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Could Digital Currencies Lead to the Disappearance of Cash from the Market?

Insights from a "Merchant-Customer" Model

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ABSTRACT: Private and public agents’ plans and actions to introduce digital currencies and other innovative payment instruments could produce some unintended consequences, including the potential disappearance of physical cash. This study employs a two-sided market model to examine how payment systems might respond to new currencies. Numerical simulations indicate that the success of a new currency hinges on a large-scale launch. However, even unsuccessful attempts could disrupt existing systems, potentially resulting in the elimination of cash. If cash plays a critical role as a safeguard, regulatory and monetary authorities should give due consideration to ensure its continued availability when payment innovations are introduced.

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WORKING PAPERS

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Contents

Executive Summary

Could cash become extinct as an unintended consequence of innovation in the payment system due to externalities in its demand by agents? This question seems particularly important in view of the dwindling use of cash as transactions switch to more modern forms of payments. The continued presence of cash in the payment system could entail some collective benefits that exceed the private gains of using it in individual retail transactions (hence the externality). While other payment instruments have their own useful attributes, cash has some pros and cons compared to other means of payments. Cash provides a high degree of anonymity and independence from intermediaries that distinguish it from digital payment methods. For instance, cash often does not generate transaction records, does not require third-party involvement, and, unlike some digital currencies, is not subject to remote control.[1] These attributes make cash a fallback option during disruptions or failures in payment systems. The implications of its reduced availability could be significant, and the role of cash may become more fully appreciated only if it were to vanish entirely, potentially leaving limited opportunities to address its absence. Since these benefits may not be fully understood or appreciated by users—and would not be fully internalized—cash could disappear from the market as an unintended consequence of innovations, such as the introduction of central-bank digital currencies, due to a lack of demand, even if such a development were to ultimately lead to a suboptimal outcome.

Using a two-sided market model that builds on previous research by the same authors, this study examines how the (equilibrium) payment system could respond to the introduction of a new instrument (or "currency") in different scales and modalities. The focus is on whether such an instrument could inadvertently lead to the extinction of currencies (payment instruments) that were previously part of a diverse payment ecosystem—comprising cash, traditional payment cards, and modern electronic money—as a result of induced shifts in demand. While, in principle, *any* currency (and possibly more than one) could be at risk of extinction (we consider this possibility in the model and in simulations), ongoing trends in demand suggest that the possibility of physical cash disappearing due to a lack of demand on either side of the market cannot be fully discounted. For this reason, throughout the paper, reference will often be made to cash extinction as the special case of interest, bearing in mind that, technically, the results discussed in the paper could apply, in principle, to any currency (before specific conditions are applied). In the same way, while we refer to the introduction of a digital currency as the most likely major "shock" that could alter the payment system in the foreseeable future, similar considerations may apply to the introduction of *any* innovative payment instrument, such as a new type of crypto asset. This issue is critical as more innovative instruments are likely to emerge in the market in the near term.

Furthermore, our study delves into the potential factors that may be driving cash extinction, including customer demand and habit changes influenced by merchants' decisions. Merchants' incentives are also pivotal. While our model excludes switching costs, users face fixed costs for each instrument, thereby discouraging

[1] For an overview of studies on the social and private costs of retail payments, see Junius *et al.* (2022).

multihoming. Unlike conventional research focusing on currency supply, our study explores currency-adoption dynamics by merchants and customers, treating the supply of currencies and the number and value of desired transactions as fixed. It probes how market agents respond to these dynamics and to each other, analyzing long-term equilibria, and currency usage patterns. Typically, on one side of the market every agent adopts a single currency, while on the other side every agent utilizes multiple currencies.

The results obtained through numerical simulations are consistent with the argument that a new “currency” is likely to be successful only if it is launched on a sufficiently large scale. Under some conditions, however, even an unsuccessful launch can disturb the initial equilibrium, leading to the extinction of currencies that were previously used (such as cash). In other cases, the new currency can replace one of the old currencies, but it is also possible that both old and new currencies continue to be used in the new equilibrium.

The introduction of digital currencies could thus potentially lead to the extinction of cash, a possibility that cannot be dismissed outright, if agents on one (or both) sides of the market cease accepting it. If this outcome is deemed to entail significant adverse externalities—not least, what would arise from the loss of “option value” if reintroducing cash after its extinction were to prove to be prohibitively costly—then it would be prudent to implement precautionary measures to prevent cash’s disappearance, even if other payment instruments offer benefits in efficiency or financial inclusion. This study presents an analytical framework highlighting that, while the payment system could evolve over various qualitatively different paths, cash extinction cannot be ruled out *a priori* and its potential consequences should therefore be carefully considered.

Introduction

The payment system landscape is rapidly changing, driven by technological innovations. Various privately issued crypto assets, such as Bitcoin or Ethereum, have already been launched in the market, with mixed success, and several jurisdictions are considering the introduction of central-bank-issued digital currencies (CBDC). Could the introduction of digital currencies and other innovative payment instruments lead to the unintended consequence of physical cash disappearing from the payment system?[2] This paper addresses this question from the perspective of the dynamic evolution of the payment system.

Cash has unique properties that make it an important safeguard during disruptions or failures in payment systems. Cash provides a high degree of anonymity, generally leaving no transaction records (exceptions include, in some jurisdictions, identification requirements for transactions above a certain threshold, and mandatory declarations for carrying amounts above some thresholds across borders), cannot be remotely controlled, unlike some digital currencies, and can be physically stored, enhancing its resilience to adverse events such as natural disasters. It plays a crucial role as an "outside option," ensuring the reliability and integrity of other payment methods.

Concerns about a "cashless society" have been voiced by various observers. Some view it positively, such as Rogoff (2016), while others, like Rochemont (2020) and Beer *et al*. (2015), warn of adverse consequences, particularly the loss of privacy. These concerns are heightened by the potential irreversibility of cash extinction because reintroducing cash in a non-cash system would be challenging and costly.

Expanding upon the groundwork laid by Maino and Pani (2024), this study constructs an elaborate analytical framework aimed at comprehensively evaluating the risk of a potential disappearance of physical cash. This research scrutinizes how a variety of factors, including diverse agents' incentives and initial conditions, intricately shape outcomes within the payment ecosystem. Through this analytical lens, the study delves into the dynamic evolution of the payment system over time, considering a spectrum of scenarios and conditions. It underscores the critical importance of carefully weighing the potential adverse impacts on the payment infrastructure, which may accompany the introduction of digital currencies. To safeguard the continued utilization of cash and to uphold the equilibrium of the payment system, the study advocates for a proactive policy approach and for the implementation of measures aimed at ensuring the sustained relevance of physical currency, especially in scenarios where the introduction of new digital currencies might inadvertently lead to the extinction of traditional cash.

This paper is organized as follows: Section 2 provides brief reviews of related studies; Section 3 presents the model in a two-currency setting; Section 4 extends the model to three currencies and examines how a two-

[2] In this paper, “payment system” means the system of payment instruments, rules, institutions, and payment intermediaries (such as banks and credit card companies) involved in the settlement of “retail transactions,” i.e., transactions (typically on a small scale) between private individuals and/or small and medium enterprises.

currency equilibrium may be disturbed by the introduction of a third currency; and Section 5 draws the main conclusions.

Related Studies

Payment instruments, or "currencies" (like cash, cards, or digital currencies (DCs)) function as platforms connecting two sides of the market: merchants (sellers) and customers (buyers). The utility of a currency depends on its features (e.g., transaction fees, interest, or reward points) and its adoption by the other side of the market. For example, merchants are more likely to accept credit cards if many customers use them. The economics of this type of service has been analyzed by the “theory of two-sided markets,” a growing area of economics that started with a few seminal papers by Armstrong (2006), Rochet and Tirole (2003) and Ellison and Fudenberg (2003; see also Rysman, 2009). Schmalensee (2002) was among the first to apply the theory to the payment system.

Like other two-sided markets, the payment system is in constant evolution, which has only accelerated in recent years with advances in new digital technologies. A major feature of this evolution has been a secular decline in the use of cash, to the point that many observers are wondering whether we are heading to a completely “cashless economy.” Various authors have documented the downward trend in the use of cash and inquired about its causes. Marszałek and Szarzec (2022) have retraced this decline to a change in demand for cash and banking services and to a change in the payment habits of customers. Amromin and Chakravorti (2007) have shown how demand for cash (proxied by demand for small-denomination currency) decreases with increased use of debit cards and greater retail market consolidation. Arango *et al.* (2015) have highlighted how merchants’ acceptance of payment cards—more than the rewards offered by credit card companies—affects what instrument customers use to make payments. Looking at the recent experience with mobile money in Uganda, Simione and Muehlschlegel (2023) found evidence that the use of mobile money is related to perceptions of the risks of carrying cash, to send or receive remittances, and to borrow and save.

What would the transition to a “cashless” economy imply? Some authors (e.g., Rogoff, 2016) advocate in its favor on the grounds that it would facilitate the fight against crime and tax evasion. Other authors have attempted to quantify the social costs of phasing out cash. For instance, using a dynamic cash inventory model that allows agents to use either cash or credit at each moment in time, Alvarez and Lippi (2017) have estimated that households’ cost of moving from their optimal cash-credit share to one where the cash share is zero would amount to a small fraction (0.005–0.025 percent) of the value of their daily consumption.

Other studies have explored the economics of the market for payment instruments. This line of research has highlighted that payment instruments carry a strong fixed cost of adoption (habits) and high switching costs that entail important lock-in effects, preventing users from responding to minor differences in costs and thereby providing significant market power to the providers of payment services. This framework has been used, for instance, to explain the determinants of “interchange fees” charged to merchants by credit card networks that enjoy a strongly monopolistic position. Bounie *et al.* (2016), for instance, found evidence that many merchants accept some credit cards out of fear of losing profitable business to competitors. This type of (in this case, negative) externalities that occur in a two-sided market is also present in our model, where both merchants' and customers' decisions on which instrument to use are affected by the risk of missing a transaction if the counterpart does not use that instrument.

This study analyzes the dynamics of a market with a three-currency payment system, extending the two-currency model used by the same authors in a previous study. While the model applied here can be used to examine a variety of situations (such as the presence of multiple equilibria, the stability of equilibria, and the boundaries of the basins of attractions of stable equilibria), the main question of interest is how a system in a stable equilibrium, with two currencies, would respond to the introduction of a third currency. This situation represents what would happen, for instance, in a jurisdiction where a DC were introduced. The focus is on whether such a change could inadvertently lead to the extinction of one (or both) of the existing currencies. In real life, the concern is that the introduction of a DC in a diverse payment ecosystem—comprising cash, traditional payment cards, and modern electronic money—where the use of physical cash has already declined significantly, could lead to the complete disappearance of cash, even if such an outcome were not an intentional policy objective. Similar questions apply to other potential payment innovations, such as the introduction of new digital currencies. This study also delves into the potential drivers of the extinction of a payment instrument (such as cash), including the factors that influence customer demand, habit changes influenced by merchants' decisions, and merchants' incentives. While our model excludes switching costs, users face fixed costs for each instrument. Unlike conventional research focusing on currency supply, our study explores currency adoption dynamics by merchants and customers, treating supply and usage as fixed.

Two-Currency "Merchant-Customer" Model

An important feature of the retail payment system is that it acts as an intermediator, facilitating the interaction between agents on two distinct sides of the market: on one side are businesses (corporate entities, small enterprises, professionals) selling goods and services to the public (the "merchants"), and, on the other side, are mainly private individuals buying these goods and services, typically for personal use (the "customers").

The incentives that an agent, on either side of the market, faces when using one or another payment instrument also depend, among other factors, on the choices made by the agents on the other side of the market. These situations are described in economic theory as "two-sided markets," and the instruments used to connect agents on the two sides (for instance, the payment instruments) are commonly called "platforms." Two-sided markets also feature a third group of participants, the platform providers (for instance, the suppliers of payment services, such as banks and credit card companies). In this study, the actions of this group of agents are taken as given and embedded in exogenous parameters. The inclusion of this category of actors, and the economic incentives that they face, could be a useful extension of this model.

The model that we present hereafter is a simplified version of the model discussed in Maino and Pani (2024), which can be more easily extended to the three-currency case. Assume only two "currencies" ("payment instruments" or "platforms") — "cash" and "cards." There is a population of consumers ("customers"), of size one (after normalization); of these, a fraction, x_c, carry cash; a fraction, x_d, carry cards; and a fraction, x_{cd}, carry both. There is also a population of sellers ("merchants"), also of size 1, of whom π_c accept only cash, π_d accept only cards, and π_{cd} accept both instruments. Customers only buy from merchants and merchants only sell to customers (there are no "peer-to-peer" transactions). A customer "carries" cash (or cards) if he is ready to provide cash (or cards) to a merchant in exchange for his merchandise; in the same way, a merchant "accepts" cash (or cards) if he is ready to sell his merchandise in exchange for cash (or cards). When a currency is accepted by a merchant or used by a customer, it is considered "used." If money changes hands in a transaction using that currency, it is deemed "spent" by the customer and "taken" by the merchant.

In this model, merchants and customers are randomly paired. Merchants decide which currency they accept, while customers choose the currency they carry, both incurring a fixed cost for usage. If the customer's currency matches the merchant's, a transaction occurs, with both parties benefitting. Otherwise, both incur losses. In transactions, merchants pay variable costs, and customers perceive variable benefits associated with the used currency. When a customer holds both currencies and the paired merchants accept both, the currency for the transaction is randomly selected with equal probability. This framework simulates the dynamics of currency usage and transactions within a market setting.

We assume, for simplicity, that all agents are risk-neutral; their decisions are based on the *net expected value* of the cost and benefits of each alternative action.

The Merchant's Problem

Merchants pay a fixed cost, p_c, for accepting cash and a fixed cost, p_d, for accepting cards. Furthermore, they pay a variable cost, q_d (idiosyncratic, uniformly distributed over [0, 1]; "d" as in "debit cards"), whenever they take cards in a transaction, and a variable cost, q_c, whenever they take cash. If no transaction takes place, merchants incur a loss equal to ξ. Apart from q_d, all other parameters are the same for all merchants.[3] A transaction is missed every time that a merchant is matched with a consumer that does not carry the currency accepted by the merchant (if the merchant accepts both currencies, a transaction always takes place). The total net expected costs incurred by a merchant are thus equal to the following:

If he accepts only cash, to

$$M_c = p_c + q_c(x_c + x_{cd}) + \xi x_d$$

If he accepts only cards, to

$$M_d = p_d + q_d(x_d + x_{cd}) + \xi x_c$$

And, if he accepts both currencies, to

$$M_{cd} = p_c + p_d + q_c\left(x_c + \frac{x_{cd}}{2}\right) + q_d\left(x_d + \frac{x_{cd}}{2}\right)$$

[3] Note that there is an asymmetry in this model between cash, whose variable costs (benefits) vary among customers (merchants), and cards, whose variable costs (benefits) are the same for all customers (merchants).

where x_c is the number of customers who carry only cash, x_d is the number of customers who carry only cards, and x_{cd} is the number of customers who carry both currencies.

Hence, a merchant prefers to accept only cash rather than to accept both currencies if and only if

$$p_c + q_c(x_d + x_{cd}) + \xi x_d < p_c + p_d + q_c\left(x_c + \frac{x_{cd}}{2}\right) + q_d(x_d + x_{cd}/2)$$

yielding

$$q_d > q_1 = \frac{q_c \frac{x_{cd}}{2} - p_d + \xi x_d}{x_d + \frac{x_{cd}}{2}}$$

In the same way, a merchant prefers to accept only cards rather than to accept both currencies if and only if

$$q_d < q_2 = q_c + \frac{p_c - (\xi - q_c)x_c}{x_{cd}/2}$$

And he prefers to accept only cash rather than only cards if and only if

$$q_d > q_3 = \frac{q_c x_{cd} + p_c - p_d - (\xi - q_c)x_c + \xi x_d}{x_d + x_{cd}}$$

It can be shown that $q_3 = (\zeta q_1 + (1 - \zeta)\, q_2)/2$, where $\zeta = x_{cd}/(x_d + x_{cd})$.

We assume that every merchant accepts at least one currency; in other words, the costs of going out of business are assumed to be larger than the lowest cost of accepting one currency under the worst circumstances, which is $\min(p_c, p_d) + \xi$. The participation constraint is therefore always satisfied.

Notice that—as stated and quantified in Bounie *et al.* (2016)—a merchant may decide to accept one currency (e.g., cards) even if it involves higher fixed and variable costs than the other, if a large number of customers carries only that currency. For instance, if ξ and x_d are sufficiently large and x_c sufficiently low, a merchant may accept cards even if $p_d > p_c$ and $q_d > q_c$.

Given the values of x_c, x_d, and x_{cd}, and of all other parameters, the support, *S*, of the distribution of q_d (the interval [0,1]) can thus be divided into segments (one, two, or three) corresponding to what currencies are accepted by a merchant whose parameter, p_c, falls in that segment. The boundaries of the segments are the values, q_1, q_2, and q_3, that fall between 0 and 1 (with q_3 always falling between q_1 and q_2) (Figure 1):

> If $q_1 < q_2$, merchants with $q_d < q_1$ accept only cards, merchants with $q_d > q_2$ accept only cash, and merchants with $q_1 < q_d < q_2$ accept both instruments.
>
> If $q_1 > q_2$, merchants with $q_d < q_3$ accept only cards, and merchants with $q_d > q_3$ accept only cash.

Since q_1, q_2, and q_3 all depend on x_c, x_d, and x_{cd}, so do the number of merchants who accept cash, cards, or both:

$$\pi_c = 1 - \max(q_1, q_3) = f_1(x_c, x_d, x_{cd}; \Omega)$$

and

$$\pi_d = \min(q_2, q_3) = f_2(x_c, x_d, x_{cd}; \Omega)$$

$$\pi_{cd} = 1 - \pi_c - \pi_d = f_3(x_c, x_d, x_{cd}; \Omega)$$

where π_c, π_d, and π_{cd} are, respectively, the number of merchants that accept only cash, only cards, or both, and f_1, f_2, and f_3 are functions of x_c, x_d, and x_{cd}, and of the set of parameters, Ω (which includes q_c, q_d, and ξ).

The Customer's Problem

The customer's problem is similar to that of the merchant, with the difference that instead of variable costs, customers perceive variable benefits from spending their currency. In the case of cash, such benefits may consist of stronger protection of privacy (cash payments do not typically leave a record) and resilience to outages and equipment malfunctioning (net of handling and withdrawal costs, such as ATM fees); for cards, they typically consist of rewards and "cashback" discounts awarded to cardholders.

Before approaching a merchant, each customer must carry at least one currency and may carry both. If he decides to carry cash, he incurs a fixed cost, c_f, in ATM or branch withdrawal fees and in time spent getting to the location; if he decides to carry cards, he must pay a fixed cost, c_d, in application and annual fees.
When the agent spends cash in a transaction, he perceives an intangible benefit, β_i, which is idiosyncratic and uniformly distributed in the interval [0, 1], which represents the combination of several attributes of cash or cards, such as privacy or convenience. Customers utilizing a credit card receive a fixed nonnegative reward, r_d.

Since not all merchants may accept both currencies, a customer who carries only one currency may fail to make the purchase if he is matched with a merchant who does not accept the same currency. Missing a purchase entails a cost, w, for the customer, which is exogenous and equal for all. The probability that a customer may be matched with a merchant who does not accept cash is equal to π_d, and the probability that he may be matched with a merchant who does not accept cards is equal to π_c. Hence, a customer that carries only cash incurs an expected net cost (net of benefits) equal to

$$C_c = c_f - \beta_i(\pi_c + \pi_{cd}) + w\pi_d$$

The net expected cost for a customer that carries only cards is, instead, equal to

$$C_d = c_d - r_d(\pi_d + \pi_{cd}) + w\pi_c$$

and the net expected cost for a customer who carries both cash and cards is equal to

$$C_{cd} = c_f + c_d - r_d(\pi_d + \pi_{cd}/2) - \beta_i(\pi_c + \pi_{cd}/2)$$

As a result, analogously to the merchant's problem, a customer prefers to carry only cash if and only if

$$\beta_i > \beta_1 = r_d - \frac{c_d - (w + r_d)\pi_d}{\pi_{cd}/2}$$

which is essentially the formula for q_d with some signs changed. In the same way, a customer prefers to carry only cards than to carry both currencies if and only if

$$\beta_i < \beta_2 = \frac{r_d \frac{\pi_{cd}}{2} + c_f - w\pi_c}{\pi_c + \pi_{cd}/2}$$

and he prefers to carry only cash, rather than only cards, if and only if

$$\beta_i > \beta_3 = \frac{r_d\pi_{cd} - c_d + c_f + (w + r_d)\pi_d - w\pi_c}{\pi_c + \pi_{cd}}$$

Again, β_3 lies between β_1 and β_2 , and the support of customers' preferences for cash (the interval [0, 1]) can be divided into segments representing each customer's preferred choice:

If $\beta_1 > \beta_2$, customers with $\beta_l > \beta_1$ carry only cash, customers with $\beta_l < \beta_1$ carry only cards, and customers with $\beta_l > \beta_i > \beta_2$ carry both currencies.

If $\beta_1 < \beta_2$, customers with $\beta_l > \beta_3$ accept only cash and customers with $\beta_l < \beta_3$ carry only cards.

Again, since β_1, β_2, and β_3 all depend on π_c, π_d, and π_{cd}, so do the number of customers who carry cash, cards, or both:

$$x_c = 1 - \max(\beta_1, \beta_3) = g_1(\pi_c, \pi_d, \pi_{cd}; \Phi)$$

and

$$x_d = \min(\beta_2, \beta_3) = g_2(\pi_c, \pi_d, \pi_{cd}; \Phi)$$

$$x_{cd} = 1 - x_c - x_d = g_3(\pi_c, \pi_d, \pi_{cd}; \Phi)$$

where g_1, g_2, and g_3 are functions of π_c, π_d, and π_{cd}, and of the set of parameter Φ (which includes β_1, β_2, and β_3, and w).

As in the case of merchants, we assume that every customer carries at least one currency. Despite the advice of minimalists who preach frugality, the costs of "refraining from shopping" are assumed to be prohibitively high, or at least higher than the lowest cost of carrying one currency, which cannot exceed $\min(c_f, c_d) + w$. The participation constraint is always satisfied also on the customers' side.

Equilibria

We first examine what equilibria emerge in this setting. The significance of equilibria lies in the fact that they can persist indefinitely. Once a system is in a state of equilibrium, that state persists until the system is disturbed by some type of shock (such as a temporary change in parameters or the introduction of a new currency) that alters the decisions of a certain number of agents, shifting the system to a different state.[4]

We define the state of the system by a pair of vectors (π, ω) (where $\pi = [\pi_c, \pi_d, \pi_{cd}]$ and $\omega = [x_c, x_d, x_{cd}]$) that describe the decisions of the agents at a given point in time.[5] A state is said to be in equilibrium (in the Nash sense) if no agent on one side of the market would change his behavior insofar as all the agents on the other side of the market kept their behavior unchanged.[6] More specifically, in equilibrium no merchant would change his decision on what type of currency he accepts insofar as customers do not change their decisions on what currency to carry, and no customer would change his decision insofar as no merchants change their decisions on what currency to accept.

In a generic state (not necessarily of equilibrium) the response of agents on one side of the market to the actions of agents on the other side is determined by the equations discussed:[7]

$$\pi_c = f_1(x_c, x_d, x_{cd}; \Omega)$$
$$\pi_d = f_2(x_c, x_d, x_{cd}; \Omega)$$
$$\pi_{cd} = f_3(x_c, x_d, x_{cd}; \Omega)$$

and

$$x_c = g_1(\pi_c, \pi_d, \pi_{cd}; \Phi)$$
$$x_d = g_2(\pi_c, \pi_d, \pi_{cd}; \Phi)$$
$$x_{cd} = g_3(\pi_c, \pi_d, \pi_{cd}; \Phi)$$

or, in vector notation,

$$\pi = f(\omega; \Omega)$$
$$\omega = g(\pi; \Phi)$$

[4] The system may, of course, be disturbed by structural changes, such as a permanent change in parameters, changes in the number of agents on either side, or the introduction of a new currency on a large scale. This type of shock is not considered here.

[5] Vectors π and ω are *feasible*—in the sense that they represent a possible state of the system—only if their values fall within two closed sets (Π and Z, respectively), identified by the definition of the two vectors. Both Π and Z are simplexes in a three-dimensional space; Π is identified by the constraints $\pi_c \geq 0$, $\pi_d \geq 0$, $\pi_{cd} \geq 0$, and $\pi_c + \pi_d + \pi_{cd} = 1$. Since $\pi_{cd} = 1 - \pi_c - \pi_d$, its elements can be represented by the couples (π_c, π_d) that satisfy the constraints, $\pi_c \geq 0$, $\pi_d \geq 0$, and $\pi_c + \pi_d \leq 1$. These couples form a triangle, Π^*, with vertices at the origin and at the points (0,1) and (1, 0). In the same way all feasible values of ω (a triangle, Z, identified by $x_c \geq 0$, $x_d \geq 0$, $x_{cd} \geq 0$, and $x_c + x_d + x_{cd} = 1$) can be represented by the elements of a triangle, Z^*, with vertices at the origin, (0,1) and (1,0). To each feasible point, π in Π, corresponds a point, π^* in Π^* (and vv.), and to each feasible point, ω in Z, corresponds a point, ω^* in Z^* (and vv.).

[6] This is the standard extension to a two-sided market setting of the Nash equilibrium concept, whereby no agent changes his behavior insofar as all other agents keep their behavior unchanged.

[7] *For a given configuration of parameters*, each feasible point, π in Π, corresponds to a point, ω in Z, that represents the customers' response to π, and each feasible point, ω in Z, corresponds to a point, π in Π, that represents the merchants' response to π. As noted above, the two responses are not necessarily consistent, in the sense that two points, π and ω, in their respective sets, may not necessarily be the best response to each other; when this occurs, the system is in equilibrium. Note that while the feasible sets, Π and Z, do not depend on the parameters of the system, the relations that match the elements of one set with those of the other do depend on the parameters, and so do the equilibria of the system.

From the above definition, the system is considered in equilibrium when these equations hold simultaneously for the same values of π and ω. – formally, when π and ω are such that the following equations hold simultaneously:

$$\pi = f(\omega;\Omega) = f(g(\pi;\Phi);\Omega) \equiv h(\pi;\tilde{\Omega})$$

and hence,

$$\omega = g(\pi;\Phi) = g(f(\omega;\Omega);\Phi) \equiv \tilde{h}(\omega;\tilde{\Omega})$$

where $\tilde{\Omega} = \Omega \cup \Phi$ is the vector (or "configuration") of all the exogenous parameters of the system. Equilibria are thus identified as the fixed points of the function *h*.

The vectors, π and ω, at any time, represent the *state* of the system at that moment. Since, in equilibrium, for a given set of parameters, π is univocally determined by ω through the function *f*, and ω is univocally determined by π through the function *g*, either of these vectors is sufficient to describe the complete state of the system in equilibrium.

Types of Equilibria

Depending on the parameters of the system, different types of equilibria may exist. The system *always* exhibits (for intuitive reasons) two *single-currency* equilibria, in which all agents on both sides of the market use only one and the same currency. Some (but not all) configurations of parameters also show equilibria in which two currencies continue to be used indefinitely. These equilibria may have different characteristics, depending on how many agents use either currency on either side of the market. Most notable (and frequent) are "corner" equilibria[8] where all agents on one side of the market use both currencies, while agents on the other side split into two groups, one using only one currency and one using the other. Other types of equilibria also exist and have been found in simulations.

Existence of Equilibria

The *existence* of equilibria has already been affirmed: both single-currency states[9] ($\{\pi^* = (1{,}0),\ \omega^* = (1{,}0)\}$ and $\{\pi^* = (0{,}1),\ \omega = 0{,}1)\}$), in which only one instrument is used, are intuitively equilibria. Since no agent uses the other instrument, no agent can derive any benefit from doing so.[10] Obviously, if merchants know that all customers carry only cash, it makes no sense for them to accept cards that nobody holds; similarly, if

[8] These equilibria lie at one of the vertices ("corners") of the set of feasible states of the system (Π or Z).

[9] Since the vector, ω, is uniquely determined (given π and Φ), the set of feasible outcomes and the equilibrium outcomes can be fully characterized by considering only the feasible, and equilibrium, values of the two-dimensional vector, π^*. The set of feasible values is represented by a plane triangle with vertices at the origin and at [0,1] and [1,0].

[10] It is assumed here that all agents incur positive fixed costs from using each instrument. If some agents incur no fixed cost, they may use that instrument even if no agent on the other side of the market uses it. However, they will have no incentive to do so. For instance, if in the absence of fixed costs, "nostalgic" customers continued to carry cash, even after all merchants had ceased accepting it, doing so would cost nothing but would also bring no benefits.

customers carry only cards, it makes no sense for merchants to incur the useless fixed costs associated with setting up the cash-handling infrastructure.

If these are the only possible equilibria—as occurs under some configurations of parameters—a situation where both instruments are used in the market is inherently unstable; sooner or later one of the two instruments displaces the other. If, instead, the system also exhibits two-platform equilibria, it is possible—although not guaranteed—that both instruments could continue to be used indefinitely.

The fact that single-platform states are states of equilibrium has another important implication—*once a currency disappears from the market, it does not reappear on its own*. Left to itself, the system would not move toward its reintroduction. Even a change in parameters would not be sufficient to produce this outcome since such states are equilibria *under any admissible set of parameters*. This justifies the use of the word "extinction" to describe the disappearance of one instrument from the market. Reversing an extinction would require *shifting the system to a different state* and resolving a difficult coordination problem—an outcome that cannot be achieved through the market mechanism alone.

We prove here, however, that two-currency equilibria also exist under some configurations of parameters.

Equilibria in which all Merchants Accept both Instruments

Assume that $\pi_c = \pi_d = 0$ (i.e., $\pi^* = (0,0)$) and therefore $\pi_{cd} = 1$; all merchants accept both instruments. How would the customers respond? From the above equations, in this case,

$$\beta_1 = r_d - 2c_d < \beta_2 = r_d + 2c_f$$

$$\beta_3 = r_d - c_d + c_f$$

Hence, from the previous section, customers with $\beta_i < \beta_3$ carry only cards and the rest carry only cash. No customer carries both currencies, since every customer knows that whichever currency he carries, all merchants will accept it. When is this an equilibrium? Since all merchants prefer using both currencies to accepting only cash, q_1 must be at least equal to 1; since $x_{cd} = 0$, q_1 diverges to either positive or negative infinity, depending on whether or not $p_d > (\xi - q_d)x_d > 0$. Only when this inequality does not hold will all merchants accept both instruments, which implies

$$p_d \leq (\xi - q_d)x_d = (\xi - q_d)\beta_3 = (\xi - q_d)(r_d - c_d + c_f)$$

In turn, since all merchants prefer using both instruments rather than using only cards, q_2 must be nonpositive because when all merchants accept both instruments, $x_{cd} = 0$, q_2 is equal to

$$q_2 = \xi - \frac{p_c}{x_c}$$

which is nonpositive if and only if

$$x_c = 1 - \beta_3 = 1 - r_d + c_d - c_f \geq \frac{p_c}{\xi}$$

If both these conditions are satisfied, a corner equilibrium on the merchants' side exists; otherwise, this state is not an equilibrium.

Example 1: If $c_f = 0.1$, $c_d = 0.2$, $r_d = 0.5$, $p_c = 0.6$, $p_d = 0.5$, $q_d = 0.5$, $w = 2$, and $\xi = 2$,[11] both conditions are met, and this state is an equilibrium. If, instead, $p_d = 0.6$ and $q_d = 0.6$, the first condition is violated, and this state is not an equilibrium.

Similar results have been found in other studies on two-sided markets (e.g., Caillaud and Jullien, 2003; Rysman, 2009): in equilibrium—or, at least, in some equilibria—agents single-home (use only one platform) on one side of the market and multi-home (use more than one platform) on the other side.

Equilibria in which all Customers Carry both Instruments

Analogous considerations apply to states in which all customers carry both instruments, i.e., where $x_c = x_d = 0$ (i.e., $\omega^* = (0,0)$) and $x_{cd} = 1$. In these states

$$q_1 = q_d + 2p_d > q_2 = q_d - 2p_c$$

$$q_3 = q_d - p_c + p_d$$

Merchants with $q_c < q_3$ accept only cash and the rest accept only cards; no merchant accepts both currencies because whatever currency he accepts, all customers carry it. Since all customers prefer carrying both currencies to carrying only cash, β_1 must be at least equal to 1. Since $\pi_{cd} = 0$, this requires

$$c_d \leq (w + r_d)\pi_d = (w + r_d)(1 - q_3) = (w + r_d)(1 - q_d + p_c - p_d)$$

In turn, since all customers prefer carrying both currencies rather than carrying only cards, β_2 must be nonpositive. Since $\pi_{cd} = 0$, β_2 is equal to

$$\beta_2 = \frac{c_f}{\pi_c} - w,$$

which is nonpositive if and only if

$$\pi_c = q_3 = q_d + p_d - p_c \geq \frac{c_f}{w}.$$

If both these conditions are satisfied, a corner equilibrium on the customers' side (binary equilibrium on the merchants' side) exists; otherwise, this state is not an equilibrium.

Example 2: If $c_f = 0.1$, $c_d = 0.2$, $r_d = 0.5$, $p_c = 0.5$, $p_d = 0.5$, $q_d = 0.5$, $w = 2$, and $\xi = 2$, both conditions are met and this state is an equilibrium. If, instead, $p_c = 0.3$ and $c_d = 0.8$, the first condition is violated and this state is not an equilibrium.

[11] The parameters in the examples were selected from a large number of simulations, run on different sets of parameters, to illustrate cases that may arise in this model that are representative of qualitatively interesting outcomes. The examples are merely hypothetical and are not based on any empirical estimation of actual demand elasticities or other parameters.

Hence, a state where all customers accept both instruments is an equilibrium under some sets of parameters and not under others.

Numerical simulations were run on 14,641 sets of parameters (all entailing $r_d = 0.5$, $q_d = 0.5$, $w = 2$, and $\xi = 2$,[12] with c_f, c_d, p_c, and p_d all multiples of 0.1, ranging from 0 to 1)[13] and 7,624 sets were identified with corner equilibria (52 percent). Of these, 1,552 (11 percent) included corner equilibria both on the merchant and on the customer side (probably an underestimation because equilibria on the customers' side that did not fall into the lattice of candidate points for π would not have been captured in the simulation). Of the total simulations, 7,017 sets did not exhibit any corner equilibrium, but 370 of these exhibited some other type of two-currency equilibria.

The simulation also identified other types of two-currency equilibria:

- Equilibria in which all customers and merchants use cash, and some also use cards;
- Equilibria in which all customers carry cards, and some also carry cash, while each merchant accepts only one currency;
- Equilibria in which all merchants accept cards, and some also accept cash, while each customer carries only one currency; and
- Equilibria in which some customers carry one currency, some carry the other, and some carry both (while merchants either do the same or all accept cards and some also accept cash) (Table 1).

Table 1. Equilibria Identified in the Simulations with Two Currencies

	Corner customer	Frontier customer	All customers carry cash, some both	All customers carry cards, some both	All customers carry cash, some cards, some both	Total
Corner merchant	0	2,928	0	0	0	2,928
Frontier merchant	6,248	0	0	154	0	6,402
All merchants accept cash, some both	0	0	161	0	5	166
All merchants accept cards, some both	0	149	0	275	0	424
All merchants accept cash, some cards, some both	0	0	0	33	0	33
Total	6,248	3,077	161	462	5	9,953

Source: Authors' simulations.

[12] The parameters, r_d and q_d, were set equal to the average corresponding parameters for β_i and q_c. The other parameters were selected, after running several simulations with different sets, at the values that were found, experimentally, to yield the qualitatively most interesting results. These values are merely hypothetical and are not based on any empirical estimation from actual data.

[13] This model includes eight different parameters (c_f, c_d, r_d, c_f, q_c, q_d,, w, and ξ). If n values are considered for each of these parameters, the number of possible combinations would be n^8, which can pose large demands on scarce computing time and resources even with modern computing equipment. Since this is a highly abstract, stylized, theoretical model, reducing the number of parameters that can assume different values can yield significant reduction, in terms of demands on resources, at little cost, in terms of results. We have allowed only four parameters to assume, independently, 11 different values each, thus yielding 11^4 = 14,641 combinations. The values of the other parameters have been arbitrarily fixed, and the choice of which parameters are allowed to take different values is also arbitrary. Since the purpose of this paper is to verify if certain outcomes can be described as a result of a theoretical model that incorporates some key characteristics of a two-sided market for currencies, this arbitrary choice does not impair the results. Widening the set of parameter configurations could yield *additional* insights that have not emerged in these simulations but would not weaken the current results.

For 6,647 sets of parameters (45 percent of the total) the simulation could find only single-currency equilibria, although some of these sets may have two-currency equilibria that are not captured by the lattice of 66 initial points used in the simulation. However, there are configurations of parameters that exhibit only single-currency equilibria, as shown by the following example:

Example 2: If $c_f = 0.2$, $c_d = 0.4$, $r_d = 0.5$, $p_c = 0.1$, $p_d = 0.5$, $q_d = 0.5$, $w = 2$, and $\xi = 2$, the only equilibrium states are those in which all agents use only one payment instrument.

The existence of such configurations has important implications: *under some conditions, only one instrument can continue to be used indefinitely.* If both instruments are initially used—as discussed in more detail in the next subsection—one of them will ultimately displace the other. Which one? In the above example (and in a more general set of configurations of parameters), either one could prevail, depending on the initial conditions and on the dynamic behavior of the system.

M-C Dynamics: Convergence Paths and Stability of Equilibria

How does the system behave out of equilibrium? To answer this question, it is necessary to specify the dynamics, that is, the “rule of motion” of the system. We assume here that customers respond *instantaneously* to the merchants’ actions, while merchants respond to consumers’ behavior *with a lag* and at different speeds. As a result, while all customers implement, in each period, their best response to the merchants’ simultaneous actions, only a fraction, *γ,* of merchants adjust their behavior in any given period, implementing their best response to the customers’ actions *in the previous period*.[14] The rest of the merchants maintain their previous actions unchanged. Hence, at any given period, *t*,

$$\omega_t = (x_c, x_d, x_{cd})_t = f(\pi_c, \pi_d, \pi_{cd})_t$$

while

$$\pi_t = (\pi_c, \pi_d, \pi_{cd})_t = \gamma g(x_c, x_d, x_{cd})_{t-1} + (1-\gamma)(\pi_c, \pi_d, \pi_{cd})_{t-1}$$

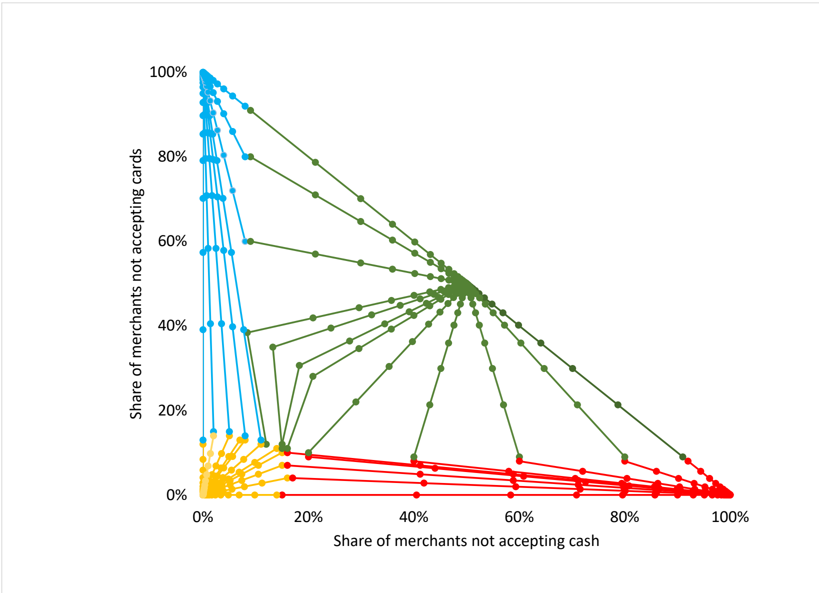
This reflects the fact that consumers are typically much more flexible than merchants. If they need cash, they can visit the nearest ATM and withdraw it almost instantly, and if they need a debit or credit card they can (in many cases) obtain one fairly rapidly.[15] Merchants, instead, need some time to set up the infrastructure they need in order to accept a new means of payment. For instance, to be able to accept payment by card, they need to have installed card readers or other specific equipment; while to accept cash, they may need to install security safes on their premises and possibly have entered a contract with a cash-handling company.

[14] In numerical simulations, we have used a value of γ equal to 0.3 (30 percent).

[15] We abstract here from deficiencies in financial inclusion that prevent some customers from opening bank accounts or obtaining debit or credit cards. This model assumes that all the population has full access to financial services, and their choice of instrument depends only on their preferences.

Note that in this specification, for a given configuration of parameters, the behavior of the system at any time depends *only* on the state of the system at that time. Every state therefore lies on a single *path* that describes how the system moves from one state to the next, as a function of time. Graphically, the system's behavior can be represented by lines on a phase diagram that join, among the possible states of the system, the states that are reached at different dates along the same path (Figure 1).

Figure 1. Phase Diagram in the Π* Set of Feasible States for Merchants

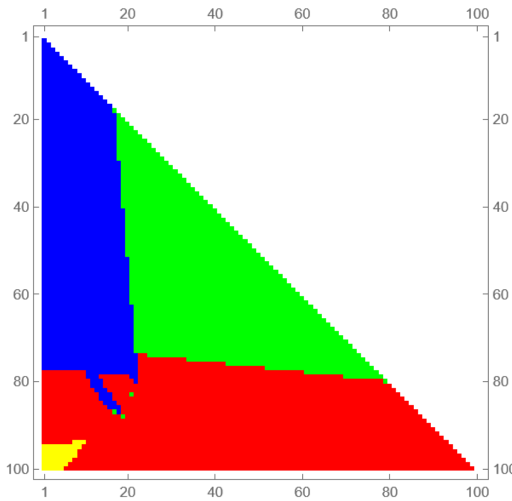


Source: Authors' simulations.
Note: Blue paths converge to a cash-only equilibrium; red paths converge to a cards-only equilibrium; yellow paths converge to an equilibrium in which all merchants accept both currencies (bottom left vertex) and green paths converge to an equilibrium where all customers carry both currencies (middle point of the hypotenuse).

Example 4: For example, if $c_f = c_d = p_c = p_d = 0.4$, $w = 2$, $r_d = 0.5$, $\xi = 3$, and $q_d = 0.2$, the system exhibits four different equilibria: the two single-currency equilibria, plus the two corner equilibria (on the customers' and on the merchants' side). Any state out of equilibrium lies on a path that converges to one of these equilibria. The set of feasible states[16] can thus be divided into four different (not necessarily connected) regions, corresponding to the "basins of attractions" of these four equilibria—defined as the set of states that lie on a path converging to the same equilibrium state (Figure 2).

[16] Represented graphically by Π* in Figures 1, 2, and 5.

Figure 2. Basins of Attractions in the Π* Set

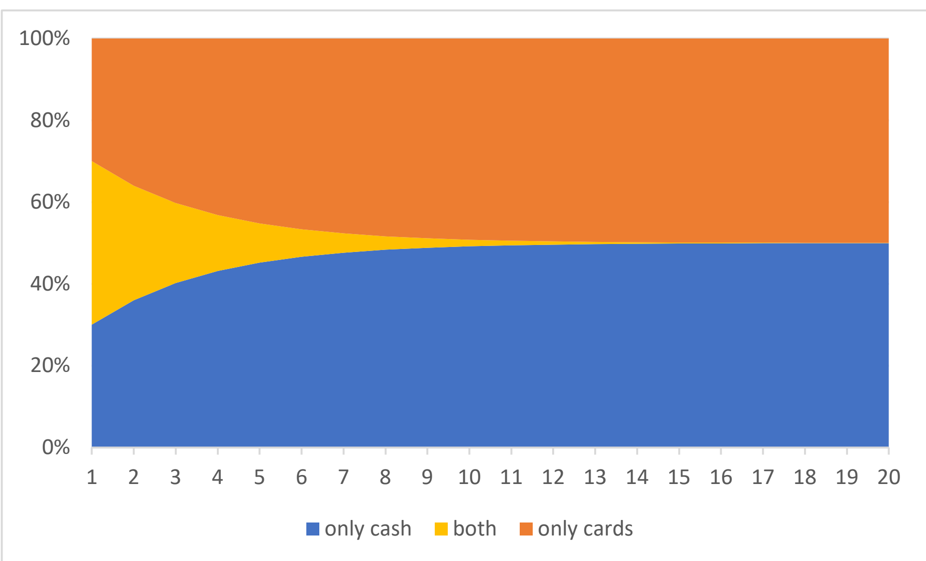


From any point in the blue region the system converges to a cash-only equilibrium; from any point in the red region the system converges to a cards-only equilibrium; from any point in the yellow region the system converges to an equilibrium in which all merchants accept both currencies (bottom left vertex); from any point in the green region the system converges to an equilibrium (on the hypotenuse) in which all customers carry both currencies. Note the green points inside the red region: since the model is discrete (merchants and customers adjust their choices in discrete jumps from one period to the next), the basins of attraction are not connected sets.

Source: Authors' simulations.
Note: The horizontal axis represents the share of merchants not accepting cash, and the vertical axis represents the share of merchants not accepting cards.

Example 5: Starting from $\pi_c = \pi_d = 0.30$ (and hence $\pi_{cd} = 0.4$), all customers carry both cash and cards. The number of merchants accepting only cash or only cards increases and the number of merchants that accept both currencies decreases. In the end, merchants split into two halves, with one accepting only cash and the other accepting only cards, while all customers continue to carry both currencies (Figure 3).

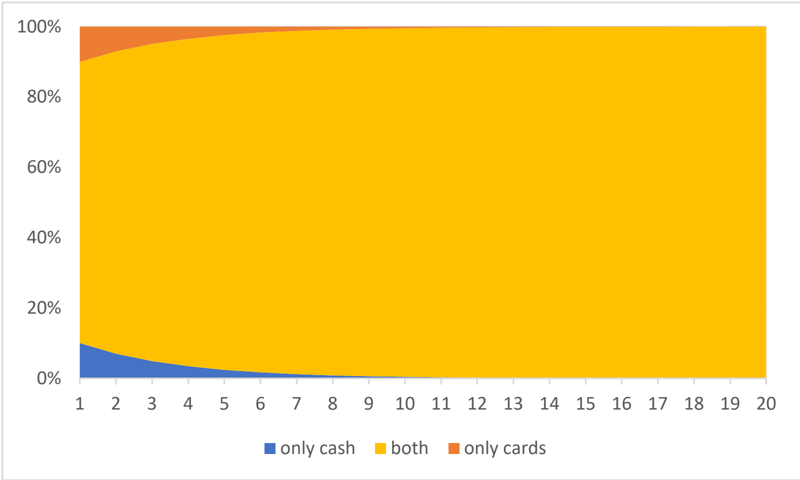
Figure 3. Currencies used by Merchants over Time, Example 5



Source: Authors' simulations.
Note: Parameters: $c_f = c_d = p_c = p_d = 0.2;\ w = 2;\ q_c = 0.5;\ \xi = 2$. Initial state: $\{\pi_c = 0.3,\ \pi_d = 0.3,\ \pi_{cd} = 0.4;\ x_c = x_d = 0,\ x_{cd} = 1\}$.

Example 6: If, instead, initially $\pi_c = \pi_d = 0.1$ (and $\pi_{cd} = 0.8$), customers split into two halves, with one carrying only cash and the other carrying only cards, while all merchants, after some time, accept both currencies (Figure 4).

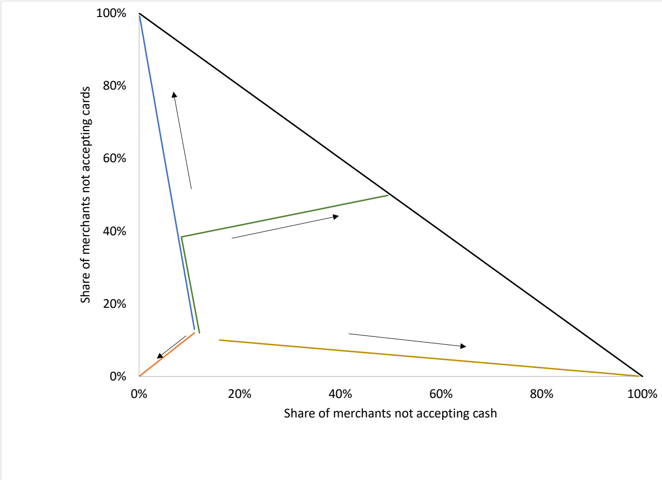
Figure 4. Currencies used by Merchants over Time, Example 6



Source: Authors' simulations.
Note: Parameters: $c_f = c_d = p_c = p_d = 0.2$; $w = 2$; $q_c = 0.5$; $\xi = 2$. Initial state: {$\pi_c = 0.1$, $\pi_d = 0.1$, $\pi_{cd} = 0.8$; $x_c = x_d = 0.5$, $x_{cd} = 0$}.

Remarkably, there are also paths converging to the two single-platform equilibria, which implies that, depending on the initial state, it is possible that both currencies may continue to be used indefinitely in the market, but it is also possible that one or the other may become extinct (Figure 5).

Figure 5. Convergence Paths Leading to Different Equilibria in the Π^* Set



Source: Authors' simulations.
Note: Parameters: $c_f = c_d = p_c = p_d = 0.2$; $w = 2$; $q_c = 0.5$; $\xi = 2$. The figure shows four different paths originating from nearby initial states that converge to four qualitatively different equilibria.

Three-Currency Model

Assume now that a third ("digital") currency, DC, is introduced. Both merchants and customers no longer have three choices, but seven possible options:

- Holding only one of three possible currencies (three options),
- Holding two of the three currencies (three more options), and
- Holding all three currencies.

The set of merchants, and the set of customers, can now be partitioned int up to seven different classes, corresponding to the number of merchants (customers) that adopts each of these seven possible options. The vectors, π and ω, defined above, now have seven dimensions, and since their elements are constrained to be nonnegative and add up to one, the sets of feasible states can be represented (among merchants and customers, respectively) by six-dimensional simplexes, with one vertex at the origin and the others at the unit value of each of the six axes in R.[6]

These choices—and the resulting equilibria and dynamics—cannot be clearly represented in two-dimensional images on paper. Hereafter we discuss briefly how the merchants' and customers' problems are modified with the introduction of a new currency. And we focus on cases of interest, notably equilibria where all currencies are used, and what happens when a two-currency equilibrium is disturbed by the introduction of a third currency, on different scales.

MERCHANTS

Assume that merchants incur a fixed cost, p_z, and a variable cost, q_z, if they accept the DC. Now they have seven different actions:

i. Accept only cash (C)

$$M_c = p_c + \xi(x_d + x_z + x_{dz}) + q_c(x_c + x_{cd} + x_{cz} + x_{cdz})$$

where x_c, x_d, and x_{cd} were defined in Section 3, while x_z, x_{cz}, x_{dz}, and x_{cdz} are the number of customers that carry, respectively, only the DC, only the DC and cash, only the DC and cards, or all three currencies. We assume here that q_c is the same for all merchants.[17]

ii. Accept only cards (D)

$$M_d = p_d + \xi(x_c + x_z + x_{cz}) + q_d(x_d + x_{cd} + x_{dz} + x_{cdz})$$

where q_d is uniformly distributed between 0 and 1 among merchants.

[17] Unlike in the two-currency model, all merchants face the same variable costs for accepting cash, while the variable costs for accepting cards differ among merchants. Here, too, there is asymmetry among cash (whose variable benefits vary among customers), cards (whose variable costs vary among merchants), and the digital currency (for whose use all merchants face the same costs and all customers face the same benefits).

iii. Accept both cash and cards (CD)

$$M_{cd} = p_c + p_d + \xi x_z + q_c(x_c + x_{cd}/2 + x_{cz} + x_{cdz}/2) + q_d(x_d + x_{cd}/2 + x_{dz} + x_{cdz}/2)$$

iv. Accept only the DC (Z): in this case, their expected costs are equal to

$$M_z = p_z + \xi(x_c + x_d + x_{cd}) + q_z(x_z + x_{cz} + x_{dz} + x_{cdz})$$

v. Accept cash and DC (CZ), incurring expected costs equal to

$$M_{cz} = p_c + p_z + \xi x_d + q_c(x_c + x_{cd} + x_{cz}/2 + x_{cdz}/2) + q_z(x_z + x_{cz}/2 + x_{dz} + x_{cdz}/2)$$

In this scenario, if the merchant is matched to a customer that carries both cash and DC, in half of the cases the transaction is paid in cash, and in the other half it is paid in DC.

vi. Accept cards and DC (DZ), incurring expected costs equal to

$$M_{dz} = p_d + p_z + \xi x_c + q_d(x_d + x_{cd} + x_{dz}/2 + x_{cdz}/2) + q_z(x_z + x_{cz} + x_{dz}/2 + x_{cdz}/2)$$

vii. Accept all instruments (CDZ), incurring expected costs equal to

$$M_{cdz} = p_c + p_d + p_z + q_c(x_c + x_{cd}/2 + x_{cz}/2 + x_{cdz}/3) + q_d(x_d + x_{cd}/2 + x_{dz}/2 + x_{cdz}/3) + q_z(x_z + x_{cz}/2 + x_{dz}/2 + x_{cdz}/3)$$

Each merchant will choose the action that yields the minimum expected costs, given his own individual cost parameter, q_c, and the share of customers that choose each option. This will determine, in turn, the number of merchants who choose any of the seven possible actions, as a function of the respective number of customers and of the parameters of the model:

$$\pi_j = f_j(x_c, x_d, x_z, x_{cd}, x_{cz}, x_{dz}, x_{cdz}; \Omega),$$

where $j = c, d, z, cd, cz, dz, cdz;$ or, in vector terms,

$$\pi = f(\omega, \Omega)$$

CUSTOMERS

The customers have a similar problem. Let c_z be the fixed cost borne by customers carrying a digital currency and let r_z be the variable benefit (interest earnings, rewards, convenience of use) that they derive from spending it. Like merchants, customers now have four additional actions:

i. Carry only cash (C)

$$C_c = c_f + w(\pi_d + \pi_z + \pi_{dz}) - \beta_i(\pi_c + \pi_{cd} + \pi_{cz} + \pi_{cdz})$$

where π_c, π_d, and π_{cd} were defined in Section 3, while π_z, π_{cz} , π_{dz}, and π_{cdz} are the number of customers that carry, respectively, only the DC, only the DC and cash, only the DC and cards, or all three currencies.

ii. Carry only cards (D)

$$C_d = c_d + w(\pi_c + \pi_z + \pi_{cz}) - r_d(\pi_d + \pi_{cd} + \pi_{dz} + \pi_{cdz})$$

iii. Carry both cash and cards (CD)

$$C_{cd} = c_f + c_d + w\pi_z - \beta_i\left(\pi_c + \frac{\pi_{cd}}{2} + \pi_{cz} + \frac{\pi_{cdz}}{2}\right) - r_d\left(\pi_d + \frac{\pi_{cd}}{2} + \pi_{dz} + \frac{\pi_{cdz}}{2}\right)$$

iv. Carry only the DC (*Z*): in this case their expected costs are equal to

$$C_z = c_z + w(\pi_c + \pi_d + \pi_{cd}) - r_z(\pi_z + \pi_{cz} + \pi_{dz} + \pi_{cdz})$$

v. Carry cash and DC (CZ), incurring expected costs equal to

$$C_{cz} = c_f + c_z + w\pi_d - \beta_i\left(\pi_c + \pi_{cd} + \frac{\pi_{cz}}{2} + \frac{\pi_{cdz}}{2}\right) - r_z\left(\pi_z + \frac{\pi_{cz}}{2} + \pi_{dz} + \frac{\pi_{cdz}}{2}\right)$$

In this scenario, if the customer is matched to a merchant that accepts both cash and DC, in half of the cases the transaction is paid in cash, and in the other half it is paid in DC.

vi Carry cards and DC (DZ), incurring expected costs equal to

$$C_{dz} = c_d + c_z + w\pi_c - r_d\left(\pi_d + \pi_{cd} + \frac{\pi_{dz}}{2} + \frac{\pi_{cdz}}{2}\right) - r_z\left(\pi_z + \pi_{cz} + \frac{\pi_{dz}}{2} + \frac{\pi_{cdz}}{2}\right)$$

vii Carry all instruments (CDZ), incurring expected costs equal to

$$C_{cdz} = c_f + c_d + c_z - \beta_i\left(\pi_c + \frac{\pi_{cd}}{2} + \frac{\pi_{cz}}{2} + \frac{\pi_{cdz}}{3}\right) - r_d\left(\pi_d + \frac{\pi_{cd}}{2} + \frac{\pi_{dz}}{2} + \frac{\pi_{cdz}}{3}\right) - r_z\left(\pi_z + \frac{\pi_{cz}}{2} + \frac{\pi_{dz}}{2} + \frac{\pi_{cdz}}{3}\right)$$

Like merchants, each customer chooses the action that yields the minimum expected costs, given his own individual benefit parameter, β_i, and the share of merchants that choose each action. The number of customers that choose any of the seven possible actions is thus a function of the respective number of merchants and of the parameters of the model:

$$x_j = g_j(\pi_c, \pi_d, \pi_z, \pi_{cd}, \pi_{cz}, \pi_{dz}, \pi_{cdz}; \Phi),$$

or, in vector terms,

$$\omega = g(\pi, \Phi)$$

Equilibria

As in the two-currency model, equilibria can be defined as states of the world where the actions of agents on each side of the market are a best response to the actions of agents on the other side; formally, such that, at the same time,

$$\pi = f(\omega, \Omega)$$

and

$$\omega = g(\pi, \Phi)$$

As in the two-currency model, if all agents on one side of the market use all the currencies in equilibrium, every agent on the other side uses only one currency; *in this model*, however, such states would not be equilibria. If all merchants accepted all currencies, customers would prefer carrying cards rather than the digital currency if and only if $c_d - r_d > c_z - r_z$, and vv.

Since this condition would be the same for all customers, one of these two instruments—either cards or the DC—would not be used in such states. As a result, merchants would not accept that instrument either. The same would occur if all customers carried all instruments (one instrument would not be accepted by any merchant and, hence, customers would stop carrying it). This is a special feature of this model and does not apply more generally. “Corner” equilibria, in which all agents on one side use all instruments and each agent on the other side uses only one (but all three are used by some agents), are possible and exist under certain conditions.

EQUILIBRIA WITH THREE CURRENCIES

To explore what equilibria could emerge in a three-currency setting, we proceeded as in the case of two currencies, running simulations on 6,561 sets of parameters, identified by fixing eight parameters and letting four other parameters vary between 0.1 and 0.9, by multiples of 0.1 (Table 2). For each set, we considered a lattice of 46,656 different starting points obtained, letting the variables $\pi_c, \pi_d, \pi_z, \pi_{cd}, \pi_{cz}, \pi_{dz}, \pi_{cdz}$ vary between 0 and 1, by multiples of 0.1, subject to the constraint that $\pi_c + \pi_d + \pi_z + \pi_{cd} + \pi_{cz} + \pi_{dz} + \pi_{cdz} \leq 1$.[18]

Table 2. Values of Parameters used in the Simulations with Three Currencies

Parameter	Value	Range
w	2	
ξ	2	
c_f		0.1–0.9
c_d		0.1–0.9
c_z	0.1	
r_d	0.5	
r_z	0.4	
p_c		0.1–0.9
p_d		0.1–0.9
p_z	0.1	
q_c	0.5	
q_z	0.3	

Source: Authors’ selection.

[18] These parameters were selected, after running several simulations with different sets, at the values that were found, experimentally, to yield the qualitatively most interesting results. These values are merely hypothetical and are not based on any empirical estimation from actual data.

These simulations identified 28,308 equilibria, including:

- 19,683 single-currency equilibria (6,561 for each currency, three for each set of parameters),
- 8,285 two-currency equilibria, and
- 340 three-currency equilibria.

In 1,399 sets of parameters, only single-currency equilibria were found; in 94 percent of the remaining 5,162 sets, only two-currency equilibria were found; while 313 sets (4.7 percent of the total) exhibited single-, two-, and three-currency equilibria (Table 3).

Table 3. Equilibria Identified in the Simulations

Number of equilibria	Only single-currency equilibria	Only single- and two-currency equilibria	Single-, two-, and three-currency equilibria	Number of sets
3	1,399	--	--	1,399
4	--	2,924	--	2,924
5	--	1,480	65	1,545
6	--	304	74	378
7	--	128	54	182
8	--	13	54	67
9	--	--	48	48
10	--	--	17	17
11	--	--	1	1
Total	1,399	4,849	313	6,561

Source: Authors' simulations.

The search identified two different types of three-currency equilibria (Table 4):[19]

- Type 1: Equilibria where all merchants accept all currencies, and each customer carries only one currency; and
- Type 2: Equilibria where all merchants accept two currencies (but not all merchants accept the same currencies), some customers carry only one currency, and the rest carry the other two.

[19] The asymmetries in the results reported in Table 4 stem from the inherent asymmetries of the model and from the different values of r_f (0.4) and q_z (0.3) used in the simulation and reported in Table 3.

▪ **Table 4. Three-currency Equilibria Identified in the Simulations**

	Type 1	Type 2.a	Type 2.b	Total
Number of equilibria	69	229	42	340
Only cash	Some customers	Some customers	--	--
Only cards	Some customers	--	--	--
Only DC	Some customers	--	Some customers	--
Cash and cards	--	Some merchants	Some customers	--
Cash and DC	--	Some merchants	Some merchants	--
Cards and DC	--	Some customers	Some merchants	--
All currencies	All merchants	--	--	--

▪ Source: Authors' simulations.

Dynamics

We assume that the laws governing the dynamics of the system are the same as those discussed above in the two-currency setting: customers adjust instantaneously to the merchants' actions while merchants respond with a lag; hence,

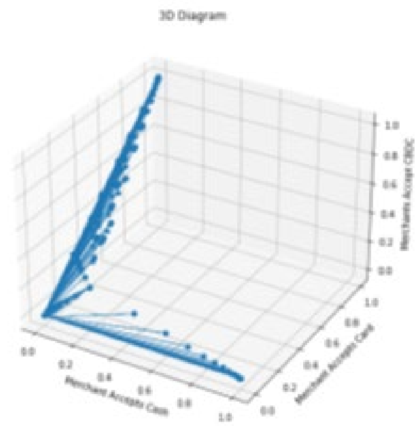
$$\pi_{t+1} = \gamma\, \pi_t + (1 - \gamma)\; f(\omega, \Phi).$$

Owing to the larger number of possible actions that can be taken by merchants and by customers, the dynamics of the system cannot be fully represented on a two-dimensional chart. Every agent on each side of the market has seven different possible actions (using only one of the three currencies, only two, or all three); the decisions of the agents on each side of the market are thus represented by seven-dimensional vectors with six degrees of freedom (since all the elements of each vector must add up to 1).

The evolution of the system is now generally described by paths in a seven-dimensional space. Since there are only six degrees of freedom, these paths can be fully represented by paths in a six-dimensional space, but not, in general, on a two-dimensional graph that can be printed on paper. As in the two-currency case, the simplex set Π (or Π^* in six dimensions) can be divided into different regions, corresponding to the basins of attractions of the different equilibria of the system.

These regions (and the paths followed by the system within them) cannot be completely represented graphically on a two-dimensional sheet. Graphs can, at best, represent the projections of such regions and paths on a two- or three-dimensional space, or some other indicator associated with them (Figure 6).

Figure 6. Possible Graphic Representations of Multi-dimensional Paths in a Three-currency Model



Source: Authors' simulations.
Note: The figure shows a three-dimensional representation of various seven-dimensional paths that all converge to the same cash-only equilibrium.

Impact of the Introduction of a Third Currency

In order to examine what happens when a third currency is introduced, we run a series of simulations based on the above specified dynamics of the system, to identify stable equilibria with cash and cards and assess how they could be disturbed by the introduction of a digital currency, on different scales.

In a set of configurations of parameters obtained by fixing (arbitrarily) seven parameters of the model and allowing the remaining four to vary by multiples of 0.1, between 0.1 and 0.9 (Table 5), we run a series of simulations to identify (a) which of these configurations exhibited a two-currency equilibrium with cash and cards, and (b) which of these equilibria are robust to small changes in the initial conditions (specifically, assuming that 1 percent of merchants would stop accepting cash or cards). We thus identified 13,122 two-currency "CD" equilibria (in which all merchants accept both currencies, while some customers carry only cash, and the others carry only cards), of which 11,160 are stable.

We then examined how the system would respond if these equilibria were disturbed by the introduction of a new currency ("digital currency," or "DC"), on three different scales:

- Small scale: the DC is initially accepted by only 1 percent of merchants,
- Medium scale: 5 percent of merchants accept the DC, and
- Large scale: the DC is accepted by 25 percent of merchants.

And under two different modalities:

- Cash complement (CC): merchants who accept the DC continue to accept the other two currencies, so that some merchants accept three currencies (the two old ones and the new) and the rest accept only the two old currencies); or

- Cash substitute (CS): merchants who accept the DC stop accepting cash, so that some merchants accept only the new currency and cards, while the others accept the two old currencies but not the new one.

Table 5. Parameters used in Simulations Featuring the Introduction of a Third Currency

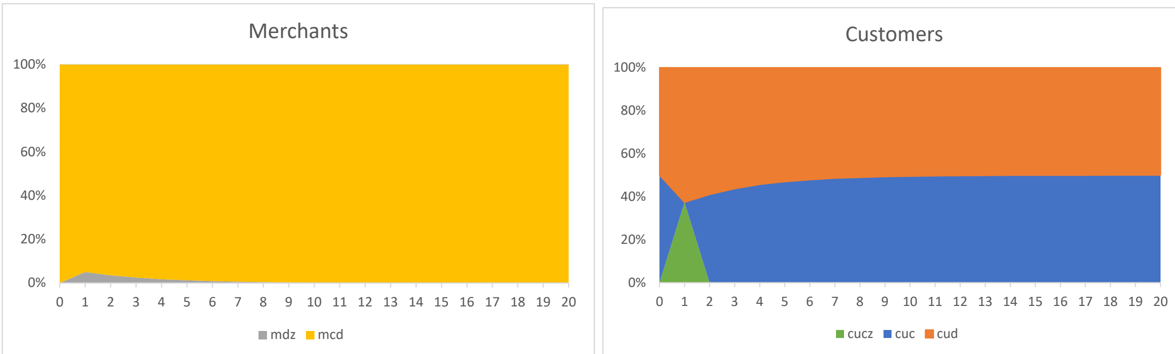
Parameter	Value	Range
w	2	
ξ	2	
c_f	0.1	
c_d		0.1–0.9
c_z	0.1	
r_d		0.1–0.9
r_z		0.1–0.9
p_c	0.2	
p_d	0.5	
p_z	0.1	
q_c		0.1–0.9
q_z		0.1–0.9

Source: Authors' selection.

The results of the simulations are reported in Table 6. Before discussing their significance, it is useful to examine a few selected examples, all entailing the launch of a DC on a medium scale as a substitute for cash, from an initial equilibrium in which all merchants accept both cash and cards. In the following simulations, a stable two-currency equilibrium that persisted until Period 0 is disturbed in Period 1 by the launch of a DC as a substitute for cash (merchants who participate in the launch cannot accept both cash and the DC in Period 1).

Example 7: the digital currency fails to take off and the system remains in the initial equilibrium. If c_d = 0.1, r_d= 0.5, r_z = 0.1, q_d = 0.1, and q_z = 0.4 (initial equilibrium: π_{CD} = 1; x_c = x_d =0.5), when some merchants stop accepting cash and start accepting the DC, some customers start carrying *both cash and the DC.* Their number is not large enough to convince more merchants to accept the digital currency. Instead, some merchants stop accepting it because (with this set of parameters) it is too costly. Over time, all merchants revert to accepting cash and cards, and customers stop carrying the DC. If the merchants who accepted the DC had continued to accept cash as well, *no* customer would have ever carried the DC, which, *in this setting*, yields lower benefits than cash to most customers (Figure 7).

Figure 7. Currencies used by Merchants and Customers over Time, Example 7



Source: Authors' simulations.
Note: Color code for line charts: blue = only cash; orange = only cards; red = only DC; yellow = cash and cards; green = cash and DC; gray = cards and DC. The horizontal axis shows periods from 0 to 20; the DC is launched in Period 1.

Example 8: the digital currency fails to take off and cash disappears. If c_d = 0.1, r_d= 0.5, r_z = 0.1, q_d = 0.1, and q_z = 0.1, from an initial equilibrium with π_{CD} = 1 (and x_c = x_d =0.5), cash would become extinct, but the DC would also disappear. When some merchants stop accepting cash and start accepting the DC, some customers start carrying both currencies. Fewer merchants then continue to accept cash, but since those who accept the DC also accept cards, customers switch to cards, not to the DC, which yields lower benefits to most of them. In the end, only cards are carried and accepted in the market (Figure 8).

Figure 8. Currencies used by Merchants and Customers over Time, Example 8

Source: Authors' simulations.
Note: Color code for line charts: blue = only cash; orange = only cards; red = only DC; yellow = cash and cards; green = cash and DC; gray = cards and DC. Color code for bar charts: blue = cash; orange = cards; green = DC. The horizontal axis shows periods from 0 to 20; the DC is launched in Period 1.

Example 9: the digital currency replaces cards. If $c_d = 0.2$, $r_d = 0.6$, $r_z = 0.7$, $q_c = 0.1$, and $q_z = 0.2$, the launch of a DC from an initial equilibrium with $\pi_{CD} = 1$ (and $x_c = x_d = 0.5$) is successful. Customers would carry it together with cards, while merchants eventually split between accepting it or cash. Eventually, cards would disappear from the market (Figure 9).

Figure 9. Currencies used by Merchants and Customers over Time, Example 9

Source: Authors' simulations.

Note: Color code for line charts: blue = only cash; orange = only cards; red = only DC; yellow = cash and cards; green = cash and DC; gray = cards and DC. Color code for bar charts: blue = cash; orange = cards; green = DC. The horizontal axis shows periods from 0 to 20; the DC is launched in Period 1.

Example 10: the digital currency takes root but the other currencies continue to be used. If $c_d = 0.1$, $r_d = 0.6$, $r_z = 0.8$, $q_c = 0.1$, and $q_z = 0.4$, the launch of the DC from an initial equilibrium with $\pi_{CD} = 1$ (and $x_c = 0.4$, $x_d = 0.6$) succeeds, but the other two currencies also continue to be used. All merchants accept cards, some also accept cash, while the rest also accept the DC. Customers, in turn, split into one group carrying only cards and another group carrying both cash and the DC (Figure 10).

Figure 10. Currencies used by Merchants and Customers over Time, Example 10

Source: Authors' simulations.
Note: Color code for line charts: blue = only cash; orange = only cards; red = only DC; yellow = cash and cards; green = cash and DC; gray = cards and DC. Color code for bar charts: blue = cash; orange = cards; green = DC. The horizontal axis shows periods from 0 to 20; the DC is launched in Period 1.

Example 11: the DC replaces the other two currencies. If $c_d = 0.1$, $r_d = 0.5$, $r_z = 0.6$, and $q_c = q_z = 0.2$, the launch of a DC from an initial equilibrium with $\pi_{CD} = 1$ (and $x_c = x_d = 0.5$) is not only successfully launched but ultimately replaces both cash and cards. Initially, an increasing number of merchants stop accepting cash and start accepting the DC instead, while continuing to accept cards. Customers respond by carrying either cards or both cash and the DC. After some time, all merchants cease accepting cash and a growing number of merchants stop accepting cards as well, thereby inducing all customers to carry only the DC (Figure 11).

Figure 11. Currencies used by Merchants and Customers over Time, Example 11

Source: Authors' simulations.

Note: Color code for line charts: blue = only cash; orange = only cards; red = only DC; yellow = cash and cards; green = cash and DC; gray = cards and DC. Color code for bar charts: blue = cash; orange = cards; green = DC. The horizontal axis shows periods from 0 to 20; the DC is launched in Period 1.

More generally, the results of the simulations highlight that:

(a) **A third currency is likely to be successful only if it is launched on a sufficiently large scale.**[20] This result, which is rather intuitive, has various implications. It means that the system is robust to "startup" innovations, such as the launch of new types of virtual currencies by new entrants—unless they are connected to entities or groups that already have a significant presence in the market (the prospected launch of Libra by Facebook could have been an example of the latter). Hence, entities planning to launch a digital currency would probably do it on a scale that is likely to upset the initial market equilibrium. This result also implies that if cash were to become extinct at some point, its reintroduction, to be successful, would also need to be conducted on a large scale.

(b) **The impact of a new currency also depends on how it is introduced.** In these simulations, if the digital currency is introduced as a complement to cash, it fails to take off even if it is launched on a

[20] In our simulation, a "sufficiently large scale" includes the "middle scale" (5 percent) and the "large scale" (25 percent).I In reality, what constitutes a "sufficiently large scale" would probably be country-specific and remains to be determined. This result is not fully proven by the simulations. A rigorous mathematical proof, which goes beyond the scope of this study, would require showing that under any feasible value of the parameters, as ε converges to zero, a shock of size ε does not lead to cash extinction.

large scale. This extreme result depends on the simplifying assumptions of the model, but this issue deserves to be investigated further. Intuitively, it is reasonable to expect that a currency introduced as a substitute for other existing payment instruments will have a more disruptive impact on the market than a currency introduced as a complement.

(c) **The introduction of a new currency can alter the market equilibrium in several qualitatively different ways:**

a. It may displace one of the exiting currencies (either cash or cards);

b. It may replace both currencies; or

c. It may continue to be used indefinitely *alongside* the other two currencies.

(d) **The launch of a new currency can alter the market equilibrium even when it is not successful.** In several cases in our simulations, the digital currency fails to take off but its failed introduction, by disturbing a vulnerable equilibrium, can shift the system to a path that leads to the eventual extinction of either cash or cards (Table 6).

Table 6. Equilibria Reached after the Introduction of a Third Currency

Final equilibrium	Outcome				
Modality		CC	CS	CC	CC
Only cash	Extinction of cards	--	--	182	182
Only cards	Extinction of cash	--	--	5,820	6,290
Only DC	Extinction of cash and cards	--	--	2,139	2,139
Cash and cards	DC launch fails, with no long-term consequences	10,044	10,044	1,650	1,180
Cash and DC	DC replaces cards	--	--	160	160
Cards and DC	DC replaces cash	--	--	--	--
All three currencies		--	--	94	93
Total		10,044	10,044	10,044	10,044

Source: Authors' simulations.

Note: CC = DC introduced as a complement to cash (at the launch, merchants can accept both); CS = DC introduced as a substitute to cash (at the launch, but not later, merchants must choose one).

Conclusions

Private sector attempts and central banks' interest in introducing digital currencies have spurred a lively debate on the potential benefits and risks that such innovations could entail. So far, this debate has centered mainly on the medium-term macroeconomic and macrofinancial impacts, exploring the potential consequences that could arise for the effectiveness of monetary policy, bank intermediation, cross-border payments, or financial inclusion. The impact on the payment system itself has received less attention. In particular, the possibility that such an introduction could provoke the disappearance of physical cash, and the potential consequences that such a disappearance could entail, has remained a comparatively marginal concern.

Considering the sharp decline in the use of cash for transaction purposes observed in the past couple of decades in most jurisdictions, the possibility that cash could become "extinct" does not seem to be negligible. Its repercussions in the long term are not fully predictable and include a tail risk of severely adverse systemic implications—stemming, for instance, from the loss of privacy. This risk is most important considering that the extinction of cash, if it were to happen, is likely to prove irreversible. Cash continues to play an important, if underestimated, role in providing users with an alternative to which they can turn if other (mostly digital) means of payment are managed improperly, unreliably, or entail an excessive risk of loss, malfunctioning, or abuse. By removing these safeguards, cash extinction could, in the long term, permit the emergence of unfavorable scenarios that could range from relatively "benign" increases in fees, user costs, and outages, to much more troublesome interferences in how users spend their balances and conduct their private lives.

This paper explores the risk of cash extinction that could stem from the introduction of innovative payment instruments, such as private or public digital currencies. Expanding a two-sided market model by the same authors (Maino and Pani, 2024), it examines how the adoption of different payment instruments by merchants and customers evolves over time in response to various incentives. The study shows that it is theoretically conceivable that multiple currencies can coexist indefinitely in equilibrium, though one currency may eventually dominate under certain conditions. Additionally, the results obtained suggest that to be successful, the launch of a new payment instrument would probably need to be carried out on a sufficiently large scale, the size of which is likely to be country-specific and remains to be estimated. This research highlights the potential disruption a new currency could cause to existing payment systems.

The findings also highlight the theoretical possibility that introducing a third currency can lead to the extinction of an existing one, such as cash, even if the new currency is not widely adopted or eventually fails. This outcome emphasizes the need for careful consideration of the potential unintended consequences of introducing digital currencies.

This specific aspect can have important policy implications. If cash plays a key safeguard role and its extinction is to be avoided, authorities in charge of regulating and supervising the payment system, and those contemplating the introduction of a public digital currency, should give due consideration to identifying and adopting measures that would ensure that cash remains a viable and utilized option in the market. This model provides an analytical framework to understand how the complex interactions between different payment instruments, merchants, and customers can potentially lead to unexpected and unintended outcomes, underscoring the importance of anticipating the broader repercussions on the payment ecosystem of major innovations, such as the introduction of new digital currencies.

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