHN INC	10	10698			
M/A-COM INC	5	309	ELOT INC	9	110			
MARION MERRELL DOW INC	5	4100	WANG LABS INC	9	2249			

#### **Model Specification**

Public debt projections under firm concentration risks are generated using a two-stage approach. We assume that individual firm's cyclical component of log revenue, denoted as  $rev_{it}$ , and pre-tax income to revenue ratio, denoted as  $pi_{it}$  jointly follow the following panel VAR(1) model

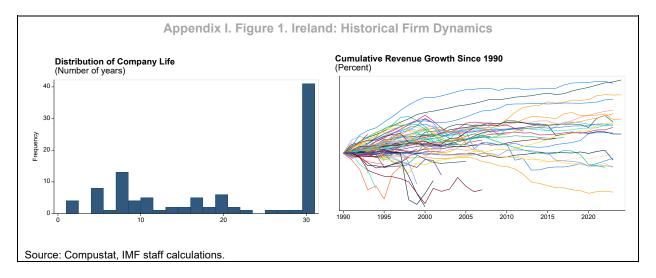
$$\begin{bmatrix} rev_{it} \\ pi_{it} \end{bmatrix} = u_i + B \begin{bmatrix} rev_{it-1} \\ pi_{it-1} \end{bmatrix} + \epsilon_{it}$$

Firm dynamics are known to be non-stationary. Top firms can exit even without a significant decline of revenue. To measure such downside risks, we add an independent Poisson exiting rule on top of the trend stationary dynamics. In the baseline model, all top firms in 1990 are prone to bankruptcy risk with a constant

arrival rate  $\lambda$ . We then match the average duration of top firms from 1990 to 2020 in the data. With constant arrival rate  $\lambda$ , the average duration is

$$\mathbb{E}[min(T,30)] = \frac{1}{\lambda} (1 - e^{-30\lambda}).$$

The following figure shows the revenue trajectory of the top 52 firms in the tech and pharmaceutical industries since 1990. The left panel shows that bankruptcy events are relatively evenly distributed across the 30 years, except during two major financial crises. The right panel shows that bankruptcy is more likely to happen when revenue declines, but there are also a few top firms that went bankrupt on a steadily growing revenue path. Our baseline model chooses a constant survival rate, but it could potentially be a function of firm specific and macro conditions. Our choice of  $\lambda=0.045$  matches the average duration of 17 years in the sample.



The top 10 MNEs contributed over 95 percent of total CIT from MNEs in Ireland. We assume that the firms' pre-tax income processes are independent, simulate the paths for each firm and aggregate them. The simulated pre-tax income of individual firm i is

$$PI_{it} = (\overline{p}i_i + \widehat{p}i_{it})exp(\widehat{rev}_{it} + \overline{rev}_{it}) * exist_{it}.$$

Where  $\bar{p}_i$  is the historical average pretax income to revenue ratio,  $\overline{rev}_{it}$  is pre-determined log trend of individual firm revenue growth.  $exist_{it}$  indicates if the firm remains in the market. Every period, an individual firm survives with probability  $1-\lambda$  or exits with probability  $\lambda$ . The hat variables are simulated variables based on the VAR estimates. Total pre-tax income from MNEs is thus the sum of  $PI_i$  over the surviving top ten MNEs, divided by the top 10 pre-tax income share in all MNEs in 2024.

$$\Pi_{\mathsf{t}} = \sum_{i=1}^{10} PI_{it}.$$

If we denote the growth rate of total MNE pre-tax income as  $\Delta \pi_t$  and keep the same assumption that

$$\Delta \tau_{it}^c = \Delta \pi_{it} = \frac{1}{\eta} \Delta y_{it} = \frac{1}{\eta} \Delta k_{it},$$

we can calculate the percentage changes of corporate income taxes using

$$\Delta \tau_t^c = s^{c,dom} \Delta \tau_t^{c,dom} + s^{c,FDI} \Delta \pi_t,$$

and similarly, for the growth of personal income taxes. The rest of the procedure remains the same as in the FDI risk model.

#### **Revenue Growth Model**

While it is plausible that countries' FDI and GNI are trend-stationary, trend-stationarity may not apply to each individual firm's revenue, given the more granular and transient nature of firm dynamics. An alternative assumption is that a firm's growth has a unit root and its revenue growth rate and income to revenue rate jointly follow a VAR process.

$$\begin{bmatrix} revg_{it} \\ pi_{it} \end{bmatrix} = u_i + B \begin{bmatrix} revg_{it-1} \\ pi_{it-1} \end{bmatrix} + \epsilon_{it}$$

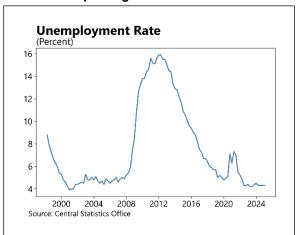
With other assumptions unchanged, we run a simulation for Ireland's domestic debt with the unit-root firm growth model. As the model is not trend-stationary, the simulation features more extreme upside and downside risks. It is also worth noting that the asymmetry between upside and downside risks will be significant in this model. The reason is that a growing debt path is caused by the decline of top MNEs' revenues, which also cause GDP to fall, amplifying the rise of the debt to GDP ratio. Conversely, a declining debt path is consistent with top MNEs' revenue growth, which contributes to higher GDP. We run a stationarity test for top companies' revenue growth. This type of model tends to generate even wider fan charts and lower debt anchors.

## C. Fiscal Reaction Function

#### **Estimating the Unemployment Gap**

Various tools could be used to estimate the natural rate of unemployment, ranging from univariate statistical filters (HP, bandpass) to full-fledged economic models exploiting structural relations

between variables. For the purposes of this note, a simple Markov switching model of unemployment is assumed. Historically, the unemployment rate dynamics have alternated between periods of low (and relatively stable) unemployment and high unemployment during crisis episodes (text figure). This suggests that a Markov model with two states may be an appropriate representation of the data generating process. Specifically, it is posited that actual unemployment fluctuates around a constant which switches between two different regimes— "low" and "high"—with certain probabilities. Formally,



$$u_t = \bar{u}_{s_t} + \epsilon_t,$$

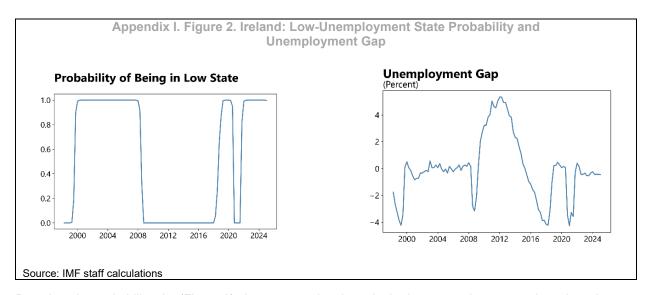
$$\epsilon_t \sim N(0, \sigma_{s_t}),$$

where  $u_t$  is the observed unemployment rate and  $\bar{u}_{s_t}$  is a constant term (the parameter of interest) which switches between two values—  $u_0$  and  $u_1$ , denoting the states of low and high unemployment, respectively. Thus, the model yields a constant natural unemployment rate which, while perhaps overly simplistic, offers the advantage of being straightforward to use and communicate. Regime changes occur with probabilities given by the transition matrix:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ 1 - p_{00} & 1 - p_{10} \end{bmatrix}.$$

The Markov switching model results suggest plausible estimates of equilibrium unemployment. Table 3 displays the results from estimating the above model. The unemployment rate in the low-unemployment state is estimated at about 4 ¾ percent, while the respective value in the high-unemployment regime is slightly over 10 ½ but with a large variance (note that the model specification allows for the variances to switch). These estimates are statistically significant and appear reasonable, given Ireland's history. The transition matrix suggests 96 percent probability of staying in the low-unemployment regime in the next period if the economy in this regime in the current period.

Dep. Variabl	.e: ILO	Unemployment	: Rates (15	 - 74 years)	Seasonally)	/ Adjusted)	No. Observations:	108
Model: Date: Time: Sample:				•	Tue, 1	Regression 17 Jun 2025 14:20:12 04-01-1998 01-01-2025	Log Likelihood AIC BIC HQIC	-186.469 384.93 401.030 391.462
Covariance 1	ype:	Regime	e O paramete	rs		approx		
	coef	std err	z	P> z	[0.025	0.975]		
const sigma2	4.7378 0.1868	0.075 0.057 Regime	62.915 3.296 1 paramete	0.000 0.001 rs	4.590 0.076	4.885 0.298		
=========	coef	std err	Z	P> z	[0.025	0.975]		
const sigma2	10.5727 12.2800	0.552 2.497 Regime tra	19.166 4.918 ansition par	0.000 0.000 ameters	9.491 7.386	11.654 17.174		
========	coef	std err	z	P> z	[0.025	0.975]		
p[0->0] p[1->0]	0.9551 0.0491	0.027 0.029	35.948 1.678	0.000	0.903 -0.008	1.007 0.107		



Based on the probability plot (Figure 2), the economy has been in the low-unemployment regime since the recovery from the financial crisis, except for a short period during the Covid-19 pandemic (Figure 2, left panel). The unemployment gap is defined as  $\widehat{u_t} = u_t - \overline{u}_{s_t}$ . Recently, it has been negative in the range of -0.3 to -0.6 percentage points which, using the Okun's law with a coefficient of -0.3 (Conefrey et al., 2015), translates into positive output gaps between 1 and 2 percent.

#### A Simple Model of Fiscal Policy

A simple model of fiscal policy which features key characteristics necessary for the derivation of a fiscal reaction function is presented below.

• *Unemployment gap.* A key assumption in what follows is that the unemployment gap can be influenced by fiscal policy. Formally, this can be expressed as follows:

$$\hat{\mathbf{u}}_{t+1} = \rho \hat{\mathbf{u}}_t + \alpha \mathbf{\eta}_t$$

where  $\eta_t$  denotes the fiscal impulse, defined as the change in the cyclically adjusted balance expressed as a ratio to potential output  $(\bar{Y}_t)$  and  $\rho$  is an autoregressive parameter.

Cyclically adjusted balance. The cyclically adjusted balance is obtained as follows (see Fedelino et al., 2009). First, the nominal primary balance PB<sub>t</sub> is decomposed into a cyclically adjusted part which represents discretionary policy and a cyclical component which is driven by the economic cycle:

$$PB_t = PB_t^{ca} + PB_t^{c}$$

For the calculation of the cyclically adjusted balance, it is assumed that the revenue (R) elasticity equals one and the expenditure (G) elasticity equals zero, a fairly standard assumption:

$$PB_t^{ca} = R_t \frac{\overline{Y}_t}{Y_t} - G_t$$

and thus, the cyclical component is

$$PB_{t}^{c} = PB_{t} - PB_{t}^{ca} = R_{t} - G_{t} - R_{t} \frac{\bar{Y}_{t}}{Y_{t}} + G_{t} = R_{t} \left( 1 - \frac{\bar{Y}_{t}}{Y_{t}} \right) = \frac{R_{t}}{Y_{t}} (Y_{t} - \bar{Y}_{t}).$$

After multiplying and dividing the right-hand side by potential output, and defining the output gap as

$$\gamma_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$$
 ,

$$P_t^c = \frac{R_t}{Y_t} \gamma_t \bar{Y}_t$$
.

The revenue to GDP ratio is relatively stable in the case of Ireland, so a simplifying assumption is made that it is constant (further denoted by  $\tau$ ), subject to random fluctuations. With this,

$$PB_t^c = \tau \gamma_t \bar{Y}_t$$

Expressing all variables in terms of potential output and using lowercase letters for the ratios yields:

$$p_t = p_t^{ca} + \tau \gamma_t.$$

Thus, the fiscal impulse can be represented as:

$$\eta_t := \Delta p_{t+1}^{ca} = p_{t+1}^{ca} - p_t^{ca} = p_{t+1} - \tau \gamma_{t+1} - (p_t - \tau \gamma_t)$$

and so, the (unadjusted) primary balance ratio in period (t+1) can be expressed as:

(2) 
$$p_{t+1} = p_t + \tau \gamma_{t+1} - \tau \gamma_t + \eta_t.$$

• Public debt. The debt evolution equation (also relative to potential output) is the standard one:

(3) 
$$b_{t+1} = \left(\frac{1+r}{1+a^p}\right)b_t - p_{t+1},$$

where r and  $g^p$  denote interest rate and potential growth rate, respectively. Combining equations (1), (2) and (3) yields a simple model of fiscal policy that captures the impact of government policies on economic activity and their implications for debt sustainability.

To derive a fiscal reaction function based on this model, it is convenient to rewrite it in a state space form. This can be done by moving the variables indexed with (t+1) to the left-hand side, forming a matrix with the respective parameters and pre-multiplying the right-hand by the inverse of that matrix. Before that, since the preferred cyclical measure is the unemployment gap rather than the output gap, the latter needs to be restated in terms of the former. This can be done by using the Okun's law:  $\hat{u_t} = \beta \gamma_t$ . Finally, a disturbance vector ( $v_t$ ) is added to capture, among other things, uncertainty and measurement errors in the unemployment equation, and shocks to the tax ratio and public debt in the primary balance and debt equations. Thus,

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{\tau}{\beta} & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_{t+1}^g \\ p_{t+1} \\ b_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ -\frac{\tau}{\beta} & 1 & 0 \\ 0 & 0 & \frac{1+r}{1+q^p} \end{bmatrix} \begin{bmatrix} u_t^g \\ p_t \\ b_t \end{bmatrix} + \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix} \eta_t + \begin{bmatrix} v_t^u \\ v_t^\tau \\ v_t^b \end{bmatrix},$$

or in vector notation:

$$A_o y_{t+1} = A_1 y_t + B_1 \eta_t + v_t,$$

$$y_{t+1} = A_0^{-1} A_1 y_t + A_0^{-1} B_1 \eta_t + A_0^{-1} v_t,$$

and finally:

$$y_{t+1} = Ay_t + B\eta_t + Cv_t, \ y_0 \ given .$$

The objective of the policymaker is to minimize the deviations of unemployment from equilibrium (or deviations of the unemployment gap from zero) and of debt from the target level. A quadratic loss function is assumed which is standard in the literature. Thus, the problem to solve is

(5) 
$$\min_{\{\eta_t\}} E_0 \sum_{t=0}^{\infty} \frac{1}{2} [(y_t - \bar{y})^{\mathsf{T}} Q(y_t - \bar{y}) + \eta_t^{\mathsf{T}} R \eta_t].$$

subject to the dynamics given by (4). In the above notation,  $\bar{y}$  stands for the vector of target values for the variables of interest and Q and R represent the relative weights assigned to the different goals, e.g.,

stabilization of unemployment vs. stabilization of debt. The inclusion of the fiscal impulse  $\eta_t$  in the objective captures the preference for stability of fiscal policy, i.e., it is undesirable to have large swings in the primary balance from year to year due to discretionary measures. The target vector has the form  $\bar{y} = \begin{bmatrix} 0, \bar{p}, \bar{b} \end{bmatrix}^T$ , where  $\bar{b}$  denotes the debt anchor and  $\bar{p}$  stands for the debt stabilizing primary balance. Clearly, the two are related and in what follows, a zero weight is assigned on  $\bar{p}$ . Thus, matrix Q has the form:

$$Q = \begin{bmatrix} w^u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & w^b \end{bmatrix}$$

and R is a positive scalar.

For calculation purposes, it is more convenient to transform the state vector, so that equilibrium is at the origin. This can be done by setting  $x_t = y_t - \bar{y}$ . Then, the objective function becomes:

(6) 
$$\min_{\{\eta_t\}} E_0 \sum_{t=0}^{\infty} \frac{1}{2} [x_t^{\mathsf{T}} Q x_t + \eta_t^{\mathsf{T}} R \eta_t],$$

and the dynamics are given by:

(7) 
$$x_{t+1} = Ax_t + B\eta_t + Cv_t + e, \text{ with } x_0 = y_0 - \bar{y},$$

where the constant term  $e = (A - I)^{-1}\bar{y}$  and I denotes the identity matrix. It can be shown that for the specific choice of  $\bar{y}$ , this constant term is zero.

The following parameters are used for the scenarios presented below: the autoregressive coefficient for the unemployment gap  $\rho$  is fixed at 0.72 (an estimate consistent with the data); the fiscal multiplier for unemployment  $\alpha$  is set at 0.2—an OLS estimate of (1) and half of the value reported by Monacelli et al. (2010);¹ the Okun's law parameter is fixed at -0.3 (Conefrey et al., 2015); and nominal potential growth and interest rate are assumed to be 4 ¼ and 2 ½, percent respectively (2 ½ percent real potential growth, 2 percent inflation and 0.5 percent equilibrium real interest rate). The tax to GNI ratio is set at 43 percent. Finally, the following assumptions are made for the weights in the objective function: the weight on the unemployment gap is 1 and that on the fiscal impulse (control variable) is fixed at 10. The significantly higher weight is justified by smoothness considerations; abrupt changes in the PB from year to year are generally undesirable. The weight on the debt anchor is 0.5 in the baseline, reflecting that Ireland does not have issues with sustainability, and some deviations from the target can be tolerated. Alternative scenarios with higher and lower weights on debt are also shown.

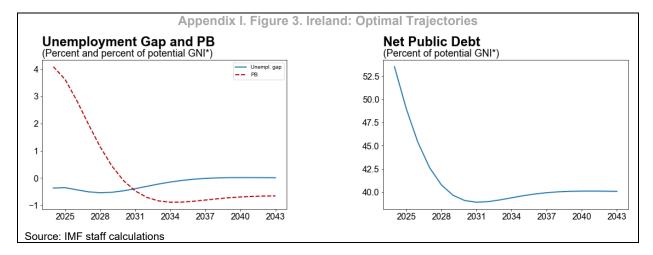
The resulting fiscal reaction function responds positively to debt and negatively to the unemployment gap. With the above parametrization, and assuming that disturbances are normally distributed with zero mean, the solution to problem (6)-(7) results in the following fiscal reaction function (FRF):

$$p_{t+1} = -0.43\hat{u}_t - 0.57(p_t - \bar{p}) + 0.15(b_t - \bar{b}).$$

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<sup>&</sup>lt;sup>1</sup> The estimates in Monacelli et al. (2010) are based on US data and with the assumed Okun's law parameter implies an output multiplier of 1 ½ which appears too high for a small open economy like Ireland. While the OLS estimate of (1) may be biased, it implies an output multiplier of ¾ which is plausible. The sensitivity of results to the multiplier value is explored below.

The rule prescribes that the primary balance should increase (fiscal tightening) when the unemployment gap is negative (the output gap is positive) and when the debt level is above the target.



## Application of this rule indeed stabilizes the variables of interest at the target levels (Figure 3). Primary

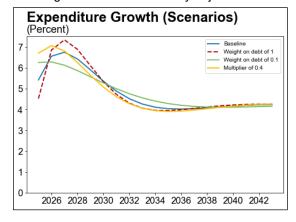
surpluses need to be maintained for the first 5 years to allow net debt to converge to the desired level. The surpluses, however, are declining gradually which, given that the revenue ratio is kept fixed, provides initially room for higher spending. Thus, the optimal PB under the constant revenue to GDP ratio

Fiscal Reaction Functions								
	û	β	$\hat{b}$					
Baseline	-0.43	-0.57	0.15					
Debt weight of 1	-0.46	-0.65	0.21					
Debt weight of 0.1	-0.36	-0.40	0.07					
Multiplier of 0.4	-0.56	-0.68	0.15					
Source: IMF staff calculations								

assumption implies that expenditure can increase at an average rate of about 6.2 percent in the first 5 years; and by an average of 5.4 percent in the first 10 years. Eventually, expenditure growth should converge to 4 ½ percent in line with the potential growth rate.

The sensitivity of the model results is examined for alternative parametrizations. The text figure shows expenditure paths when higher (1.0) and lower (0.1) weights are assigned to the debt stability objective and

when the fiscal multiplier for employment is increased to 0.4 as in Monacelli et al. (2010). The coefficients of the FRF are shown in the text table ("hats" above the variables denote deviations from targets). As expected, when the weight on the debt objective is increased to 1, the FRF suggests a stronger reaction to the debt deviations. A higher fiscal multiplier for unemployment increases somewhat the response to the unemployment gap; the reaction to debt remains as in the baseline. The time profile of expenditure growth under the alternative assumptions varies somewhat but the differences in the



average growth rates in the first 5 years are less than 0.5 ppts; these differences essentially disappear when a 10- year period is considered.

#### Robustness to Uncertainty

The results presented in the previous section are derived under the assumption of normal distribution of disturbances. It is well known that in this case the certainty equivalence principle holds and the solution to the stochastic linear quadratic control problem coincides with that of the corresponding deterministic problem using the expected value of the noise. The normality assumption, however, may not necessarily hold. Moreover, from a decision-making point of view, it is desirable that the policy actions achieve the intended results not just on average but also in a range of circumstances.

#### Two approaches to robustness are considered below.

Minimax optimal. The first approach is along the lines of Hansen and Sargent (2008) and entails
specification of a loss function similar to the one considered previously, with an additional term capturing
the preference for robustness.

Consider again problem (6)-(7) with the additional constraint:

$$\sum_{t=0}^{\infty} v_t^{\mathsf{T}} v_t \leq \delta.$$

This problem can be turned into a related minimax problem (see Hansen and Sargent, 2008) of the form:

(8) 
$$\min_{\{u_t\}} \max_{\{v_t\}} \sum_{t=0}^{\infty} \frac{1}{2} [x_t^{\mathsf{T}} Q \mathbf{x}_t + \mathbf{\eta}_t^{\mathsf{T}} R \mathbf{\eta}_t - \theta v_t^{\mathsf{T}} v_t]$$

subject to (7). Solution to problem (7)-(8) is given by the pair of linear rules:

$$\eta_t = -Kx_t, v_t = Hx_t.$$

the first one of which is the fiscal reaction function of interest. The same values for Q and R are used as before. As for the new parameter  $\theta$ , it needs to be chosen sufficiently high, so that it is above a threshold value to have a solution.<sup>2</sup> For the problem at hand, this threshold is around 62 and the value of 70 is used in the simulations below.

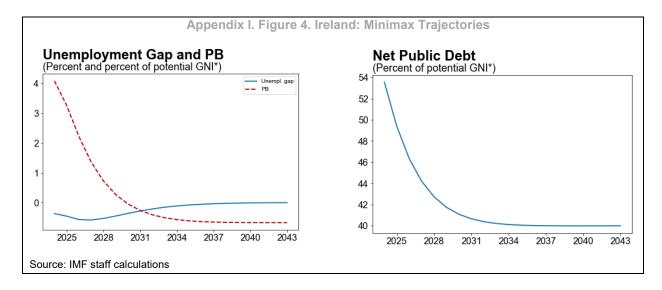
The FRF associated with the minimax rule is

$$p_{t+1} = -0.73\hat{u}_t - 1.09(p_t - \bar{p}) + 0.29(b_t - \bar{b}),$$

and it prescribes a stronger response to deviations from the target values.

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<sup>&</sup>lt;sup>2</sup> Hansen and Sargent (2008) provide the criterion  $\log \det (\theta I - C^{\mathsf{T}}PC) < \infty$  or equivalently, the eigenvalues of  $(\theta I - C^{\mathsf{T}}PC)$  should be positive.

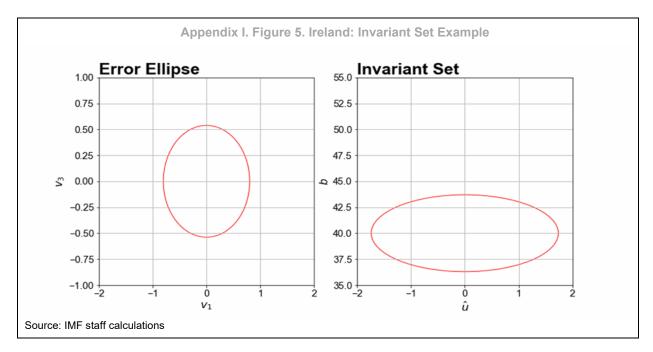


Implementing this rule suggests an average expenditure growth of about 6.3 and 5.3 percent in the first 5 and 10 years, respectively. Growth is also higher in the initial years, which causes the unemployment gap to deteriorate but over time, the differences become negligible.

Robust stabilization. The second approach to robustness does not require explicit formulation of an objective and is based on stabilization techniques. In loose terms, the goal is to find a feedback rule and a region around equilibrium that ensures stability (an invariant set) in the sense that if the system state falls in this region and the rule is applied, it can be kept there indefinitely as long as the shocks are within certain bounds. Ideally, this region should be made as small as possible and if an occasional large shock throws the state outside the invariant set, consistently applying the rule should bring the state back again (see, for example, Boyd et al. 1994; Khlebnikov et al., 2011). It is convenient to work with ellipsoidal sets for disturbances. While no probabilistic assumptions are needed (only boundedness), one could think of these sets as confidence regions for errors drawn from an elliptical distribution (e.g., multivariate normal or t-distribution). The idea is illustrated in Figure 5. The left panel shows a scatter plot of disturbances drawn from a multivariate normal distribution and the 80 percent confidence ellipse. 4 This error set essentially requires the stabilizing rule (FRF in this case) to be robust to persistent shocks or other additive uncertainties in the unemployment equation of around 0.75 ppts and in the debt equation of around 0.5 ppts in absolute value. The right panel of the figure shows the calculated stability region and the unemployment-debt pairs obtained using the robust FRF when shocks are as in the left panel. Starting from the initial condition (-0.3 percent unemployment gap and 53.5 percent net debt to GNI\*), applying the FRF steers the system to the invariant set. Because of the additive disturbances, the state cannot be exactly at equilibrium, but it is kept close to it.

<sup>&</sup>lt;sup>3</sup> In the cited references and elsewhere, the disturbances are assumed to bounded by 1 in the norm. It is relatively straightforward to extend the framework to an arbitrary ellipsoidal set.

<sup>&</sup>lt;sup>4</sup> Note that the system is three-dimensional, and the presented plots are 2D projections of the 3D ellipsoids.



An additional advantage of this approach is that it can accommodate parametric uncertainty as well. Parametric uncertainty can be modeled in different ways. For example, it can be represented in terms of time-varying parameters:

$$y_{t+1} = (A + \Delta A(t))y_t + (B + \Delta B(t))\eta_t + (C + \Delta C(t))v_t.$$

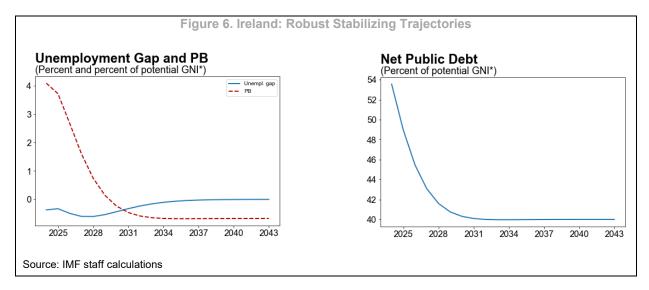
or as a set of alternative models (indexed by i):

$$y_{t+1} = A^i y_t + B^i \eta_t + C^i v_t.$$

Below, the second approach is pursued. This is motivated by a scenario where the windfall CIT from MNEs disappears and tax ratio drops from the current 43 percent to about 38 percent. In the above framework, this is modeled as two separate system matrices  $A^1$  and  $A^2$  differing in the value of  $\tau$ . The resulting FRF when applying this framework is:

$$p_{t+1} = -0.53\hat{u}_t - 0.89(p_t - \bar{p}) + 0.28(b_t - \bar{b}),$$

unemployment gap, primary balance and debt (suppressing the additive noise) are shown in Figure 6.



The robust stabilizing rule implies somewhat lower expenditure growth in the first year but in terms of average, the result is not different from the optimal and minimax rules -6.4 percent in the first 5 years and 5.4 percent in the first 10 years.

